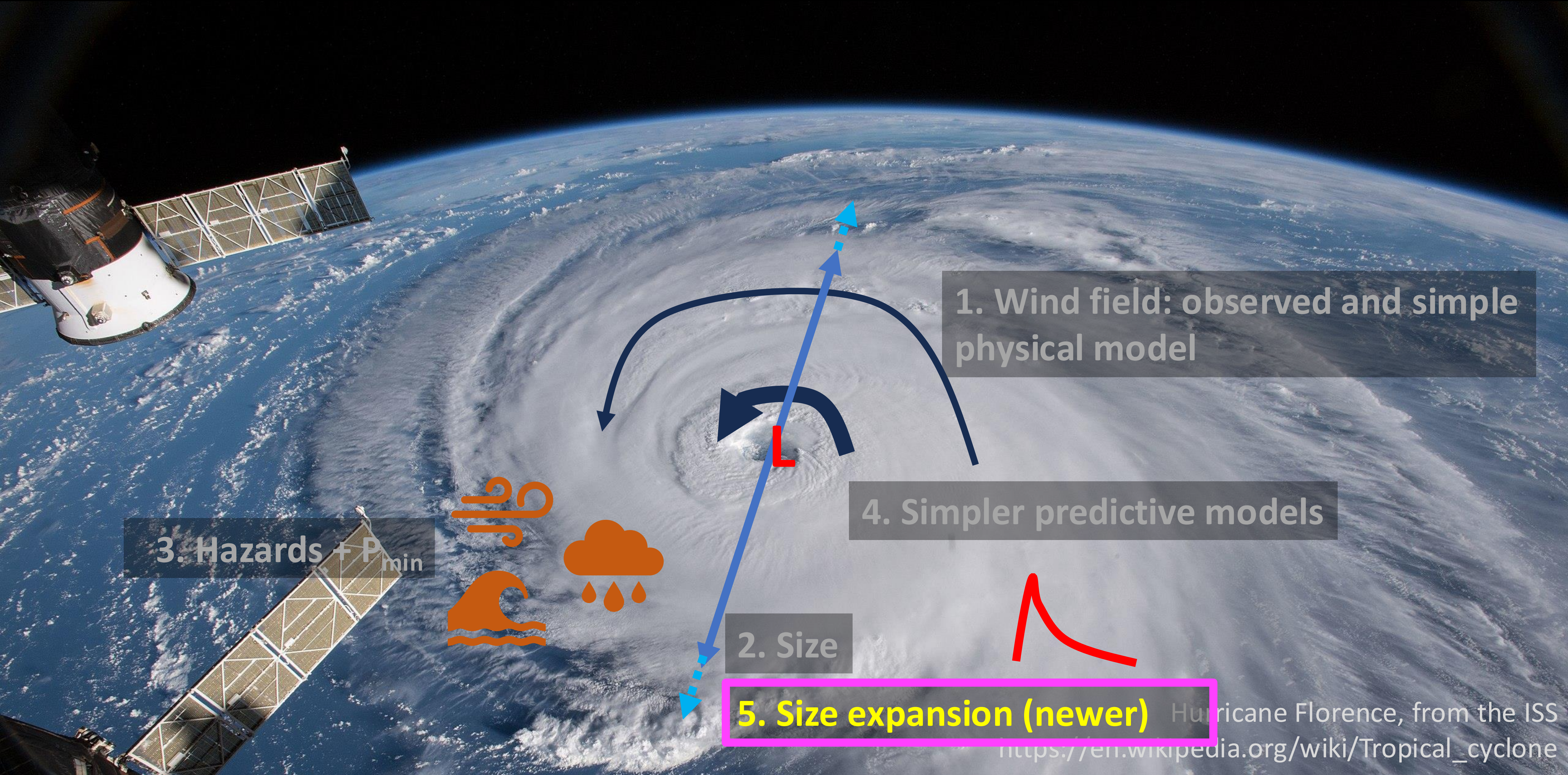
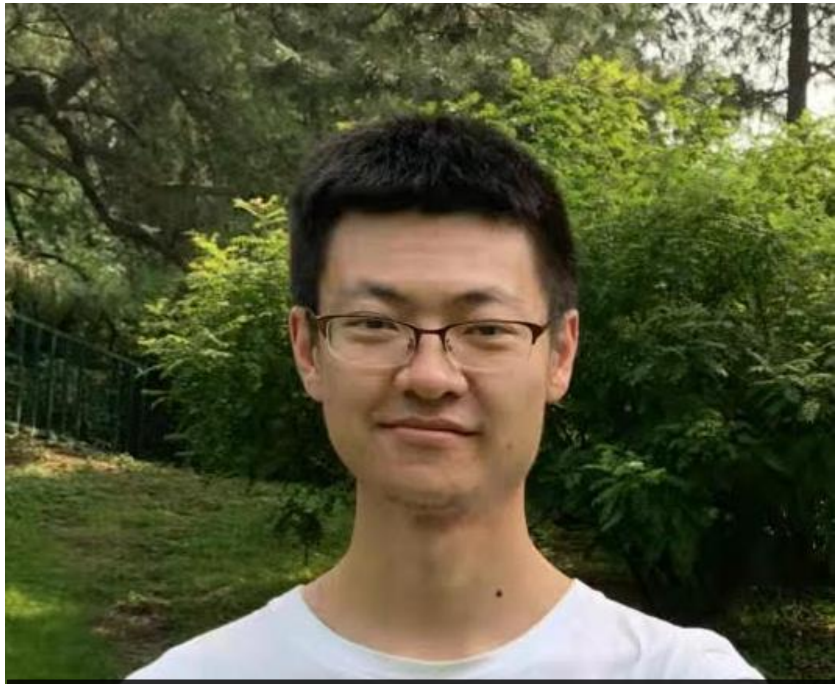
Roadmap

0. Intro: two recent events



5. Tropical cyclone expansion

- Observations
  f-plane theory
- response to warming
- (sphere theory)

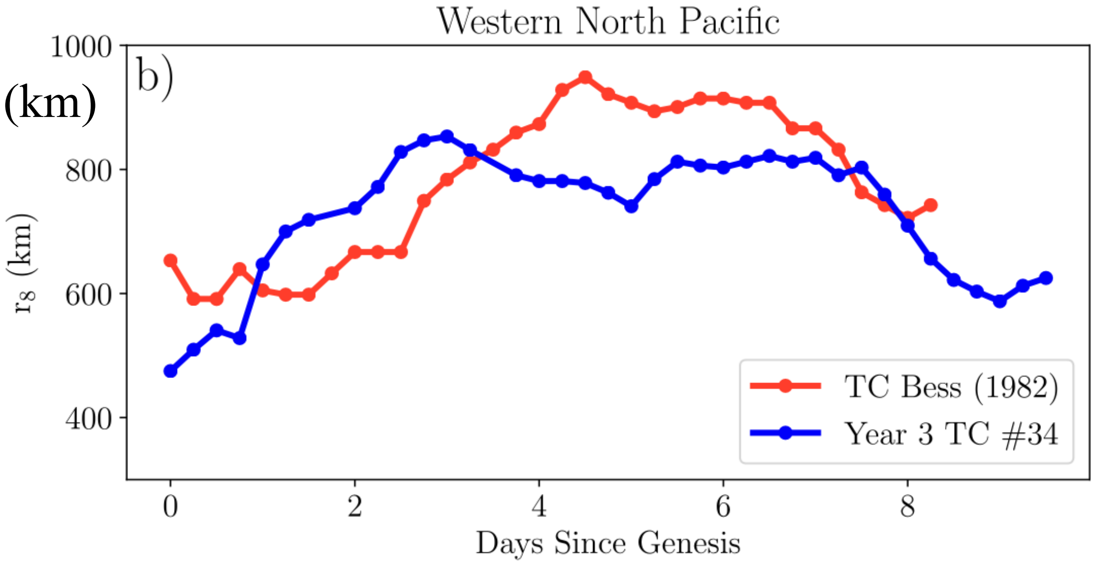

Postdoc Danyang Wang

Observations

Storms on Earth typically expand slowly with time, sometimes quickly

Reanalysis (red), GFDL HiFLOR (blue)

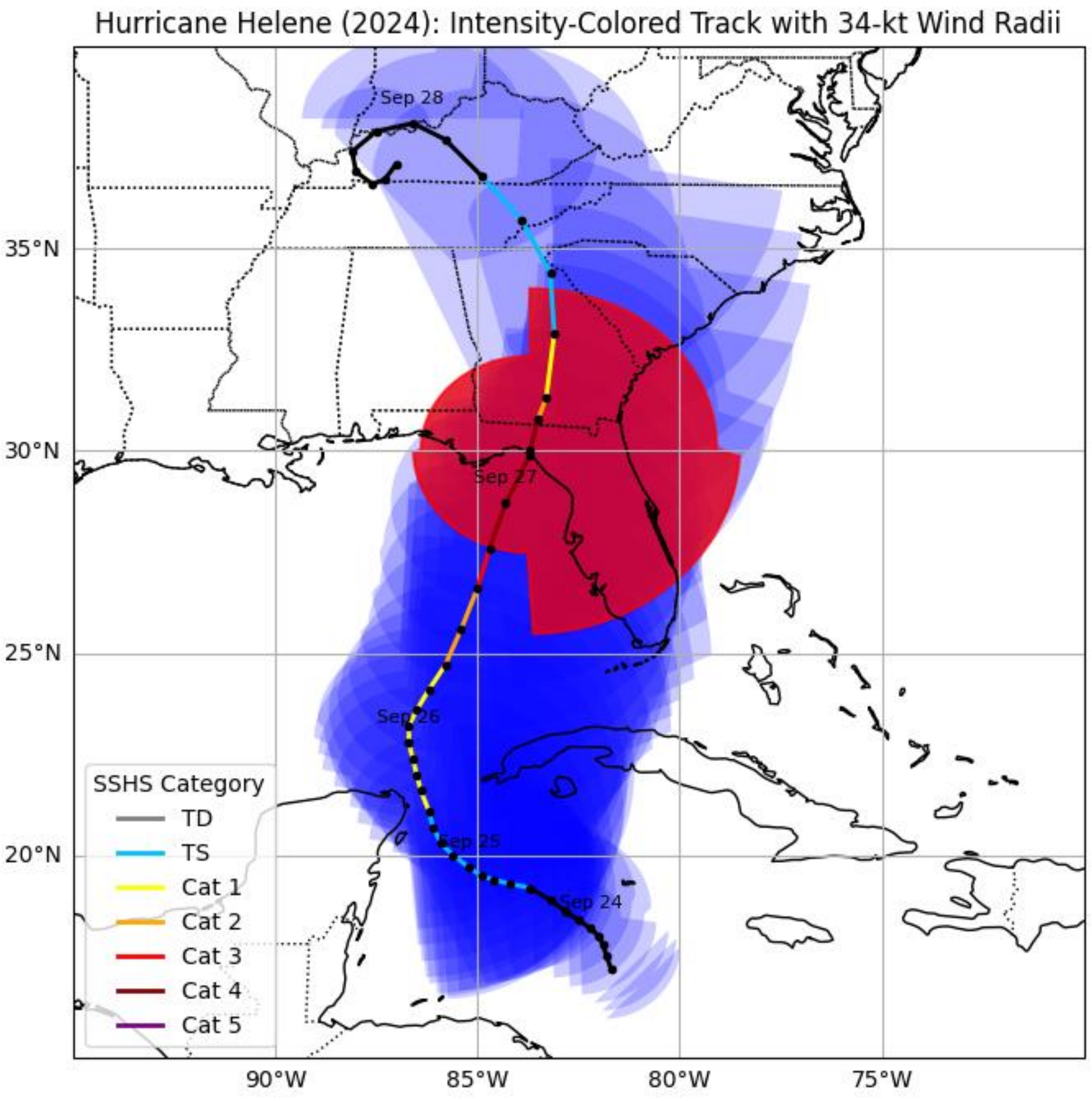
r8 (km)



time (days)

Schenkel et al. (2018)

Storms typically expand slowly with time, sometimes quickly



Helene's rapid expansion made wind, surge, and flooding dramatically worse

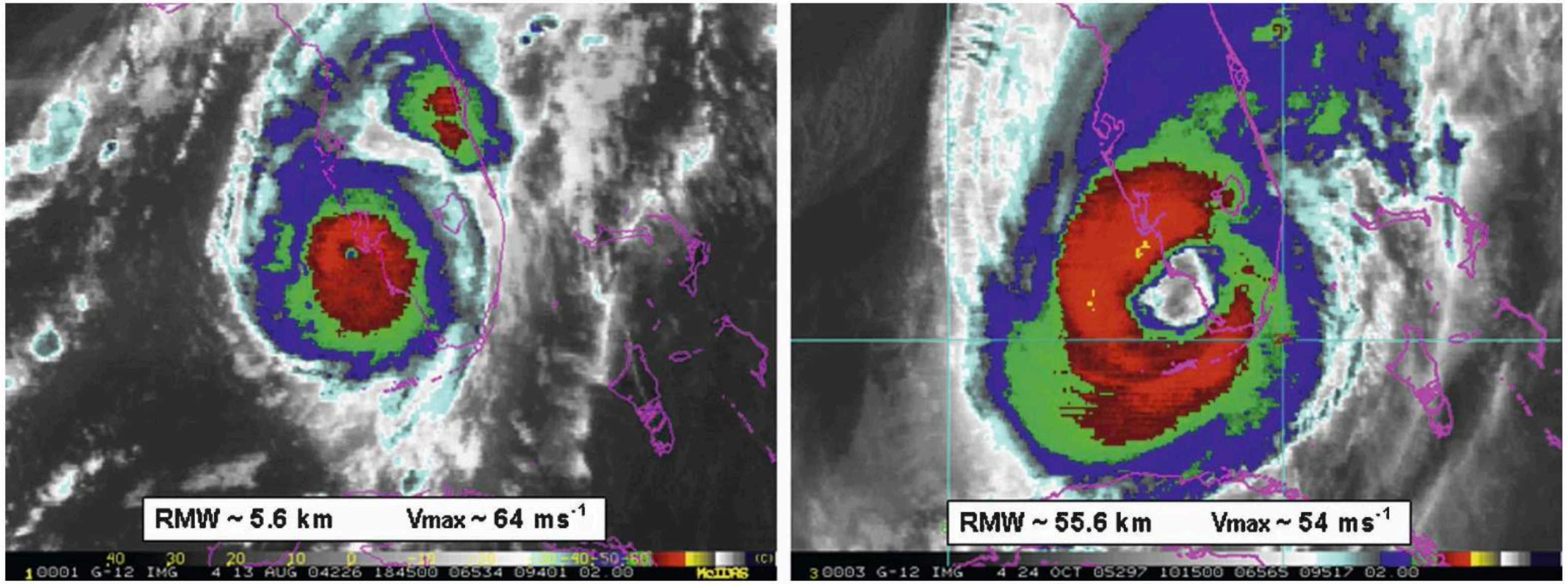
Large storms often experience a rapid expansion period

Li+ (2022)

Rapid Growth of Outer Size of Tropical Cyclones: A New Perspective on Their Destructive Potential

Storms often expand more quickly convection enhanced outside of the eyewall (multiple pathways)

Maclay et al. (2008, MWR) Tropical Cyclone Inner-Core Kinetic Energy Evolution



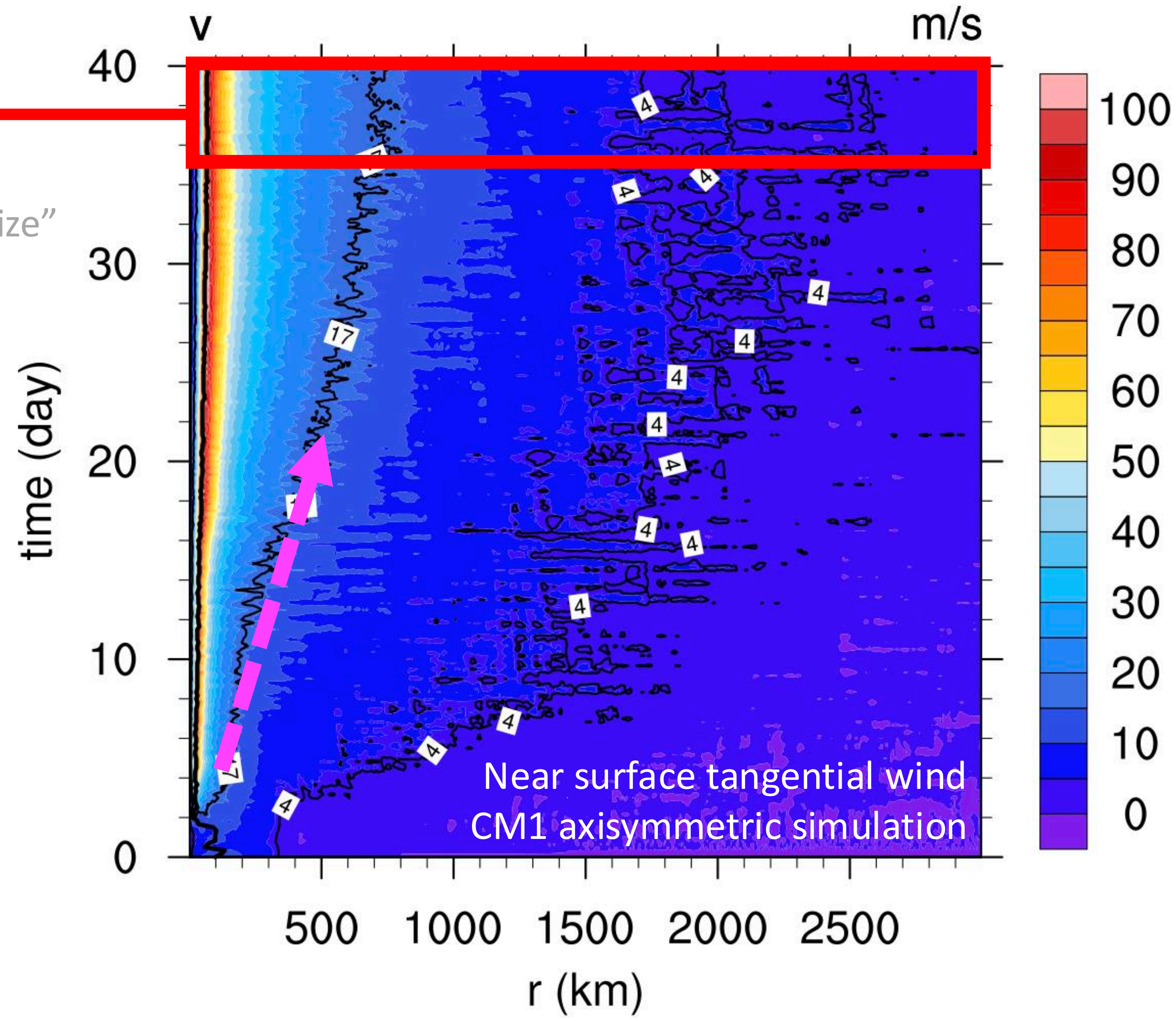
- SEF/ERC
- Moderate shear
- Land interaction

Modeling studies too
e.g. Xu and Wang (2010)

f-plane models: TCs expand from some initial size toward an equilibrium "potential size" over a timescale of O(10 days) – much slower than intensity!

Equilibrium size

Wang et al. (2022, JAS)
"Tropical cyclone potential size"



See also:
Fudeyasu and Wang (2011)
Chavas and Emanuel (2014)
Martinez et al. (2020)

Expansion theory:
f-plane

VOLUME 81 JOURNAL OF THE ATMOSPHERIC SCIENCES JULY 2024

An Analytical Model for Tropical Cyclone Outer-Size Expansion on the f Plane

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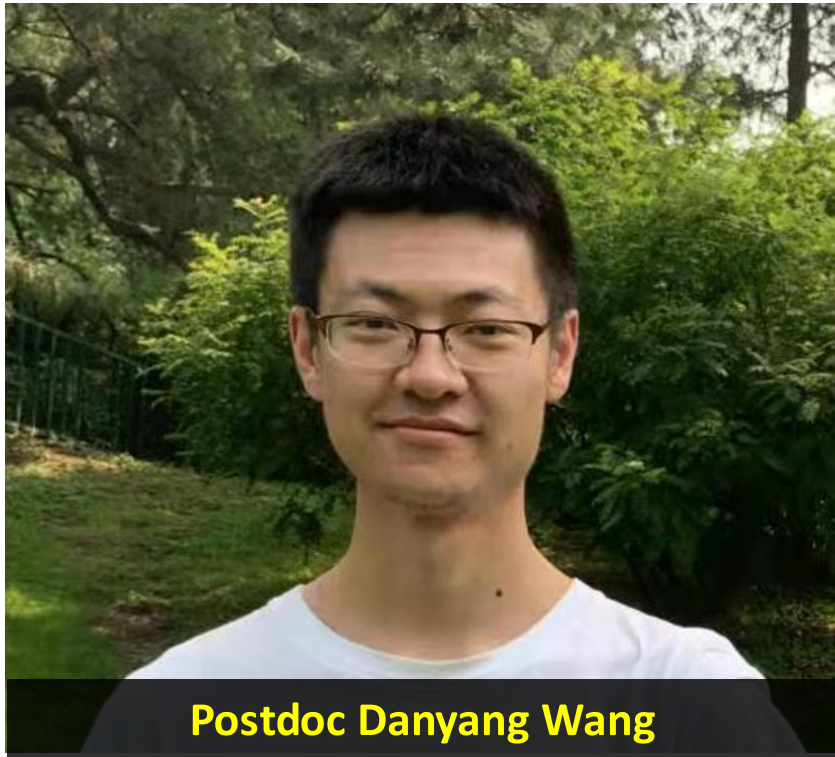
(Manuscript received 10 May 2023, in final form 6 March 2024, accepted 7 April 2024)



Basic relations

$$\frac{\partial v}{\partial t} \approx -f u_t - C_d(\mu v_t)^2/h_w$$

Import of planetary vorticity and surface friction (constant)

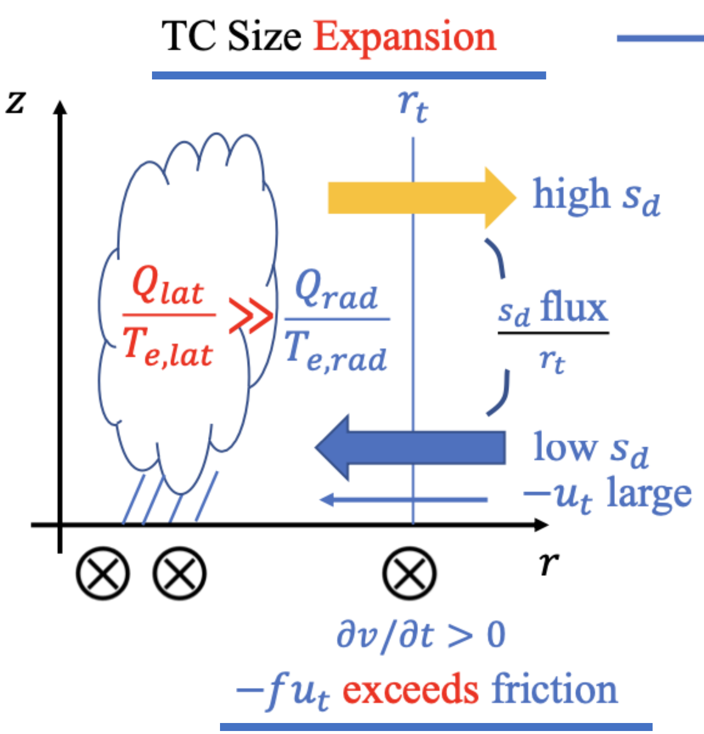
in the subsidence region

$$\frac{dr_t}{dt} = \frac{\partial v}{\partial t}/(-\frac{\partial v}{\partial r}), \quad r = r_t$$

r_t the radius of a fixed tangential velocity v_t at the top of boundary layer
local spin-up rate of tangential wind and slope of wind profile (physics known: Emanuel 2004 model)

Thus the main task is to predict the low-level radial velocity u_t

Predicting low-level radial velocity u_t



Volume integrated dry-entropy budget

$$\underbrace{\cancel{\frac{\partial S}{\partial t}}}_{0} = \frac{Q_{lat}}{T_{e,lat}} - \frac{Q_{rad}}{T_{e,rad}} + \underbrace{\cancel{\dot{S}_{res}}}_{0} + \mathcal{F}_r + \underbrace{\cancel{\mathcal{F}_u}}_{0}$$

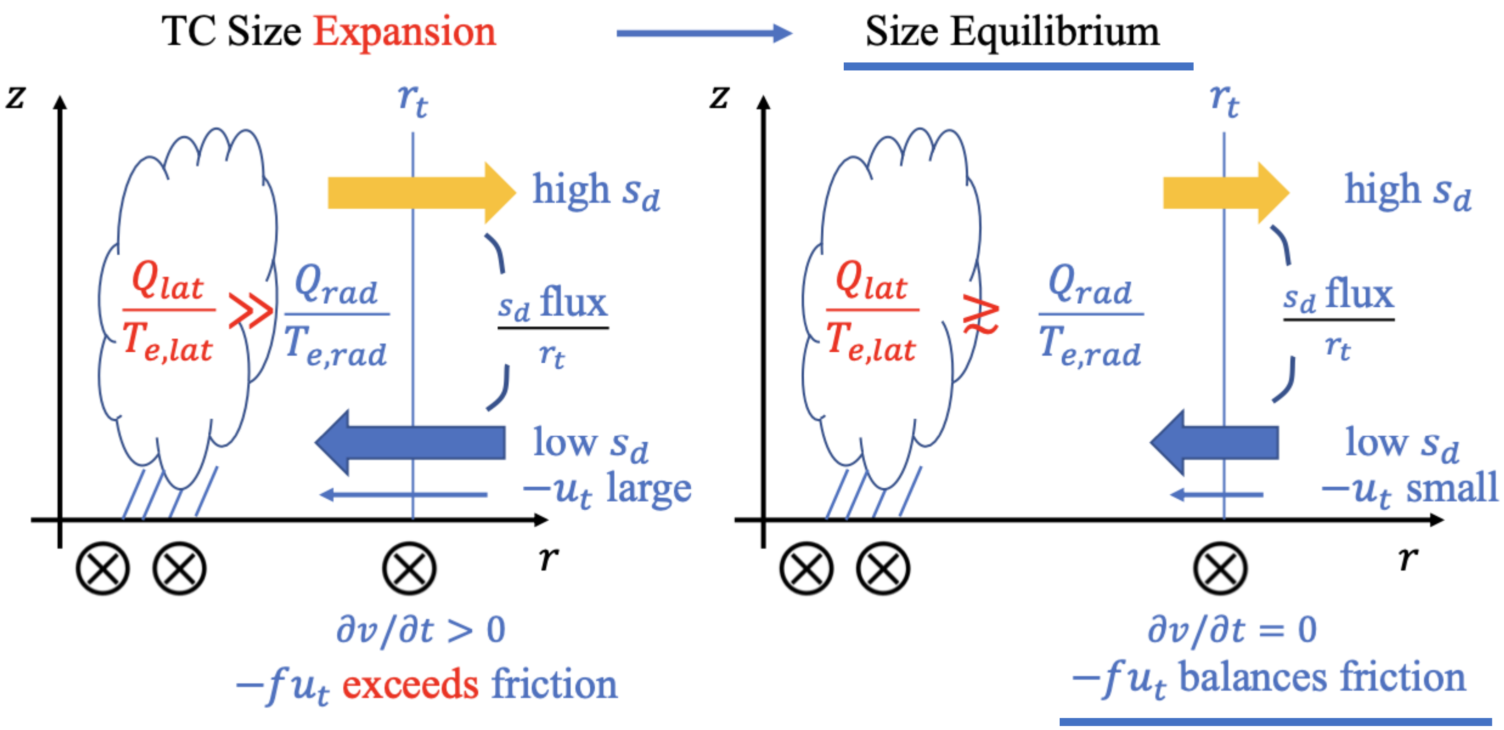
Balance: Condensational (latent) heating, radiative cooling, radial transport

$$-u_t = \frac{1}{2\pi r_t h_w \rho_i}\left(\frac{Q_{lat}}{T_{e,lat}\Delta s_d} - \frac{Q_{rad}}{T_{e,rad}\Delta s_d}\right)$$

Q_{lat} : Volume-integrated latent heating
Q_{rad}: Volume-integrated radiative cooling

Bulk free-tropospheric dry static stability, Δs_d: the dry entropy difference between tropopause and surface, $\Delta s_d \approx L_v q_{vs}/T_s$

Why approach a size equilibrium?



Assumption based on simulations: $Q_{lat} \propto r_t$ increases more slowly with size than $Q_{rad} \propto r_t^2$. Thus the storm expands towards an equilibrium size

$$-u_t = \frac{1}{2\pi r_t h_w \rho_i}\left(\frac{Q_{lat}}{T_{e,lat}\Delta s_d} - \frac{Q_{rad}}{T_{e,rad}\Delta s_d}\right)$$

Q_{lat} : Volume-integrated latent heating
Q_{rad}: Volume-integrated radiative cooling

Δs_d the dry entropy difference between tropopause and surface, $\Delta s_d \approx L_v q_{vs}/T_s$

Solution

$$\frac{dr_t}{dt} = \frac{r_{t,eq} - r_t}{\tau_{rt}}$$

$r_{t,eq}$: equilibrium size, a constant (predicted environmentally)

τ_{rt} : time scale, function of r_t (predicted environmentally)

$$r_{t,eq} = [f\frac{1}{2\pi r_t \rho_i}\frac{Q_{lat}}{T_{e,lat}\Delta s_d} - C_d(\mu v_t)^2]/(\frac{1}{2}fc_p\frac{\Delta p}{\rho_i g}\frac{Q_{cool}}{T_{e,rad}\Delta s_d})$$

$$\tau_{rt} = (-\frac{\partial v}{\partial r})\frac{h_w}{fB}$$

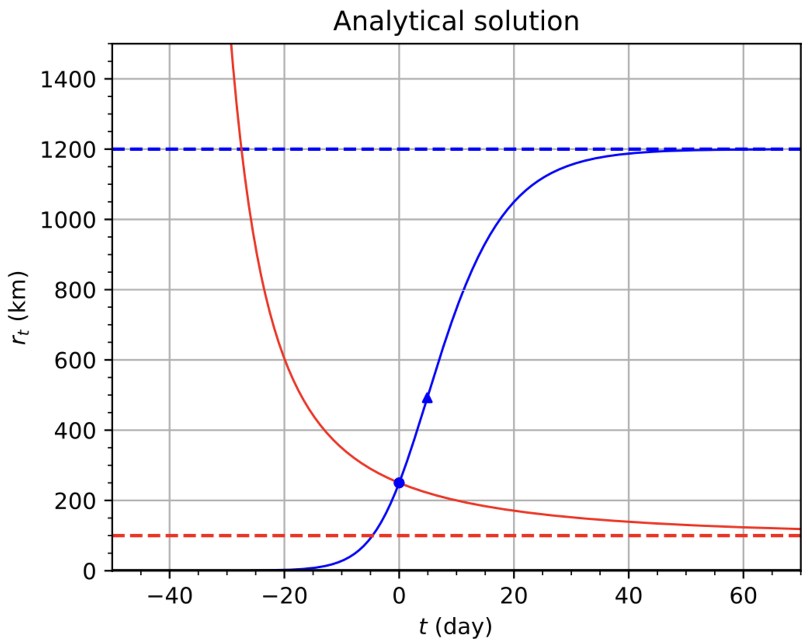
$$(\frac{\partial v}{\partial r})^{-1}\bigg|_{v=v_t} = \frac{\partial r}{\partial v}\bigg|_{v=v_t} = -\frac{2r_t v_t \sigma \xi_0}{2r_t + \xi_0 v_t^2},$$

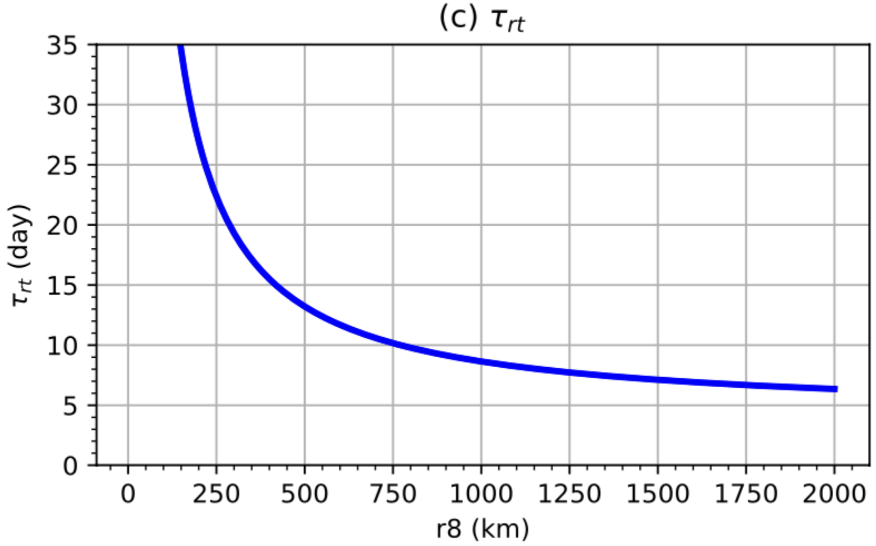
$$\Rightarrow \quad t - t_0 = \frac{1}{2\xi_0 v_t \frac{fB\sigma}{h_w}}[-(2+\frac{\xi_0 v_t^2}{r_{t,eq}})\ln(\frac{r_{t,eq}-r_t}{r_{t,eq}-r_{t0}}) + \frac{\xi_0 v_t^2}{r_{t,eq}}\ln(\frac{r_t}{r_{t0}})], \quad r_t > 0 \text{ and } r_t \neq r_{t,eq}$$

The full analytical solution of the size expansion model on the f-plane, $t(r_t)$, with all input parameters external

Basic behavior

Set v_t=8 m/s, so that r_t can be symbolically replaced by r8





Given environmental parameters and $r_{t,eq}$, model predicts both expansion and shrinking of the TC depending on whether present size is smaller or larger than $r_{t,eq}$.

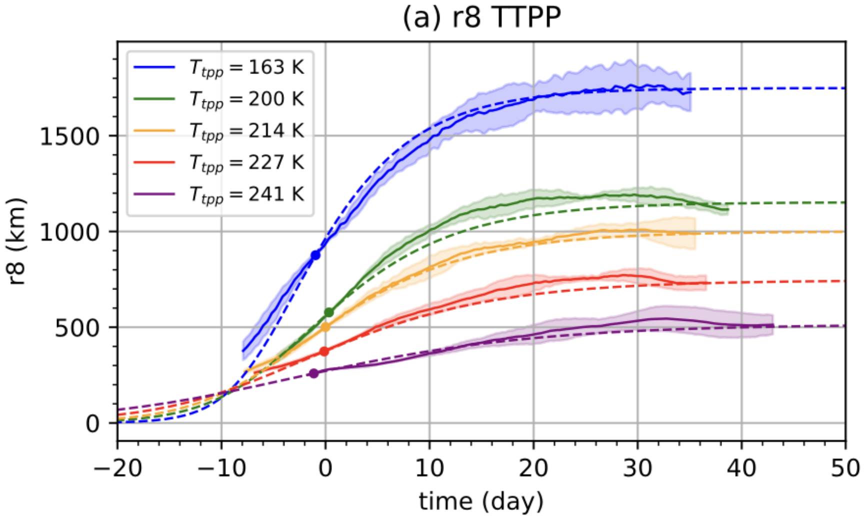
Time scale, 10-15 days, decreases with r8

Overall comparison with simulations: varying T_{tpp}

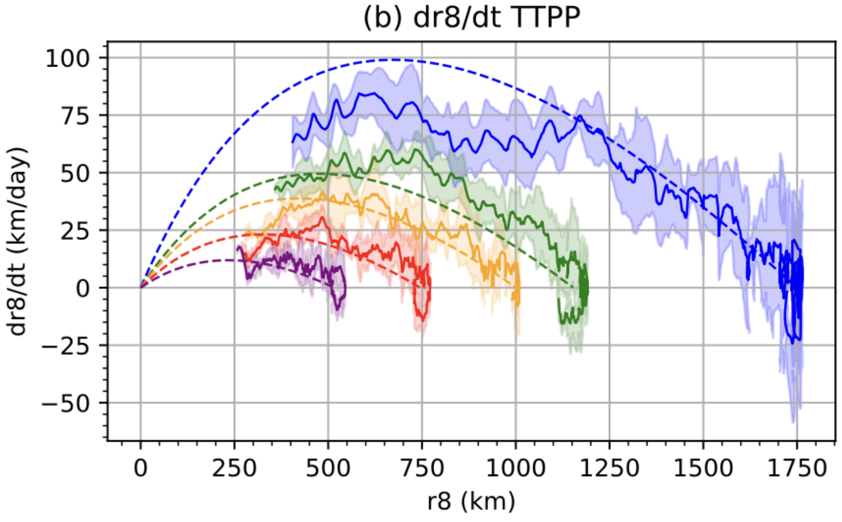
$r_{t,eq}$ set as ensemble-mean CM1 simulated equilibrium r8

Latitude: 20°N

Solid and shading: CM1 simulation ensemble-mean and 1 standard deviation;
Dashed: analytical model



Evolution of r8 (radius of 8 m/s tangential wind at 950 m of altitude)



dr8/dt as a function of r8

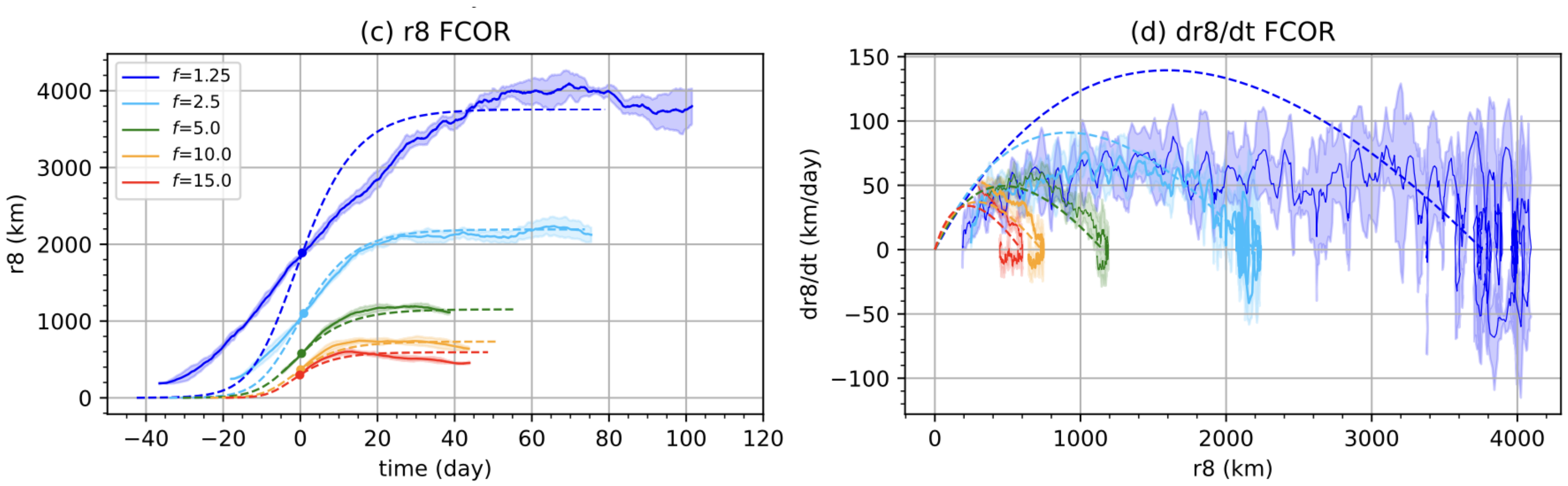
- Simulated expansion rate increases with decreasing T_{tpp} and thus with increasing potential intensity.

- Analytical model prediction matches CM1 fairly well, successfully predicting an expansion rate of tens of kilometers per day

Overall comparison with simulations: varying f

$r_{t,eq}$ set as ensemble-mean CM1 simulated equilibrium r8

Solid and shading: CM1 simulation ensemble-mean and 1 standard deviation;
Dashed: analytical model



- Analytical model prediction matches CM1 fairly well, except for overestimating expansion rate for f at 5°N and 10°N.

Evolution of r8 (radius of 8 m/s tangential wind at 950 m of altitude)

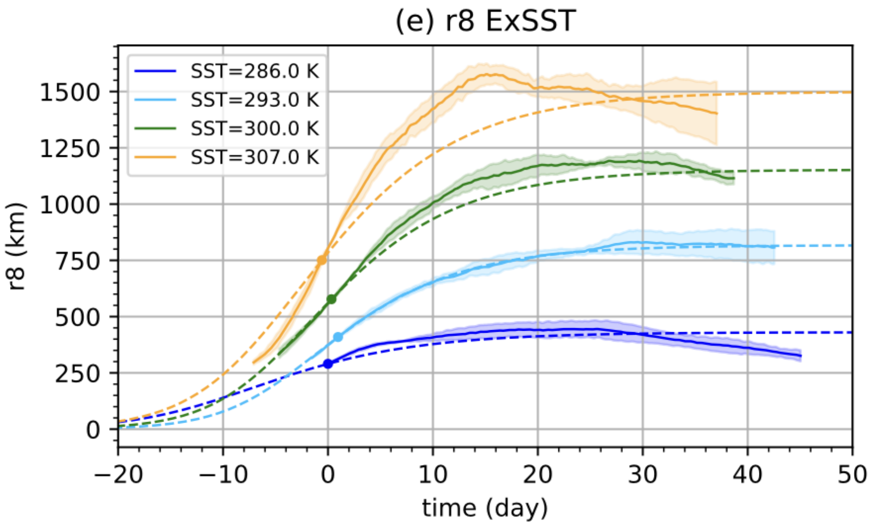
dr8/dt as a function of r8

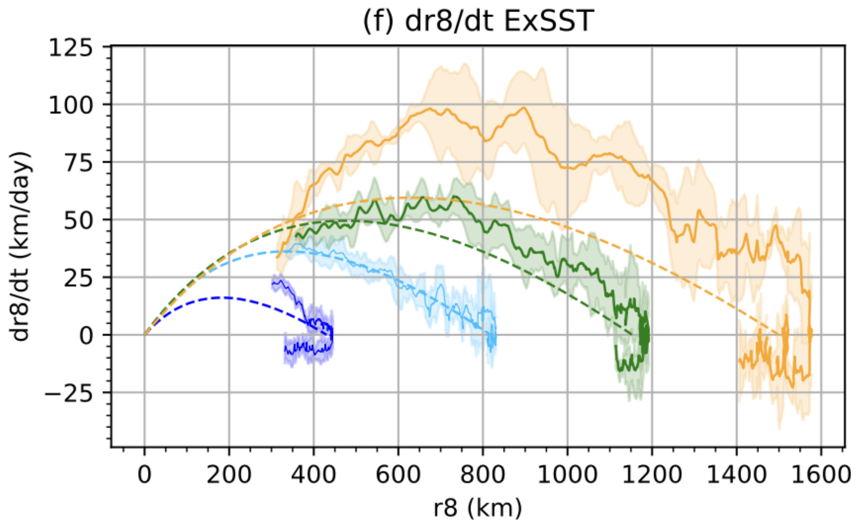
Overall comparison with simulations: varying SST

$r_{t,eq}$ set as ensemble-mean CM1 simulated equilibrium r8

Latitude: 20°N

Solid and shading: CM1 simulation ensemble-mean and 1 standard deviation;
Dashed: analytical model





Evolution of r8 (radius of 8 m/s tangential wind at 950 m of altitude)

dr8/dt as a function of r8

- Simulated expansion rate and equilibrium size increases with SST and thus with increasing potential intensity.
- This behavior is qualitatively correctly captured by expansion model, except for quantitatively underestimating expansion rate for SST=307 K.

Response to warming: mean vs. local warming

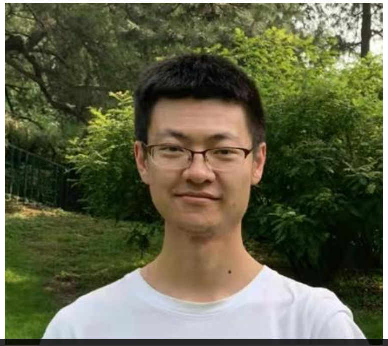
PNAS RESEARCH ARTICLE | EARTH, ATMOSPHERIC, AND PLANETARY SCIENCES

Check for updates

Tropical cyclones expand faster at warmer relative sea surface temperature

Danyang Wang[a,1,2], Daniel R. Chavas[a,1], and Benjamin A. Schenkel[b,c,d]

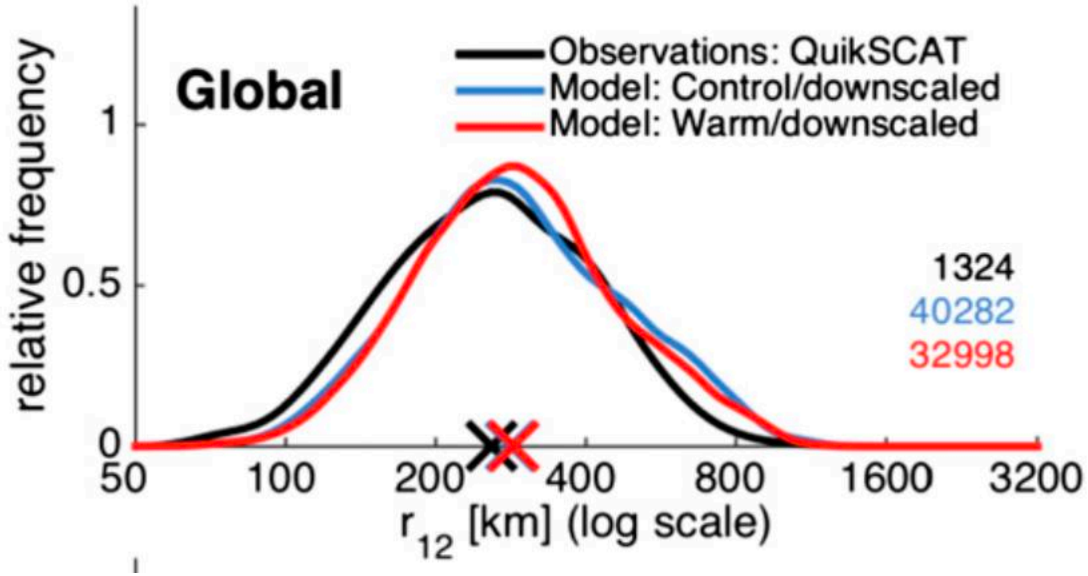
Edited by Richard Rotunno, National Center for Atmospheric Research, Boulder, CO; received November 25, 2024; accepted July 21, 2025



Lead: Danyang Wang
Purdue postdoc

Ben Schenkel
OU/CIWRO

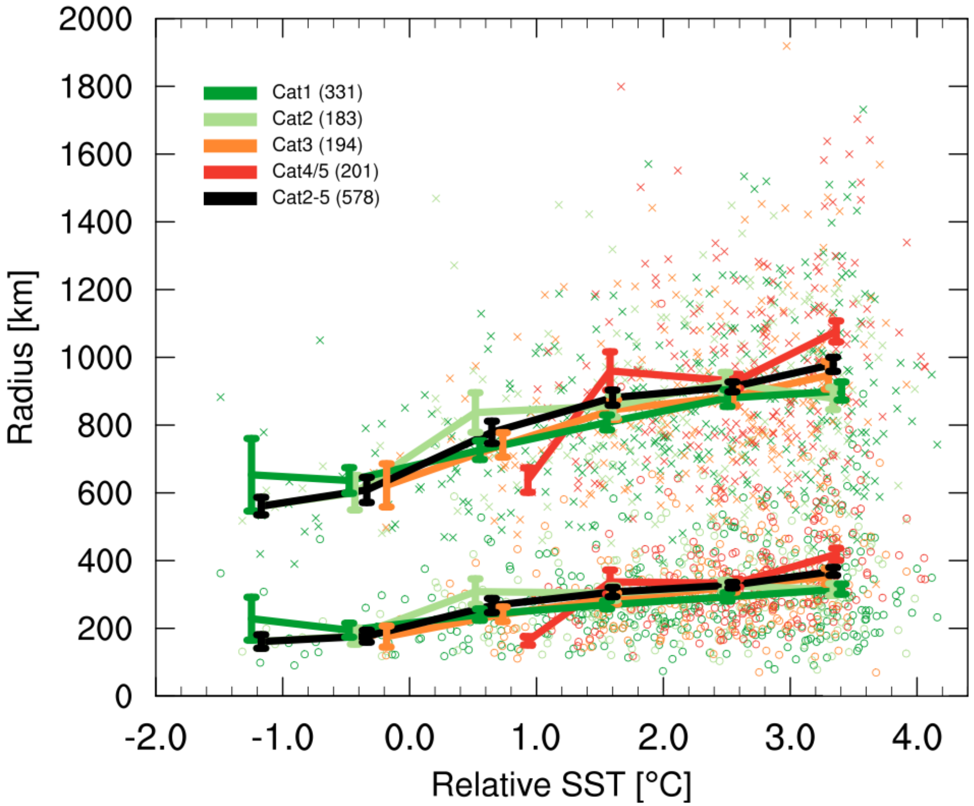
Size stays ~constant with mean SST warming



Knutson et al. (2015)
Global Projections of Intense Tropical Cyclone Activity for the Late Twenty-First Century from Dynamical Downscaling of CMIP5/RCP4.5 Scenarios

Schenkel+ (2023)
North Atlantic tropical cyclone outer size and structure remain unchanged by the late 21st century
Stansfield and Reed (2021)
Tropical Cyclone Precipitation Response to Surface Warming in Aquaplanet Simulations With Uniform Thermal Forcing

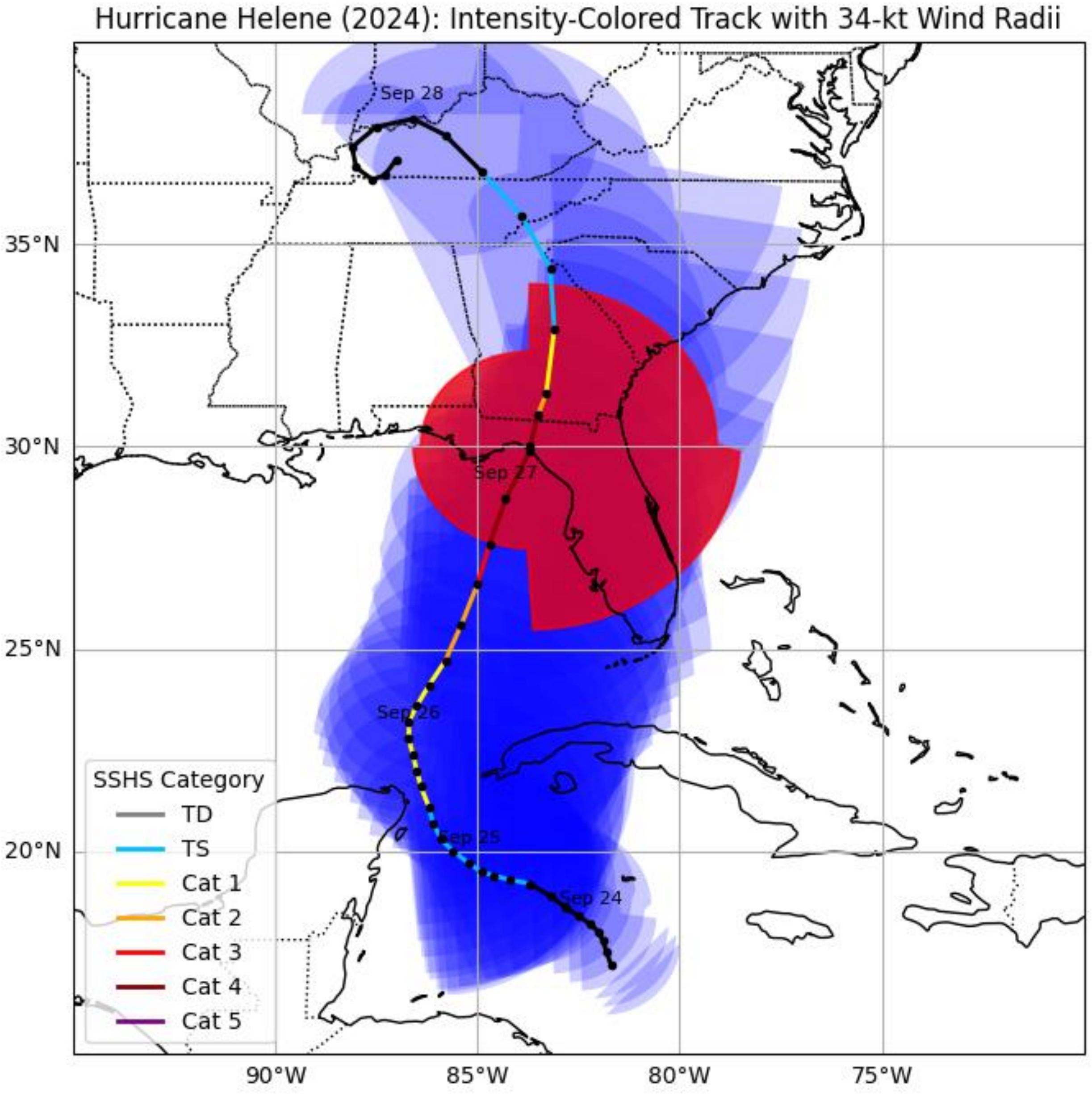
Size increases with local SST warming



Chavas et al. (2016)
Observed Tropical Cyclone Size Revisited

West Pacific has warmest water and the largest storms!

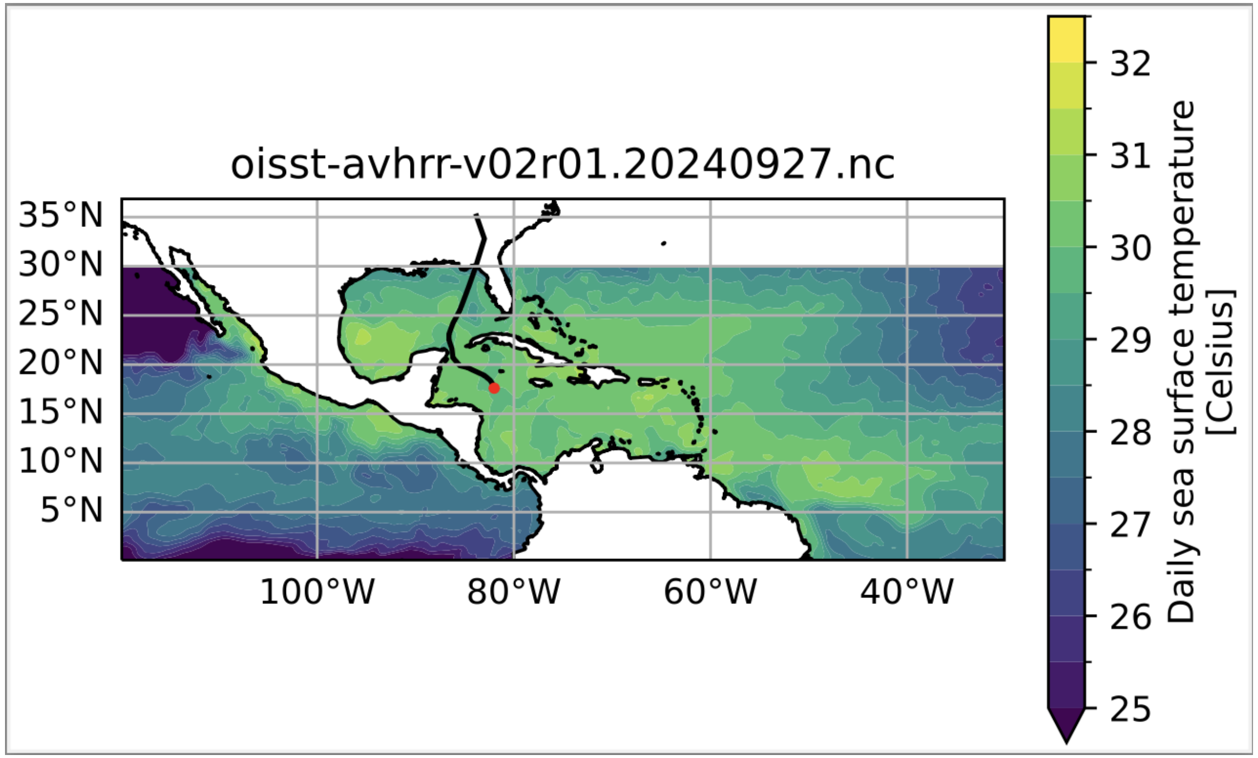
Helene expanded rapidly in 1-2 days → among largest landfall storms



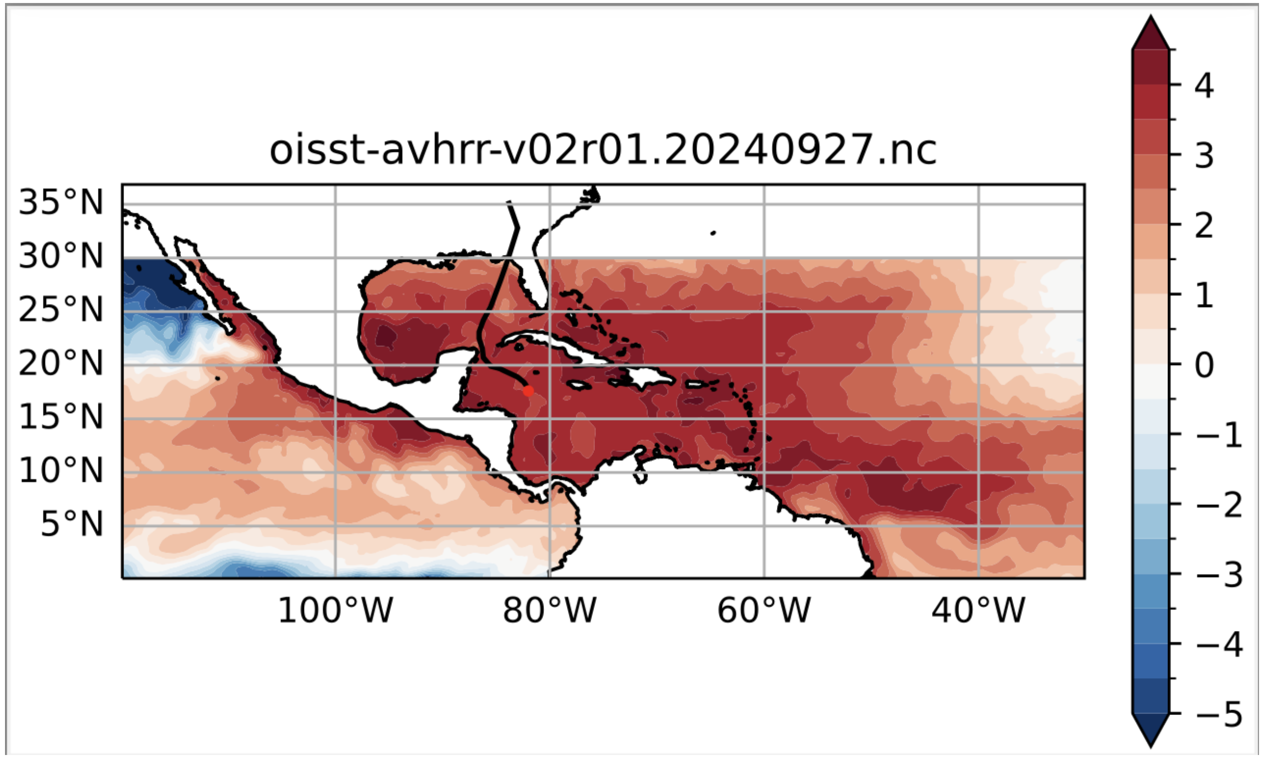
This made Helene's wind, surge, and flooding dramatically worse

The Caribbean/Gulf was very warm

Absolute SST



"Relative SST": SST difference from tropical-mean



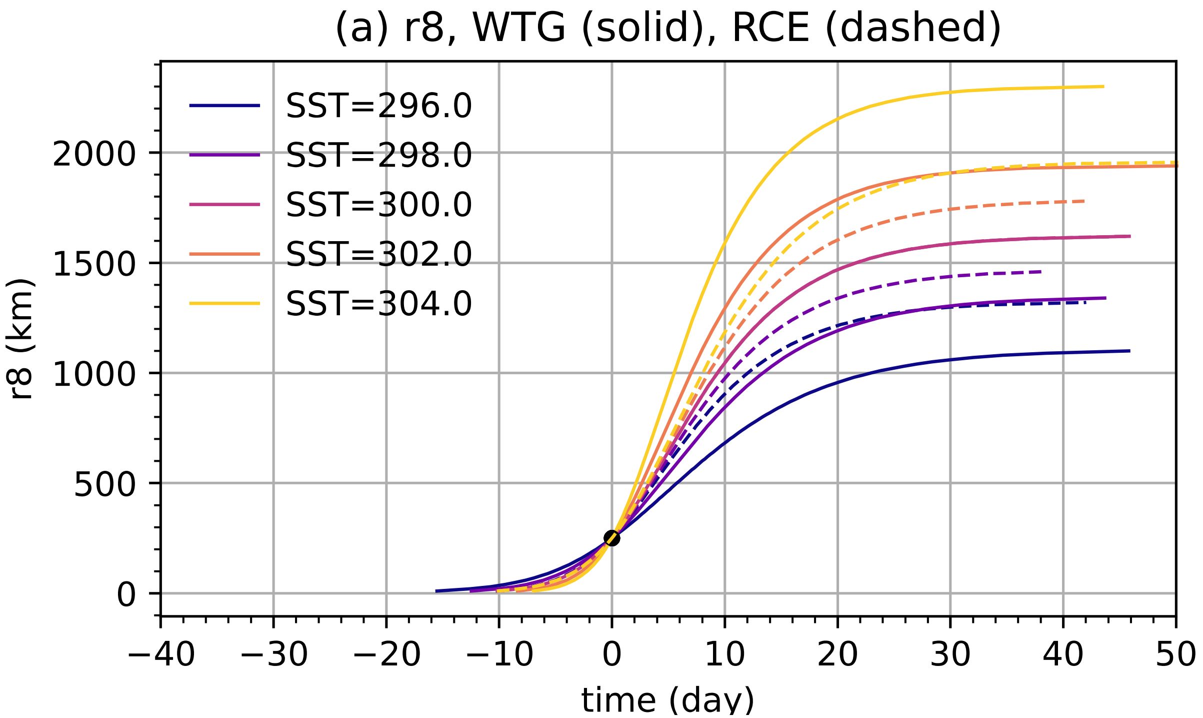
Motivation

Hurricanes intensify much more rapidly at warmer SST
(Ramsay et al. 2020, Garner 2023, Bhatia et al. 2022)

Could hurricanes also expand more rapidly at warmer SST too?
Absolute SST (mean warming) vs. relative SST (local warming)?

Consistent with theory

Implication from Wang and Chavas (2024) model

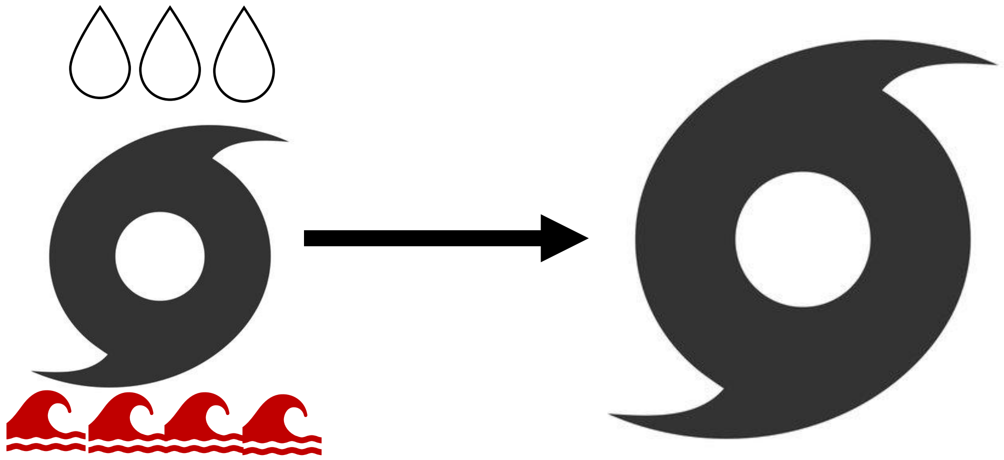


Expansion much less sensitive to mean warming because atmospheric static stability increases too.

→ Less inflow needed to export the same amount of energy from the TC.

Hypothesis: **expansion rate** increases with local warming

1) New theory: local warming most important
Wang and Chavas (2024)
An analytical model for tropical cyclone size expansion on the f-plane



2) Observations:
i. *Anything* that enhances convection outside the inner core causes storm to expand faster
Maclay+ (2009) Tropical Cyclone Inner-Core Kinetic Energy Evolution
ii. Much more convection over locally-warmer water in general
Peters and Neelin (2006), Emanuel (2019)
iii. Larger rainfall area over locally warmer water
Lin+ (2015) Tropical cyclone rainfall area controlled by relative sea surface temperature

Wang, Chavas, Schenkel (2025), under revision

Goal

Test how TC expansion rate changes with local vs. mean warming

- **Historical data: local warming (mean warming has been small)**
- uniform-SST aquaplanet simulation data: pure mean warming

Wang, Chavas, Schenkel (2025), under revision

Data and method

Historical:

- r17: Extended Best-Track data, IBTrACS data
- r8: reanalysis data (Gori et al. 2023)
- SST: OISST
- Relative SST (rSST): areal-mean SST within TC r17 or r8, subtracted by tropical-mean (30°S-30°N) value
- Expansion rate in 48-h time window (also test 24-h)
- Basins: North Atlantic (NA), East Pacific (EP), Western North Pacific (WP)
- Quality control: We focus our analysis on TCs equatorward of 30°N and sufficiently far from land to minimize effects of land or extratropical interaction.

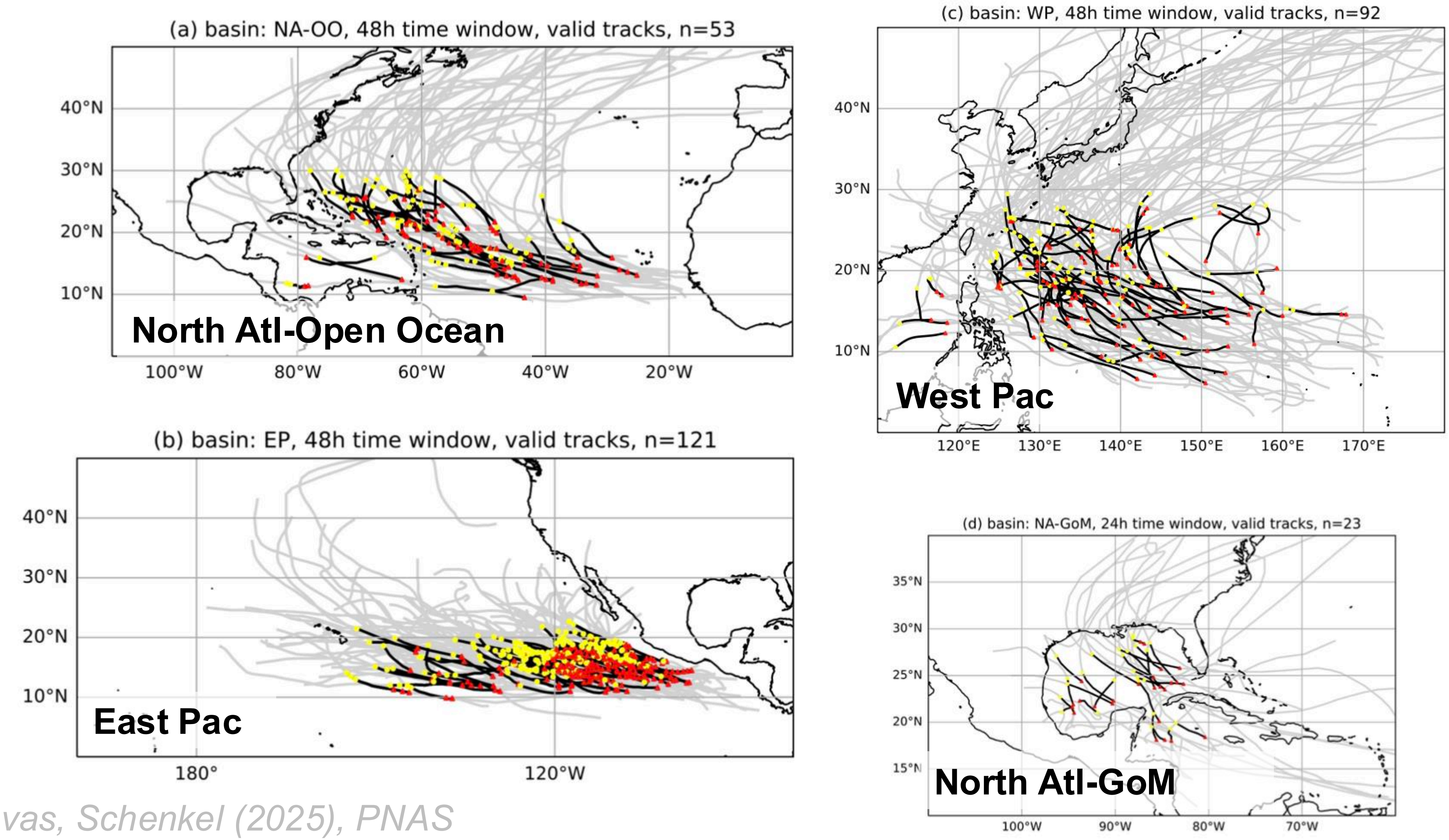
Aquaplanet:

- Data: r8 from Stansfield and Reed (2021) simulations, SST=295-305 K
- Expansion rate in 48-h time window

Wang, Chavas, Schenkel (2025), under revision

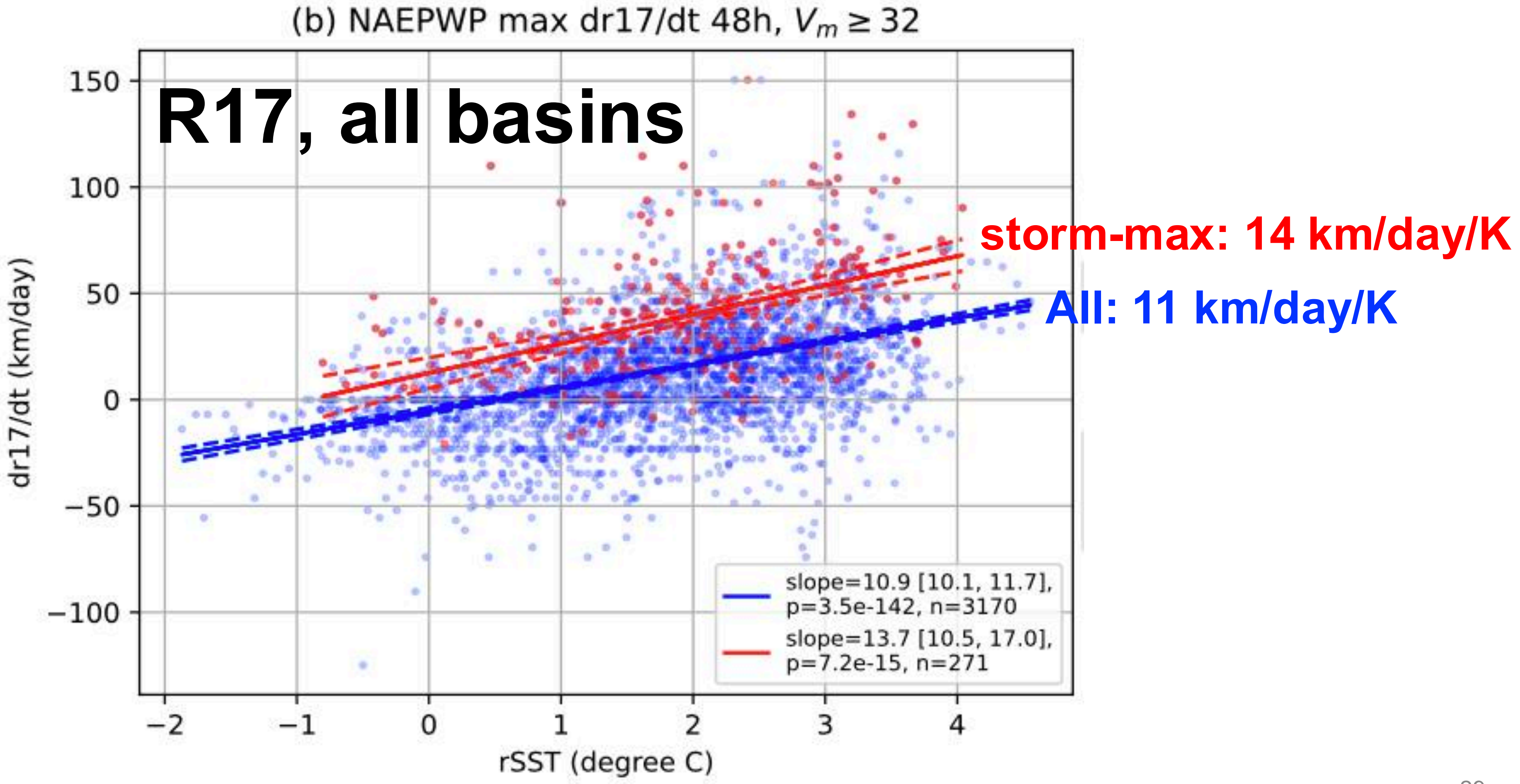
High-quality subset far from coast, south of 30N

R17 (Ext Best Track)



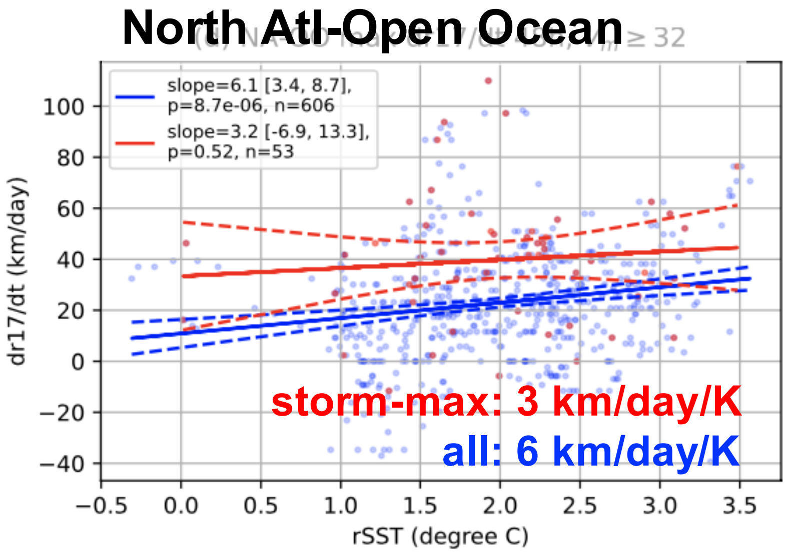
Wang, Chavas, Schenkel (2025), PNAS

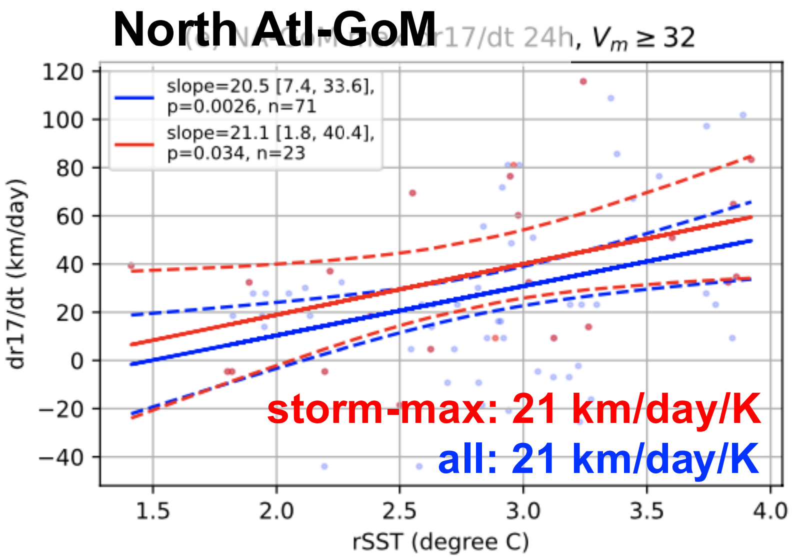
R17 (EBT) expands faster over locally warmer water



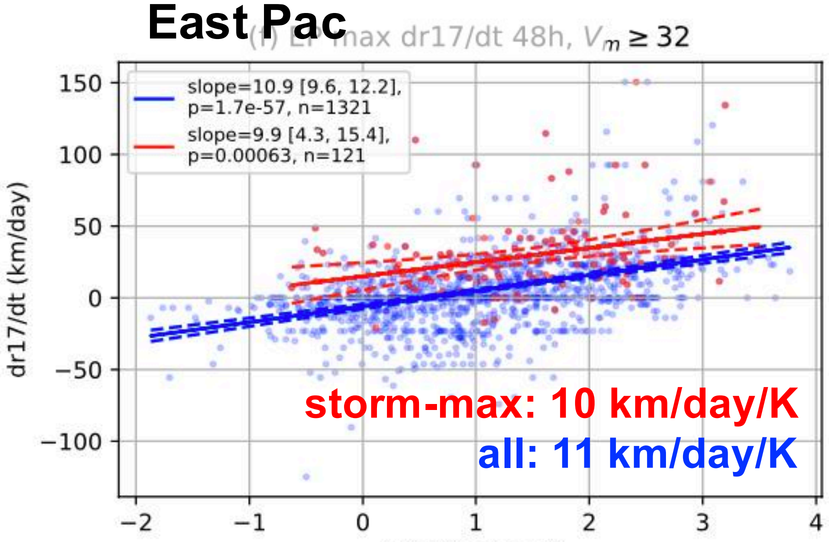
Wang, Chavas, Schenkel (2025), PNAS

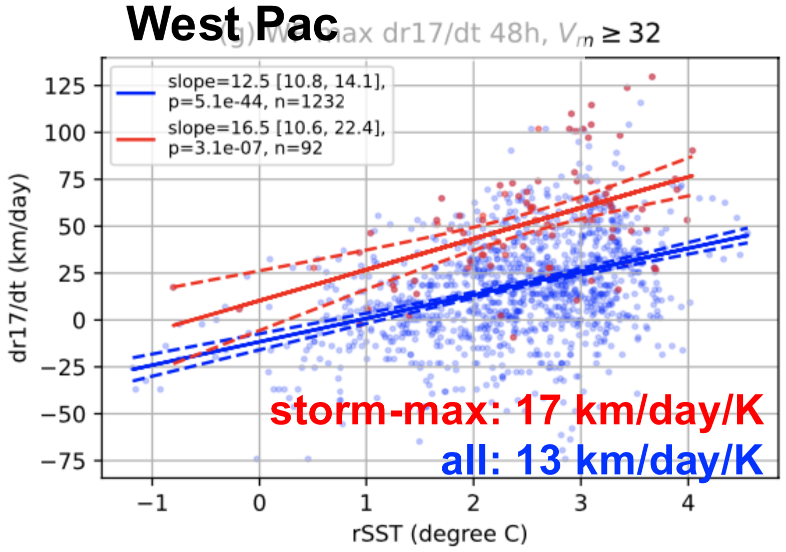
Result holds across across basins





Weaker signal in
North Atlantic – Open Ocean

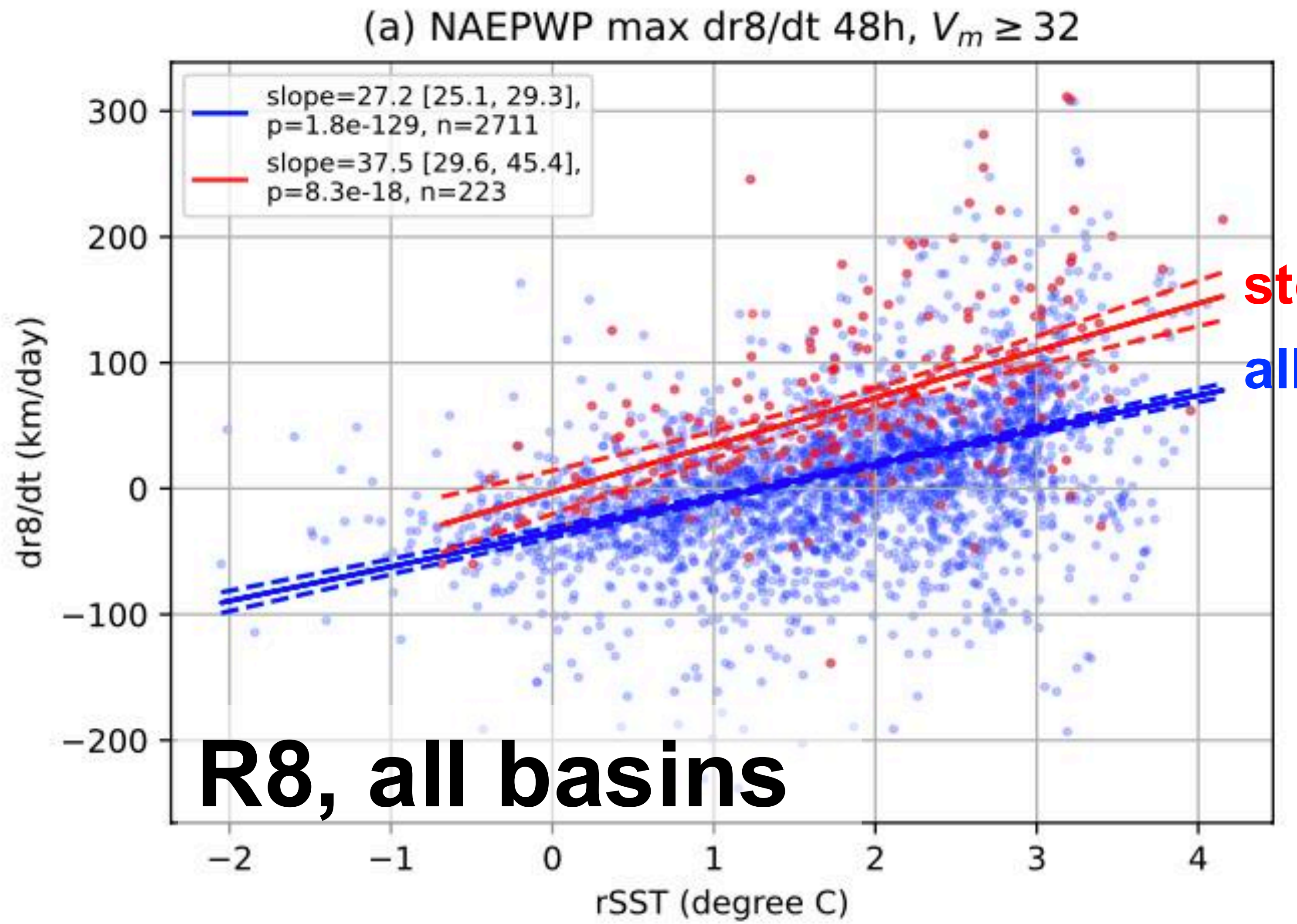




*Secondary eyewall formation: Removing periods with SEF does not change qualitative result, but SEF does temporarily accelerate expansion

Wang, Chavas, Schenkel (2025), PNAS

R8 (ERA5) similar

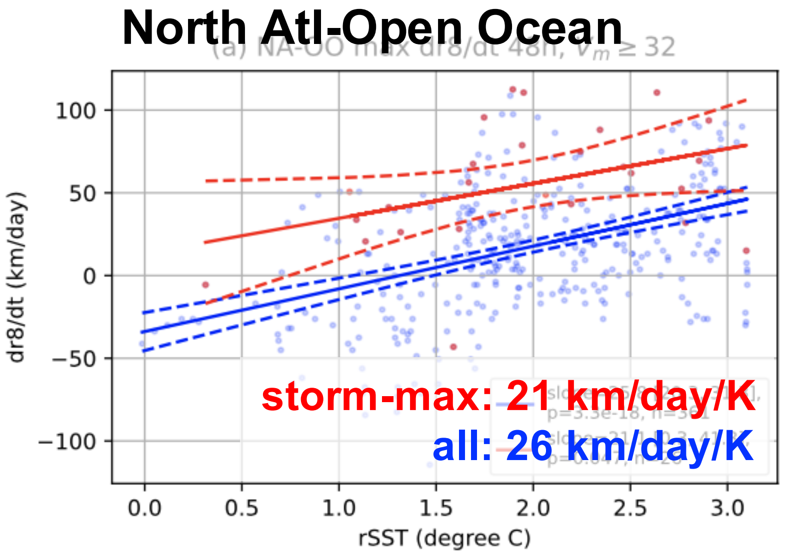


storm-max: 38 km/day/K

all: 27 km/day/K

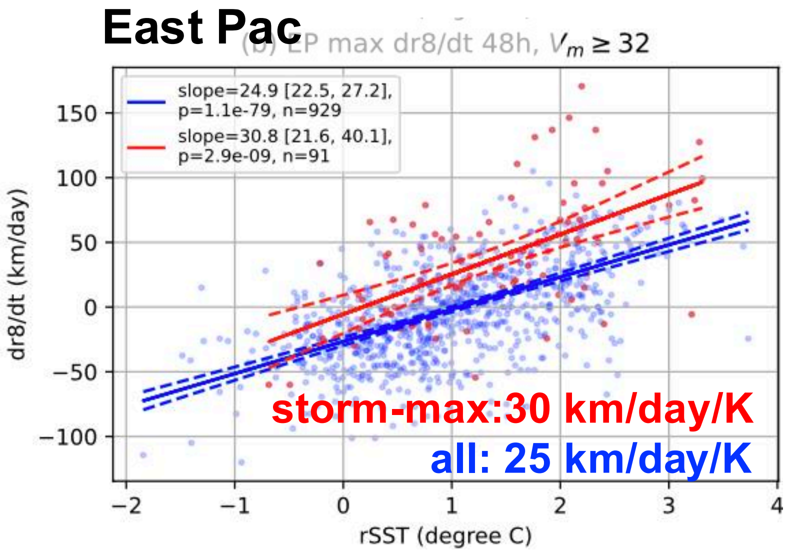
Wang, Chavas, Schenkel (2025), PNAS

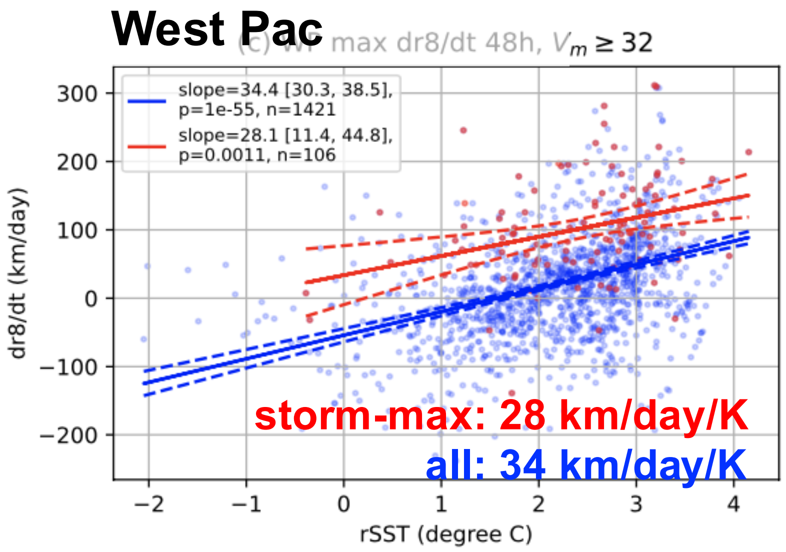
R8 (ERA5) similar – basins



Signal stronger in
North Atlantic – Open Ocean

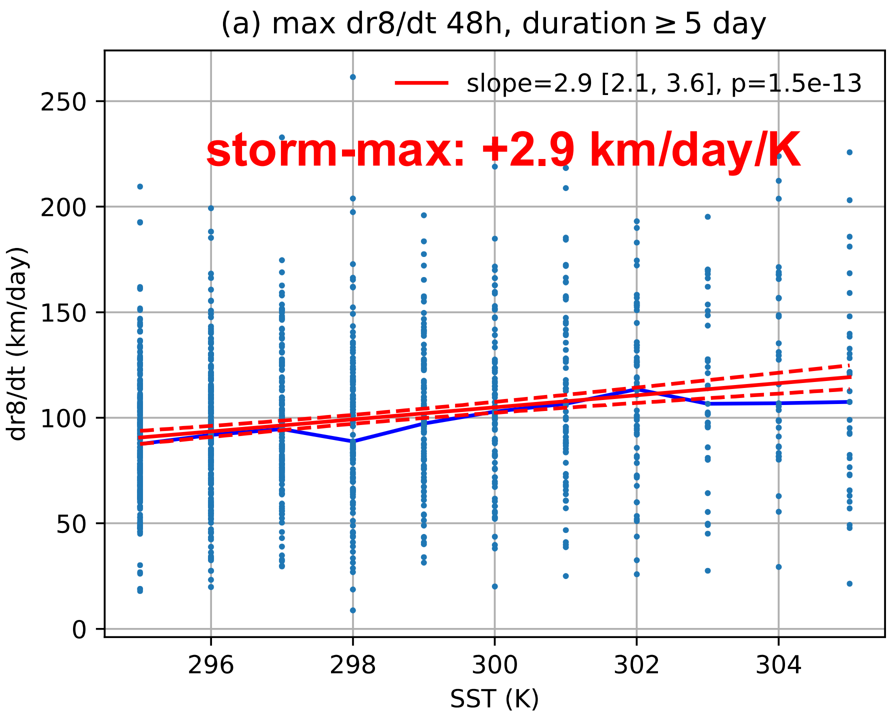
Not enough data in GoM
(R8 is much larger – not enough space)

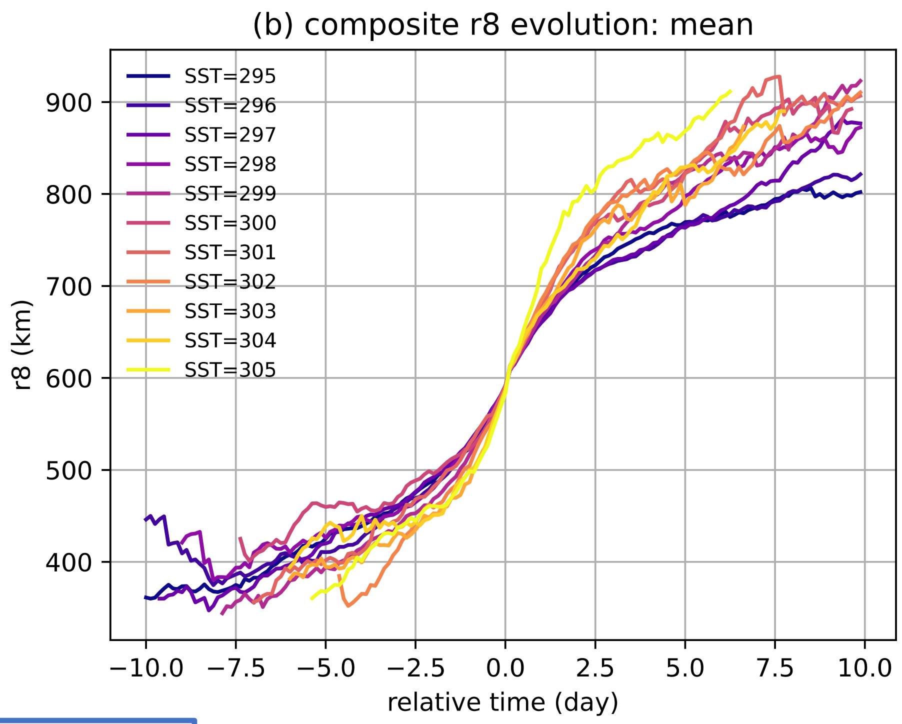




Wang, Chavas, Schenkel (2025), PNAS

Mean warming: aquaplanet experiments





Uniform global SST
Stansfield and Reed (2021)

Much smaller than observed dependence on local warming of ~38 km/day/K

Additional evidence to confirm it really is local warming (relative SST) that matters

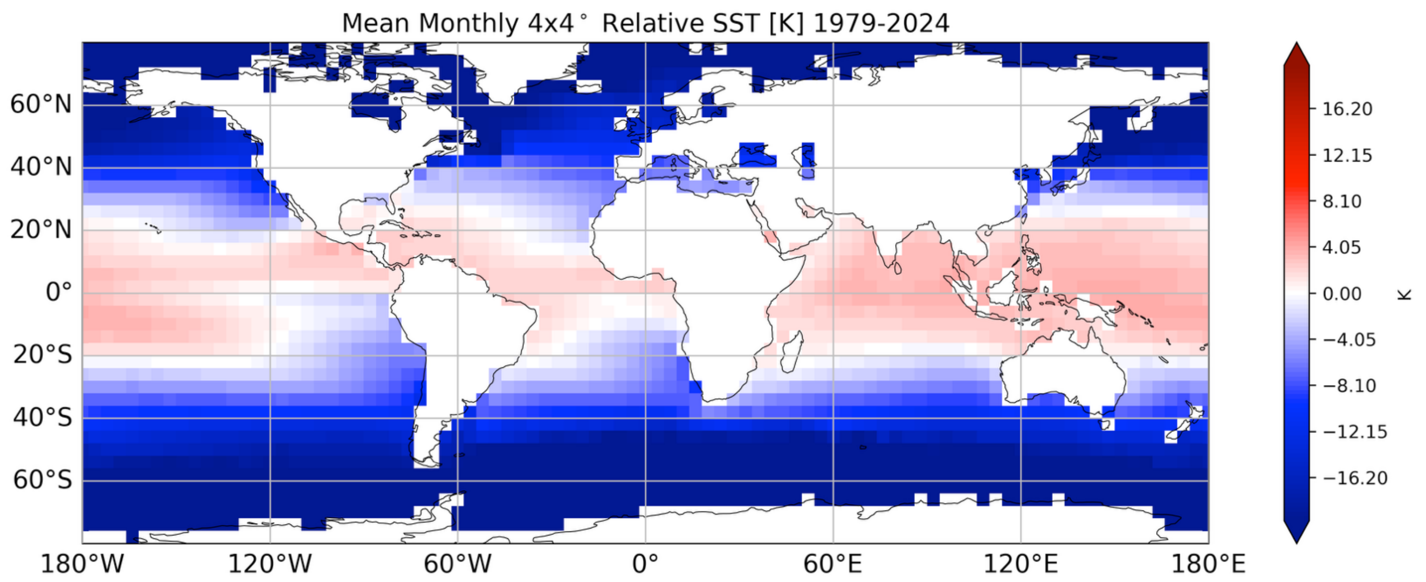
Wang, Chavas, Schenkel (2025), PNAS

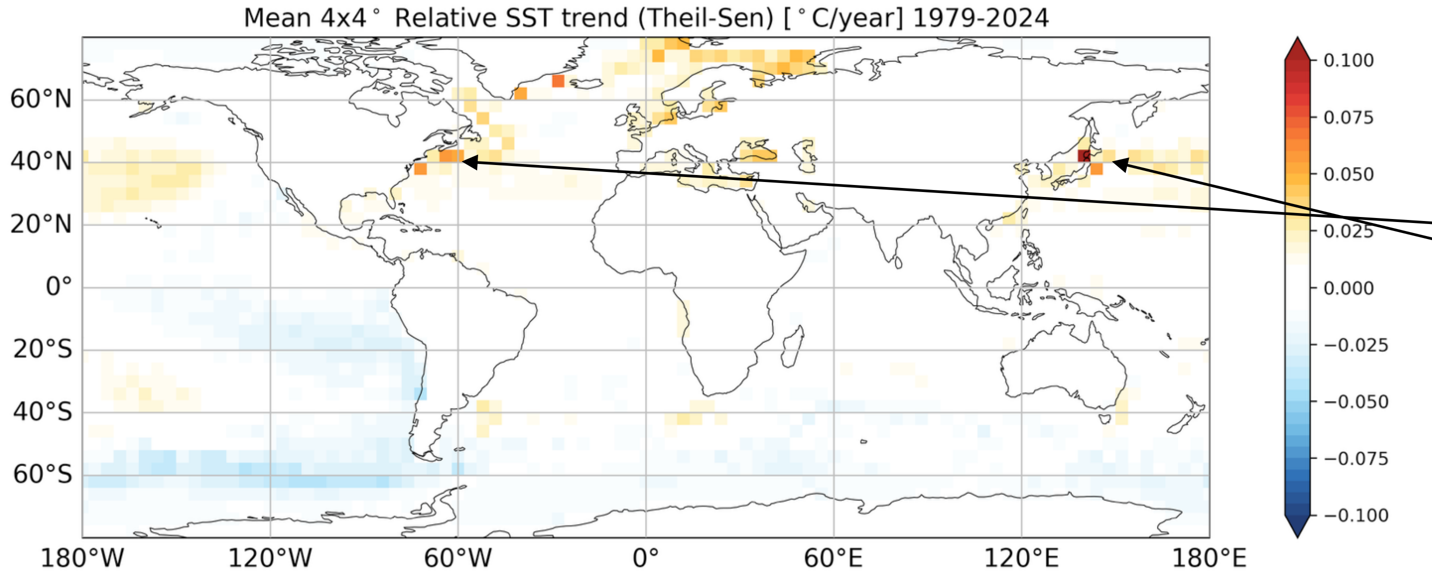
What might this mean for size itself?

Size *not* expected to change much overall

But regions that become locally warmer *might* have larger storms

Global trends in rSST



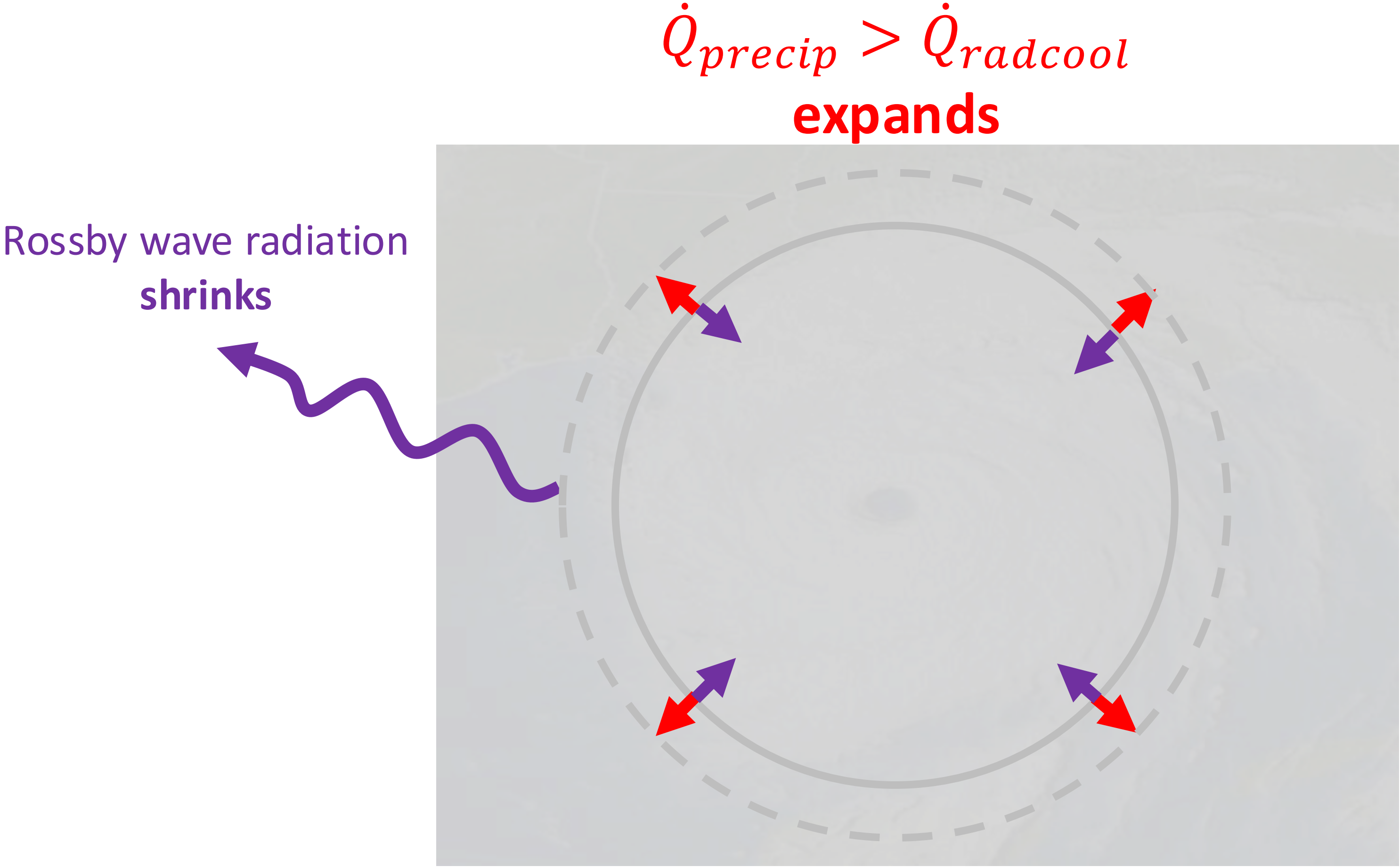


relSST trends confined to **near-coastal subtropical waters** (true globally too)

→ more favorable for TCs in general
Wang and Toumi (2021), Balaguru+ (2023)

Size too? Balaguru+ (2025)

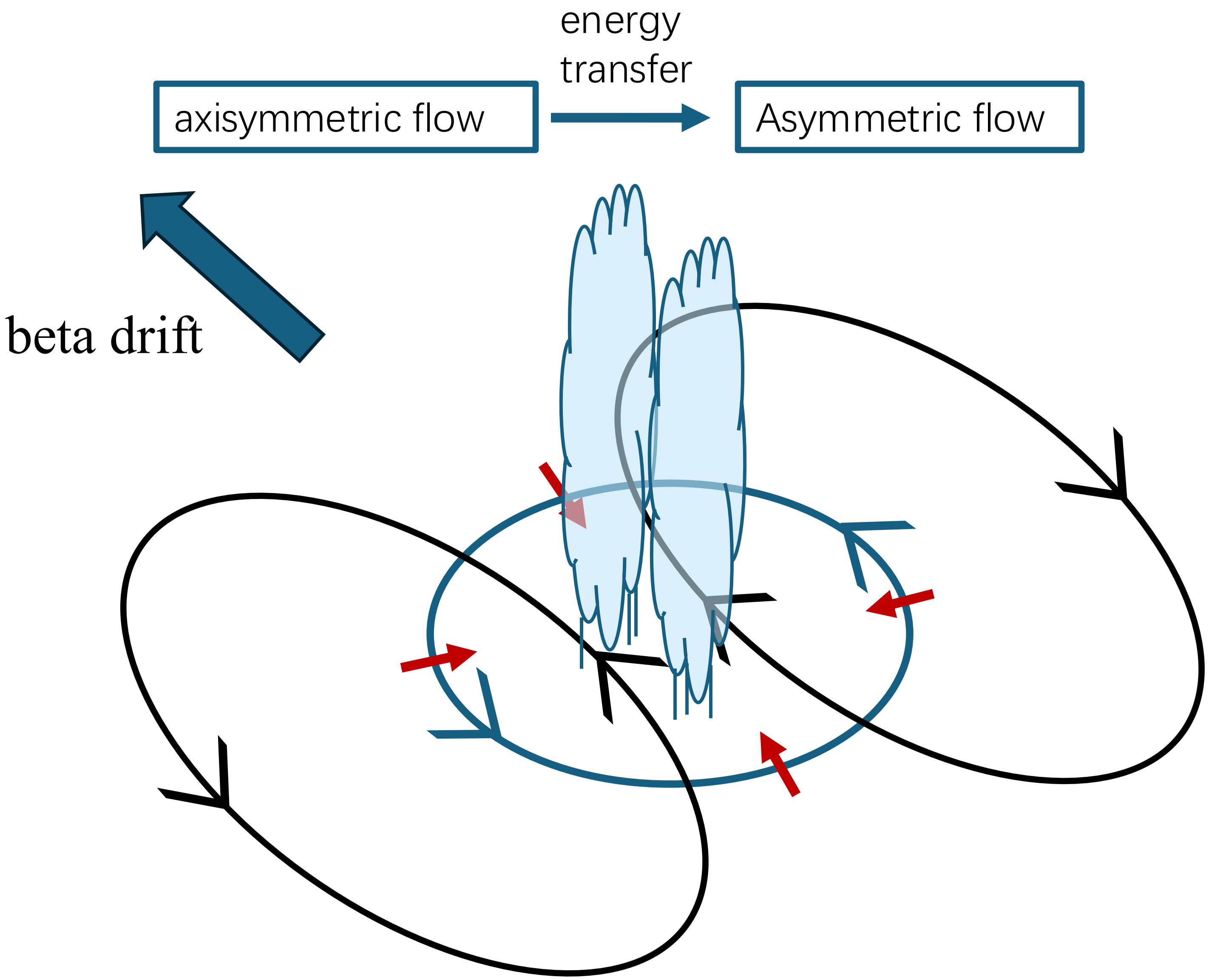
Sphere theory (with beta)


$\dot{Q}_{precip} > \dot{Q}_{radcool}$
expands
Rossby wave radiation
shrinks

An analytical model for tropical cyclone size expansion on the sphere

Danyang Wang,[a] Daniel R. Chavas[a]

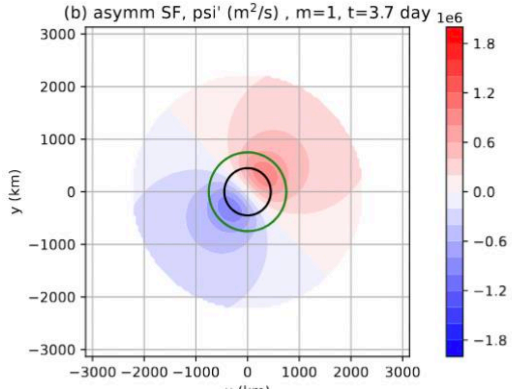
JAS, Under revision



Rossby wave drag (Rhines)

$$\frac{\partial v_s}{\partial t} \approx -f_c u_t \boxed{-\overline{f u_g}} - \mathrm{sgn}(v_t) C_d (\mu v_t)^2 / h_w, \quad \text{at} \quad r = r_t$$

New term –
azimuthal-mean
radial flux of
vorticity
(0 if f constant)

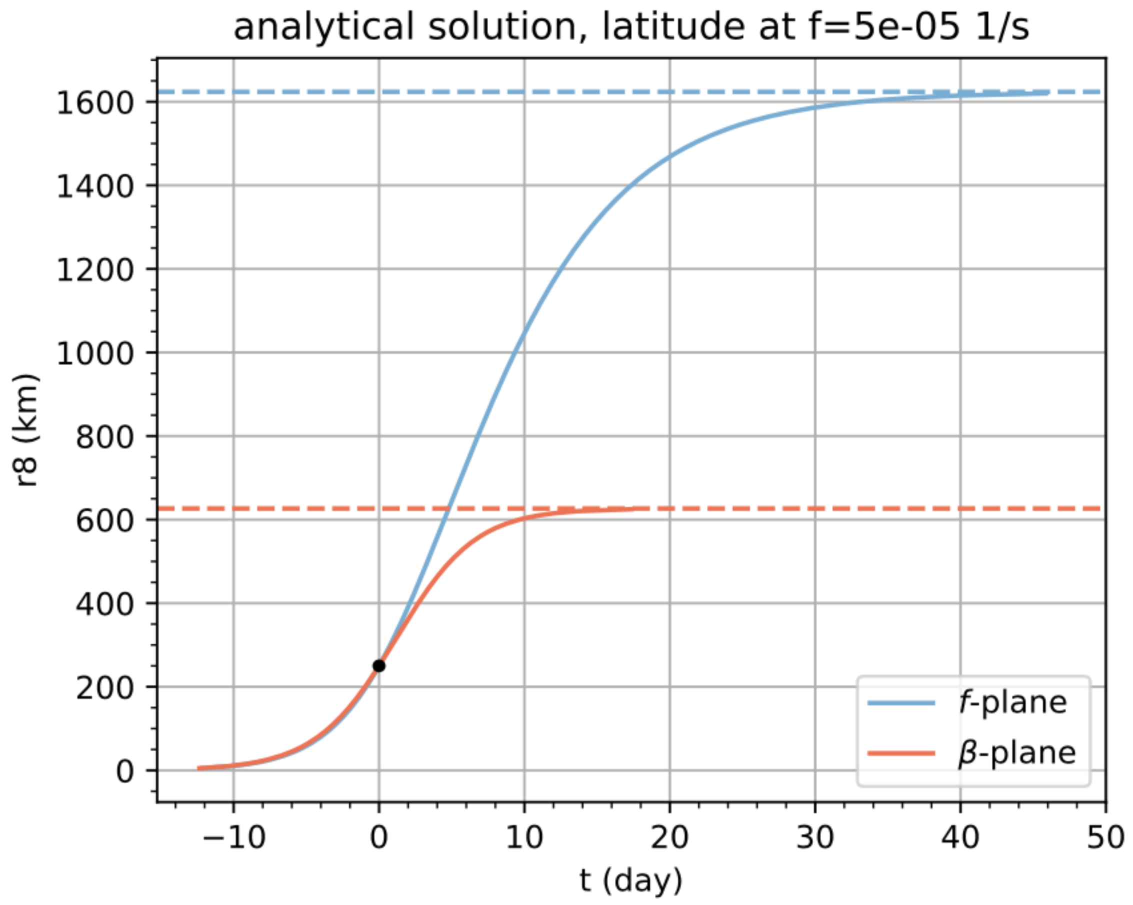


$$-\overline{f u_g} = -\frac{1}{2}\beta\Phi_g \cos(\vartheta_g)$$

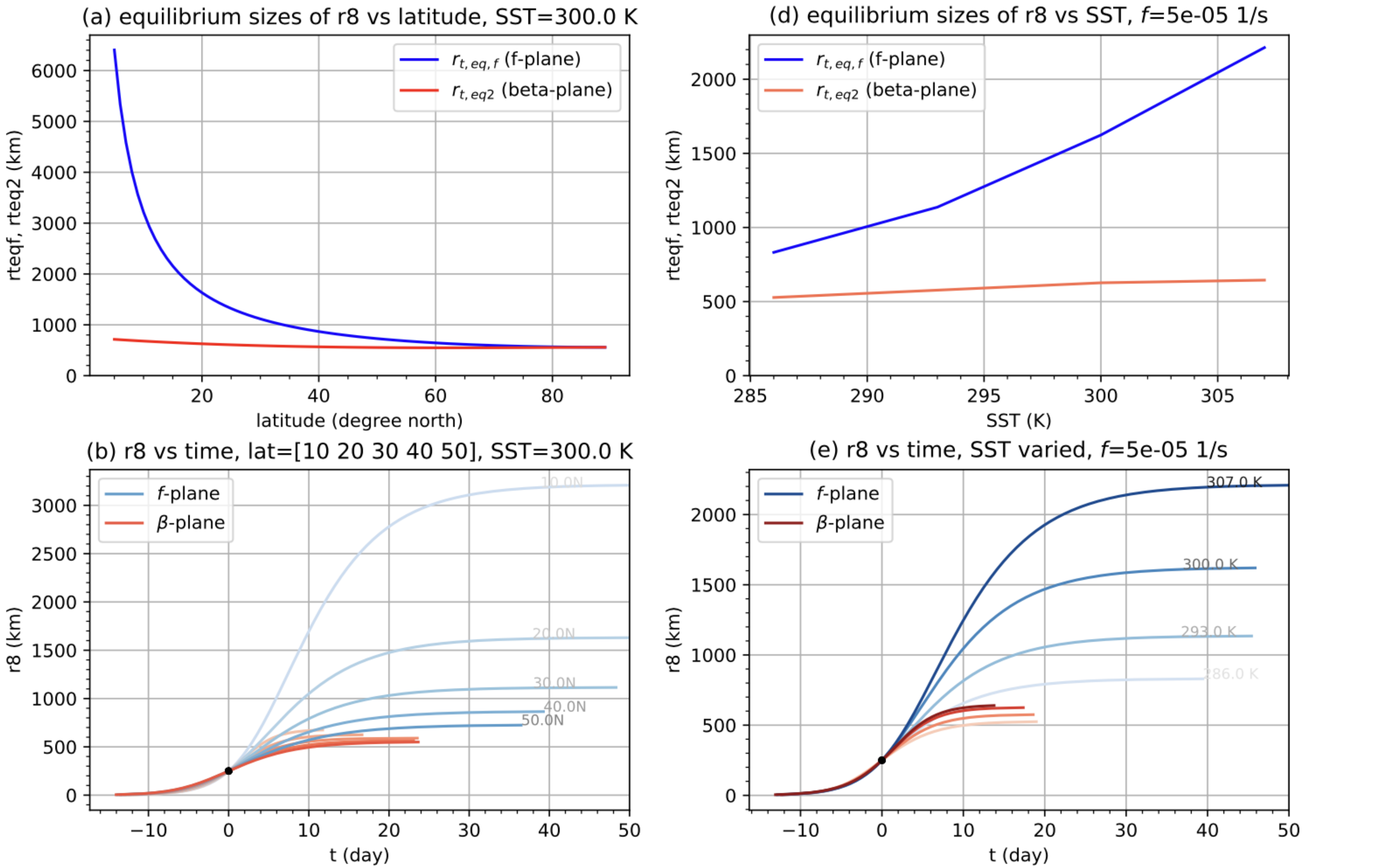
$$\psi_g(r,\vartheta,t) = \Phi_g(r,t)\cos[\vartheta_g(t) - \vartheta]$$

Beta gyre streamfunction

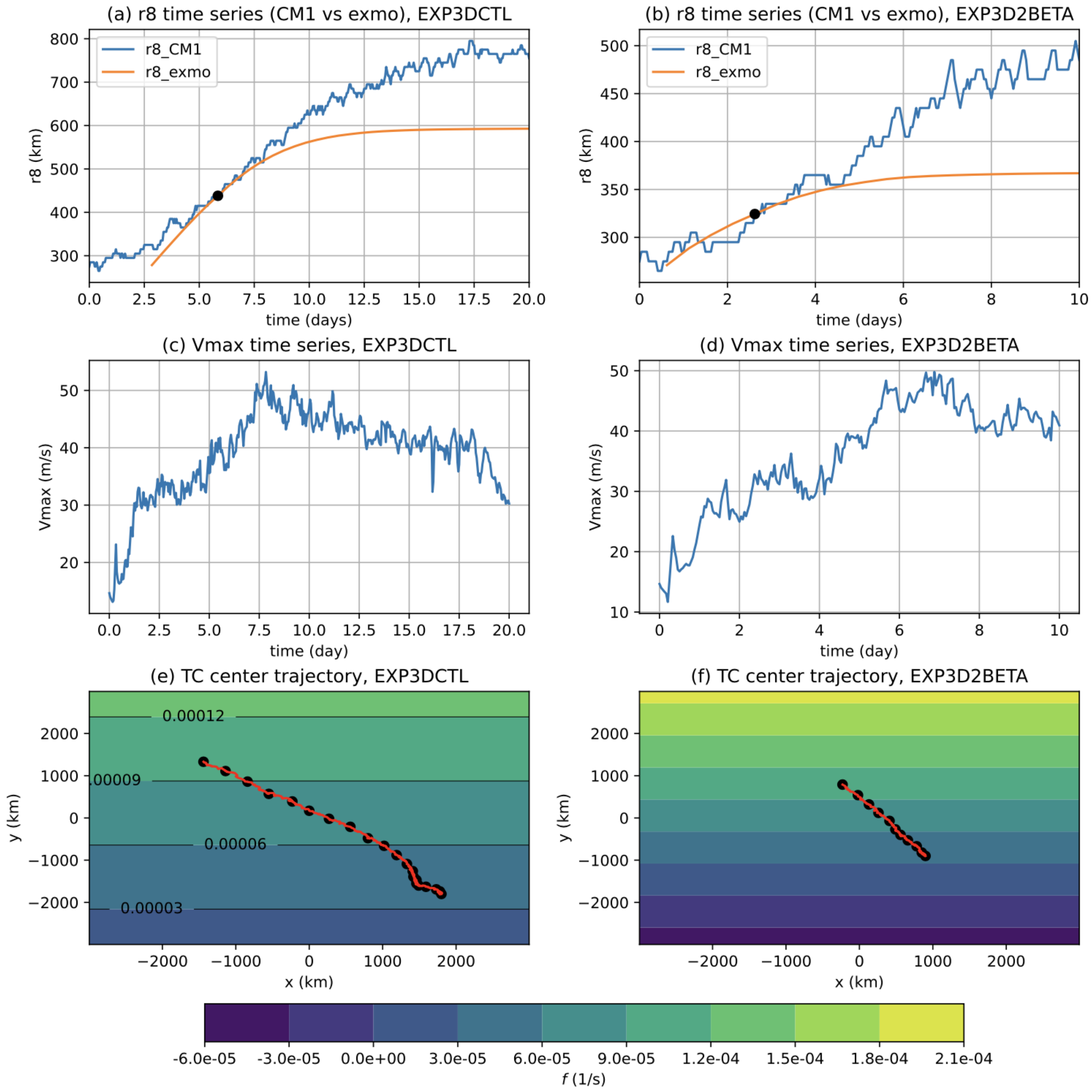
Analytic solution: wavenumber-1 Fourier-Bessel expansion



analytical solution, latitude at f=5e-05 1/s
r8 (km)
t (day)
f-plane
β-plane



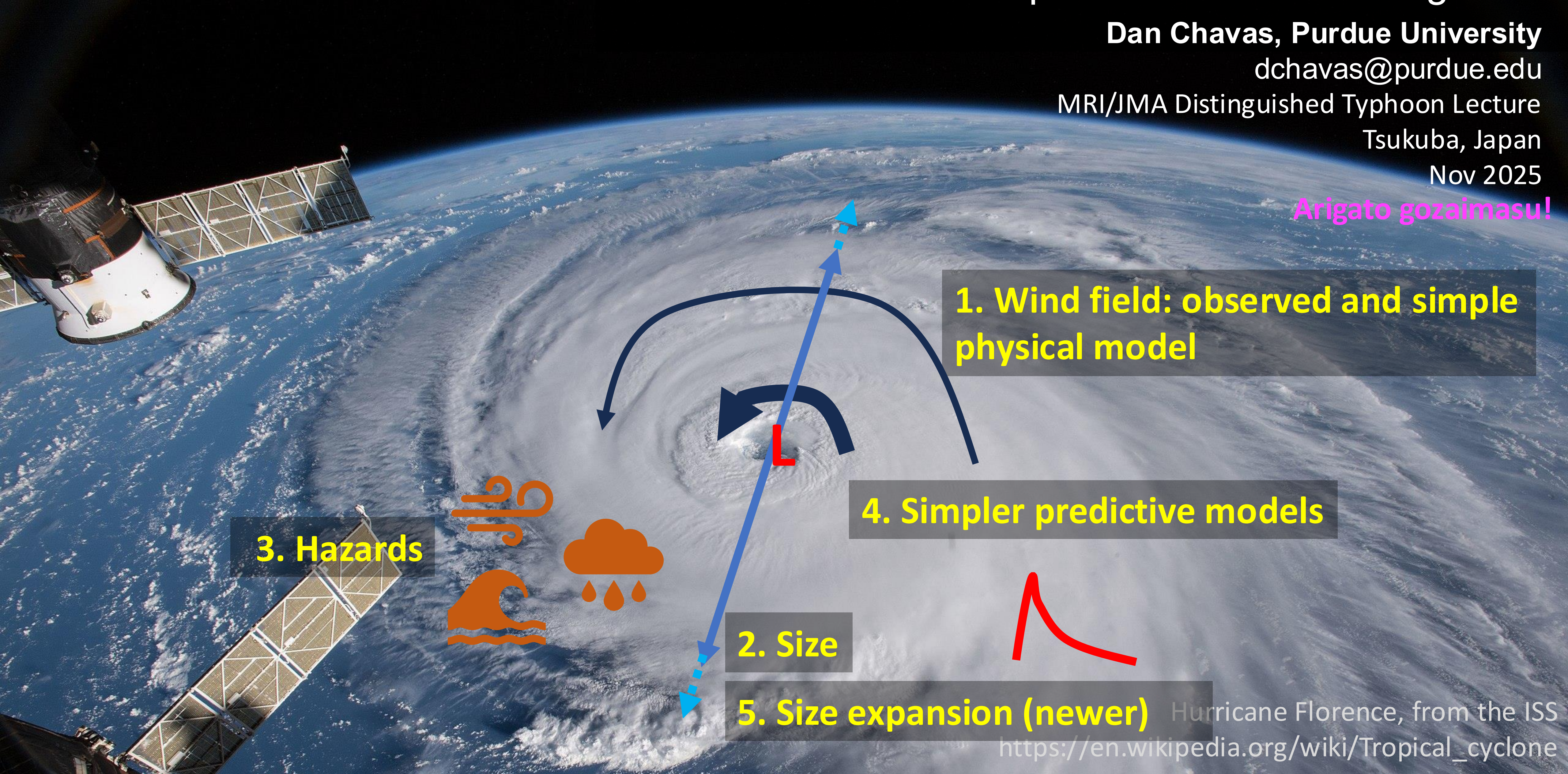
(a) equilibrium sizes of r8 vs latitude, SST=300.0 K
$r_{t,eq,f}$ (f-plane)
$r_{t,eq2}$ (beta-plane)
rteqf, rteq2 (km)
latitude (degree north)
(d) equilibrium sizes of r8 vs SST, f=5e-05 1/s
$r_{t,eq,f}$ (f-plane)
$r_{t,eq2}$ (beta-plane)
rteqf, rteq2 (km)
SST (K)
(b) r8 vs time, lat=[10 20 30 40 50], SST=300.0 K
f-plane
β-plane
10.0N
20.0N
30.0N
40.0N
50.0N
r8 (km)
t (day)
(e) r8 vs time, SST varied, f=5e-05 1/s
f-plane
β-plane
307.0 K
300.0 K
293.0 K
286.0 K
r8 (km)
t (day)


(a) r8 time series (CM1 vs exmo), EXP3DCTL
r8_CM1
r8_exmo
r8 (km)
time (days)
(b) r8 time series (CM1 vs exmo), EXP3D2BETA
r8_CM1
r8_exmo
r8 (km)
time (days)
(c) Vmax time series, EXP3DCTL
Vmax (m/s)
time (day)
(d) Vmax time series, EXP3D2BETA
Vmax (m/s)
time (day)
(e) TC center trajectory, EXP3DCTL
0.00012
.00009
0.00006
0.00003
y (km)
x (km)
(f) TC center trajectory, EXP3D2BETA
y (km)
x (km)
-6.0e-05
-3.0e-05
0.0e+00
3.0e-05
6.0e-05
9.0e-05
1.2e-04
1.5e-04
1.8e-04
2.1e-04
f (1/s)

The tropical cyclone wind field: from simple theory to real-world prediction in a warming world

0. Intro: two recent events

Dan Chavas, Purdue University
dchavas@purdue.edu
MRI/JMA Distinguished Typhoon Lecture
Tsukuba, Japan
Nov 2025
Arigato gozaimasu!



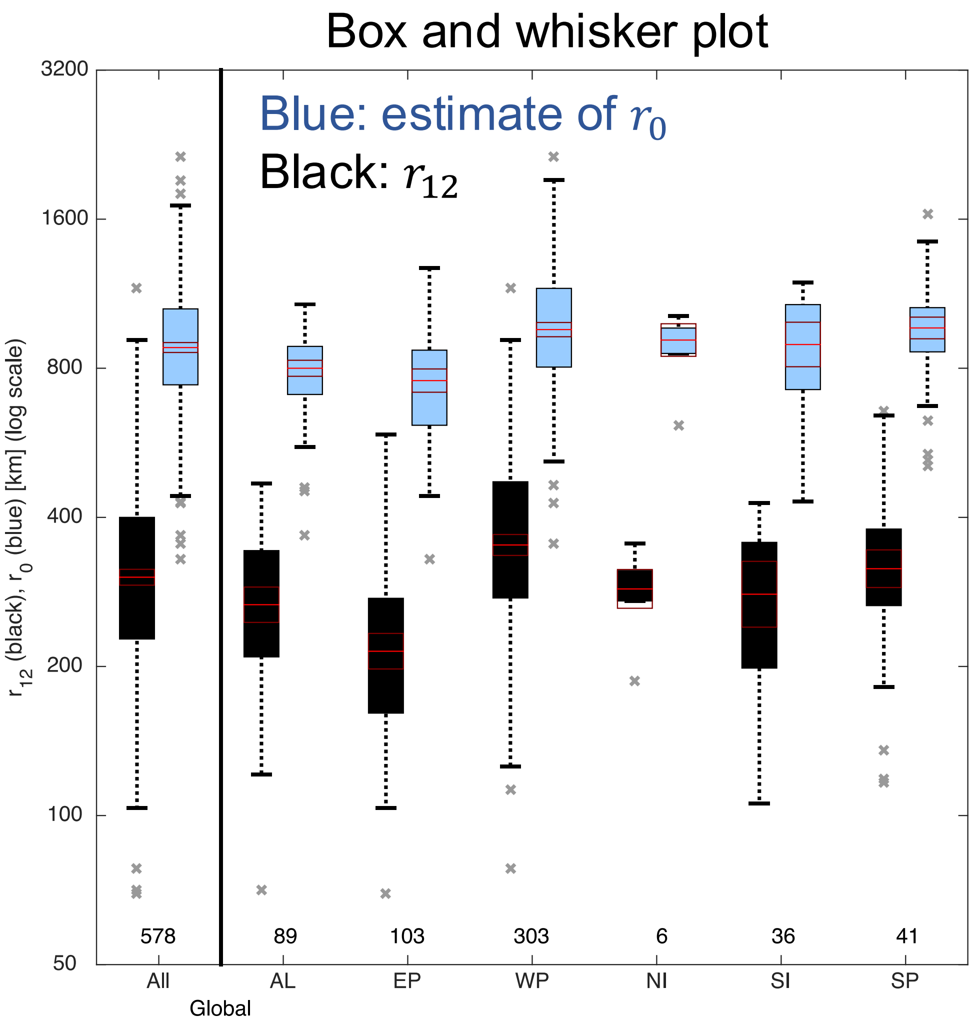
Hurricane Florence, from the ISS
https://en.wikipedia.org/wiki/Tropical_cyclone

Arigato gozaimasu

- EXTRA

Largest storms in West Pac, smallest in East Pac

Outer size distribution is approximately **log-normal**



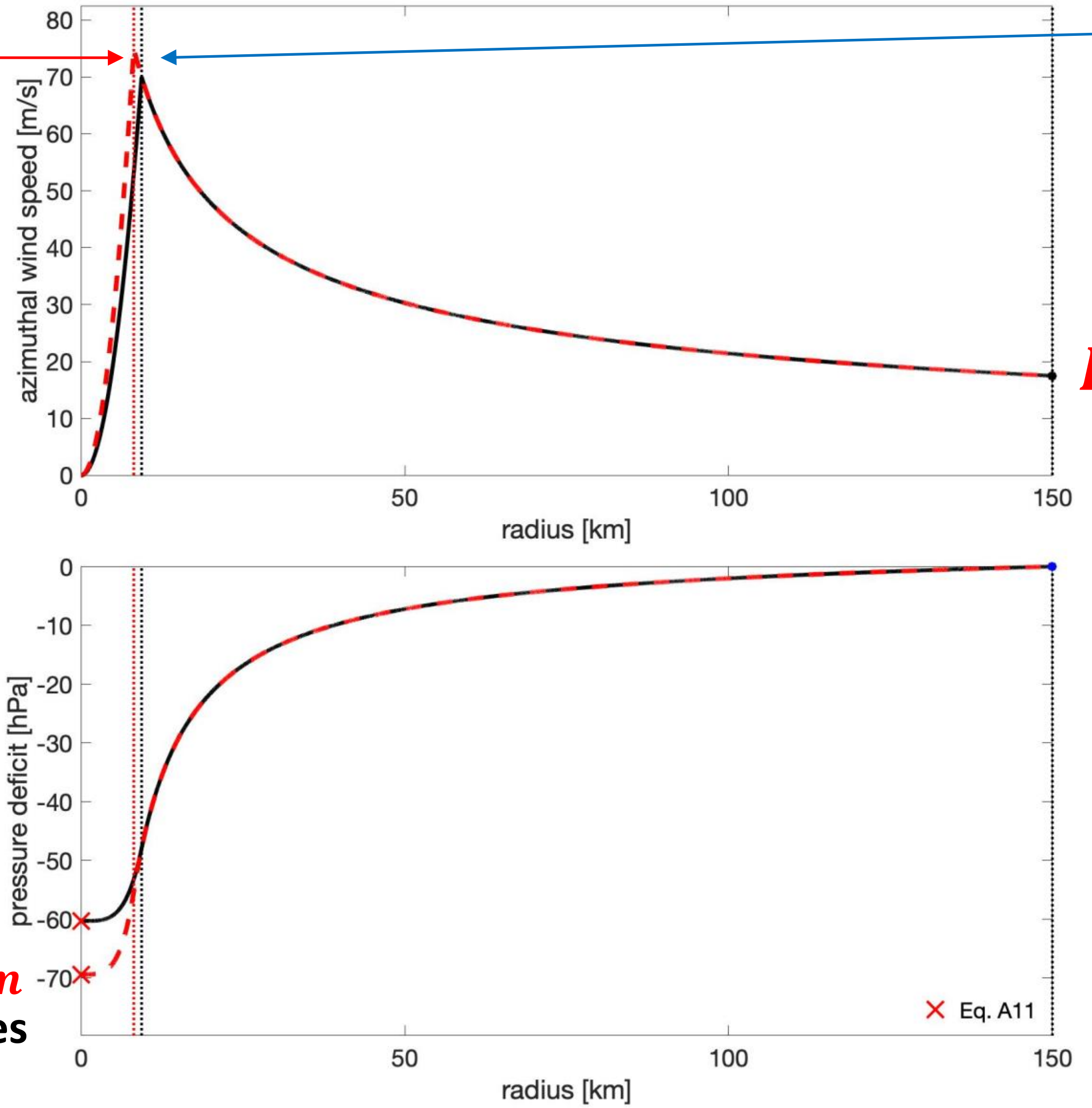
Chavas et al. (2016)

Size (R34kt) is the missing piece

V_{max}

increases slowly with global warming

(many studies)



Note: also predicts that R_{max} should get smaller, which has been shown in studies

Chen+ (2022, JC), Tran+ (2022, EF)
Chavas and Lin (2016)

R_{34kt}

stays constant with warming

Real world:
Knutson+ (2015, JC), Schenkel+ (2023, JC)

Aquaplanet:
Stansfield and Reed (2021, JGR-A)

Physics:
Chavas and Reed (2019, JAS), Lu and Chavas (2022, JAS)

P_{min}

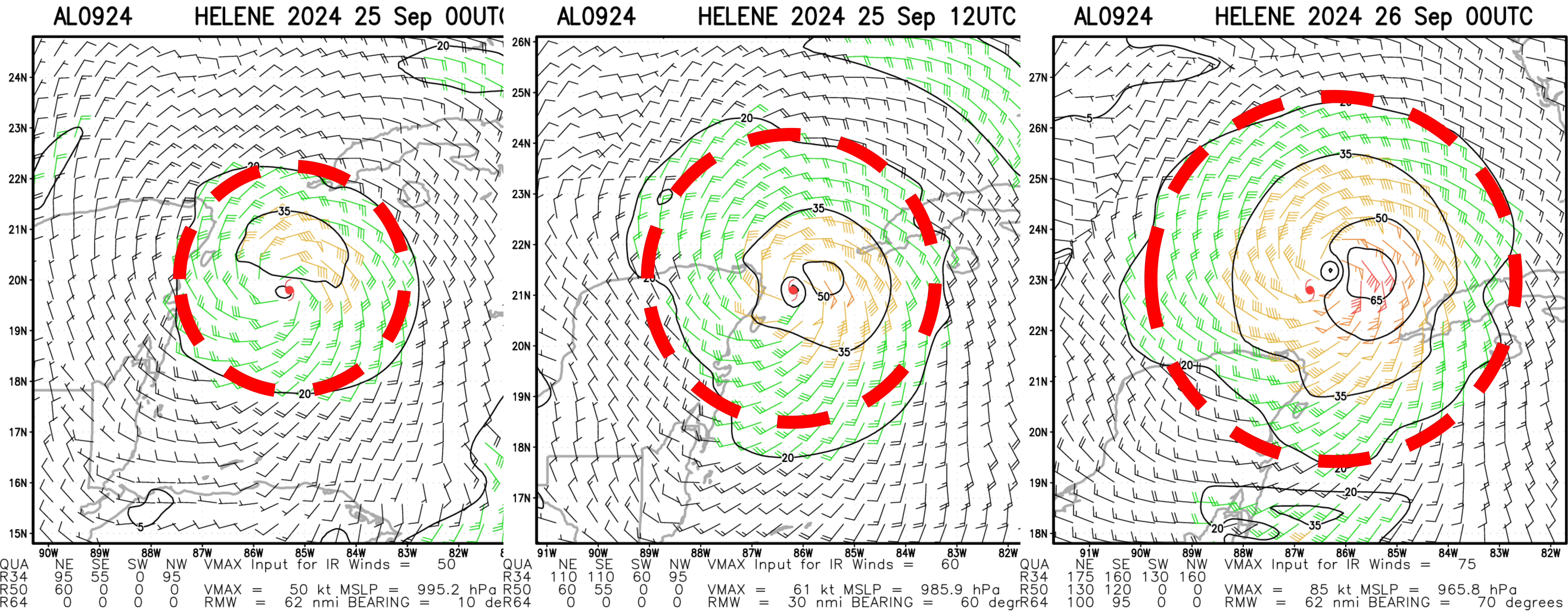
→ **lower pressures**

(effect of changes in latitude are relatively small)

Helene nearly doubled in size in 24 hours

+12 hours

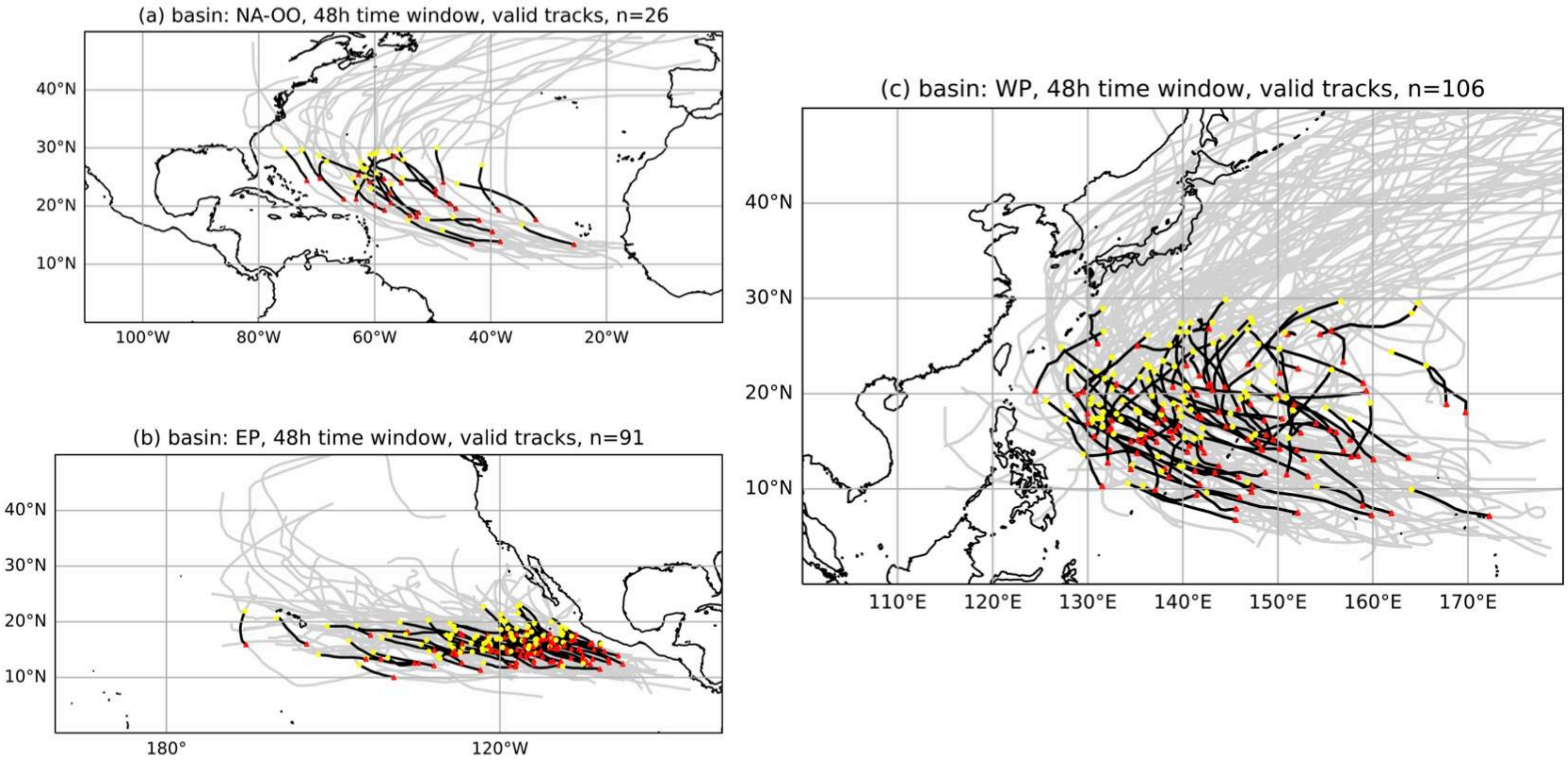
+24 hours



Larger storm = greater wind/surge/rainfall

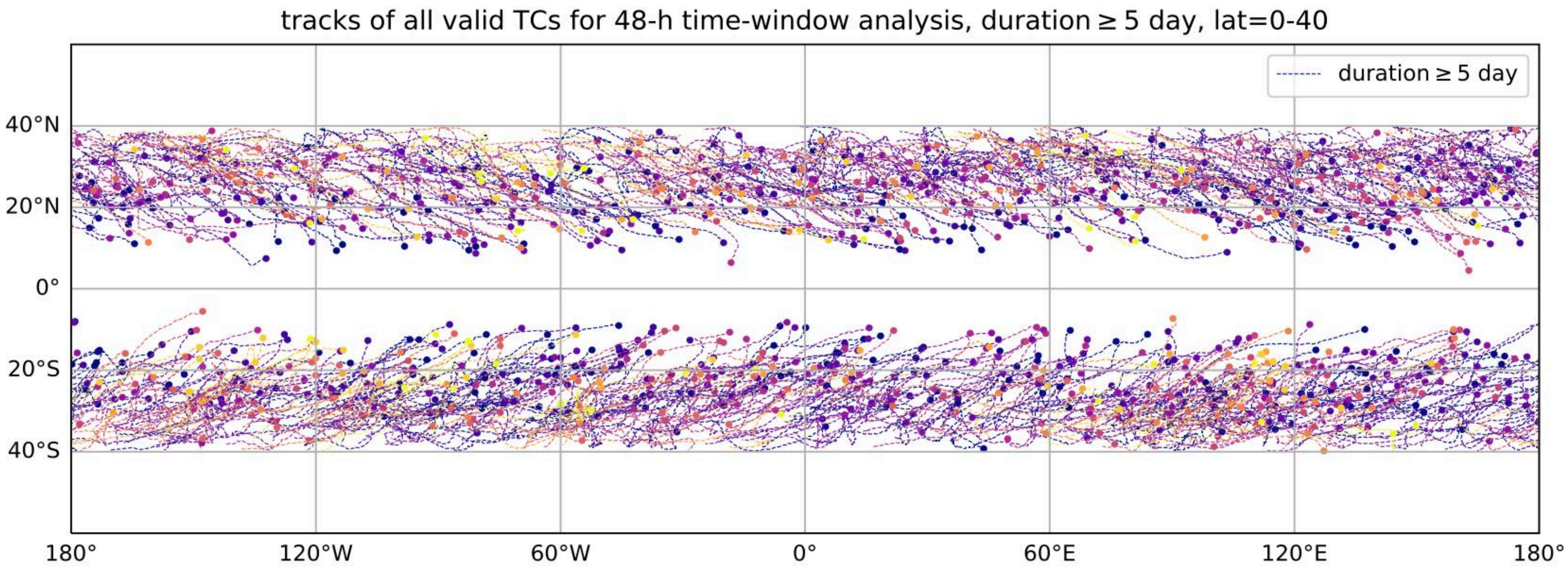
High-quality subset far from coast, below 30N

R8 (ERA5 reanalysis)



Wang, Chavas, Schenkel (2025), under revision

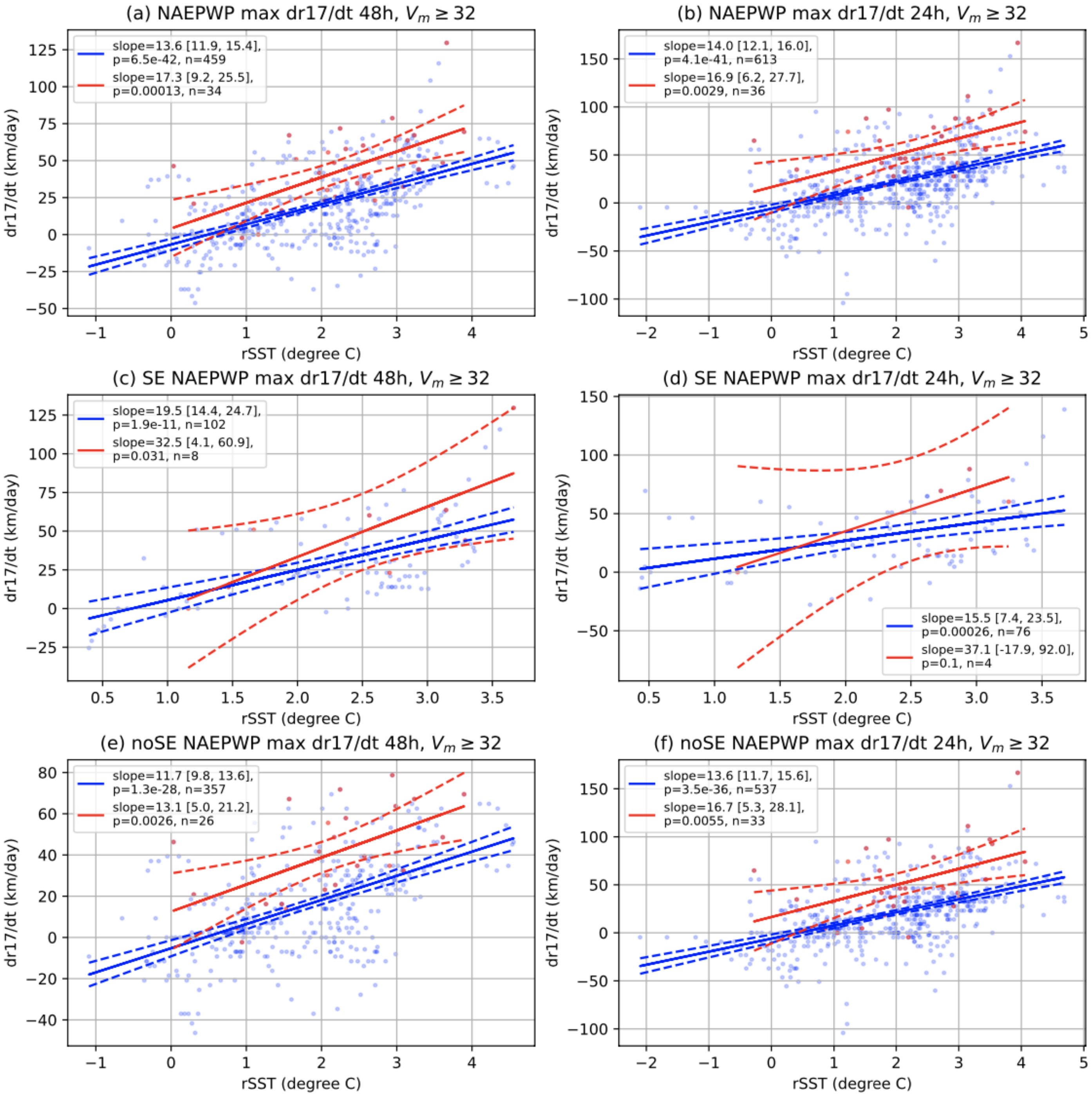
Aquaplanet tracks

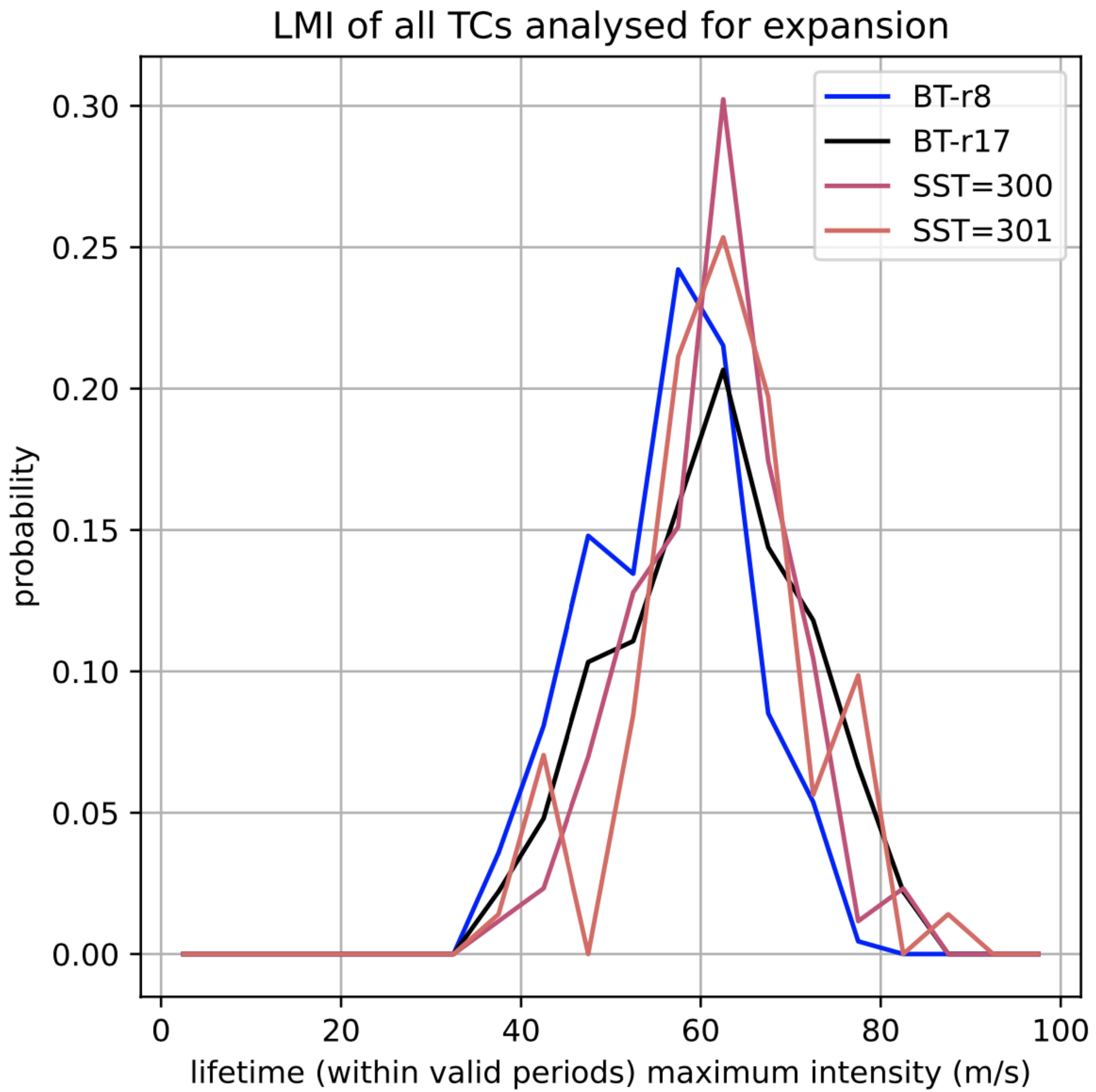


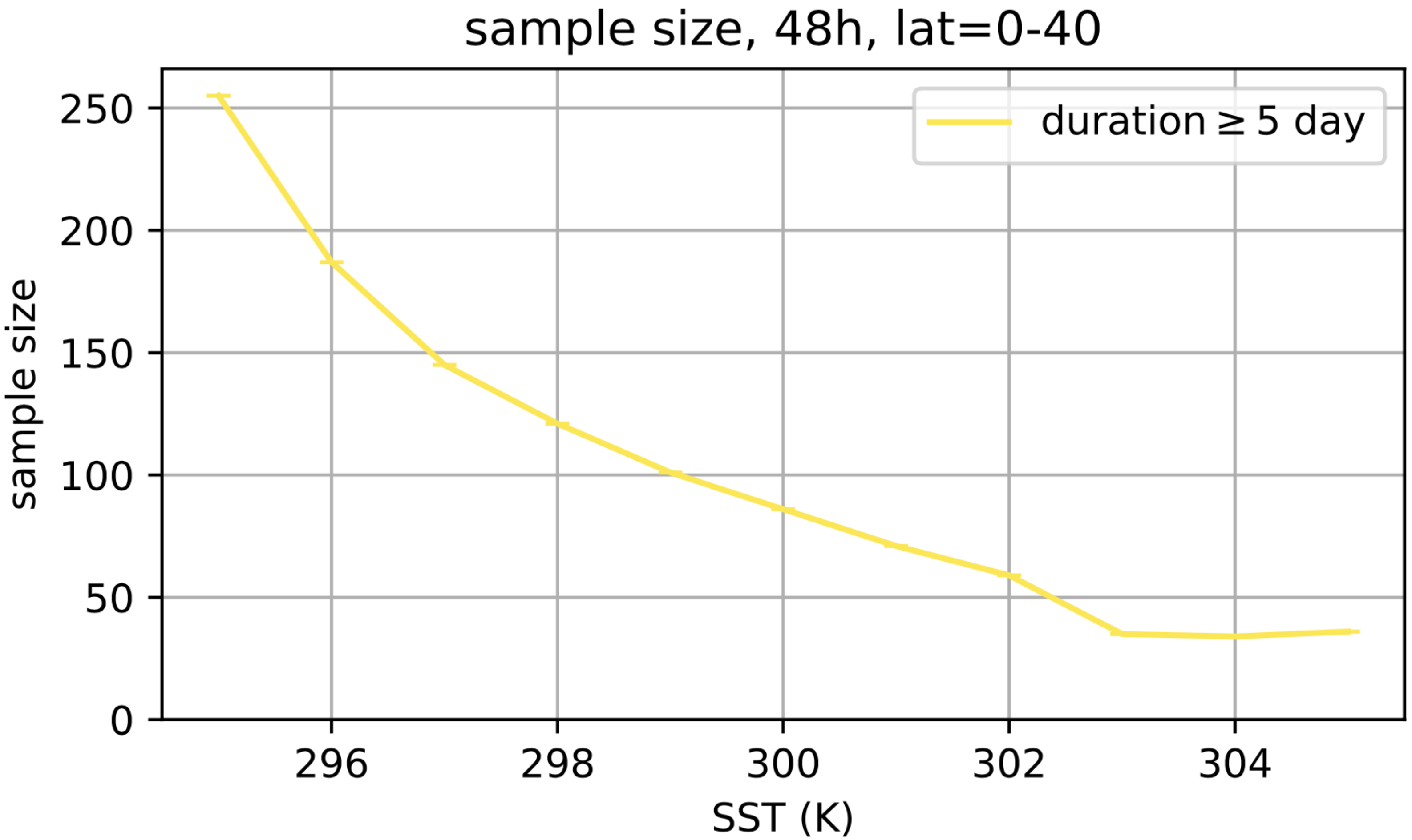
Wang, Chavas, Schenkel (2025), under revision

Results do not depend on occurrence of secondary eyewall formation,

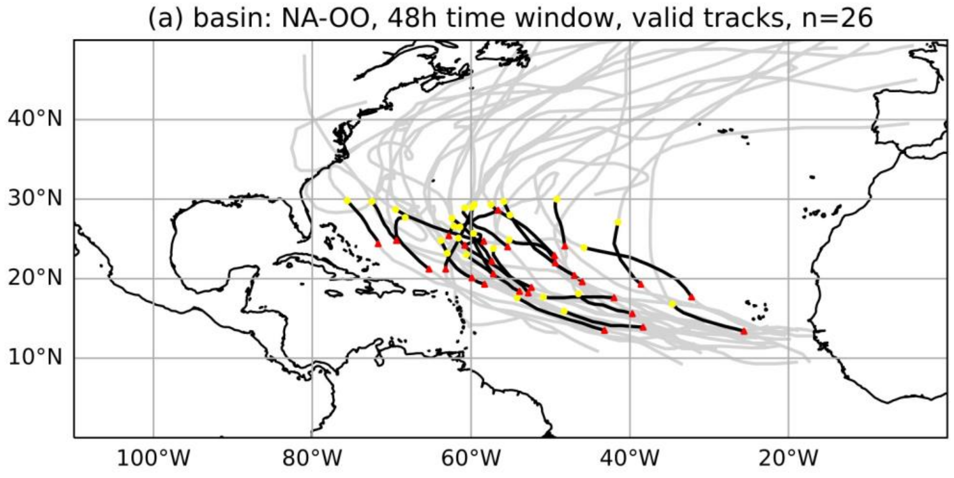
But SEF does temporarily accelerate expansion

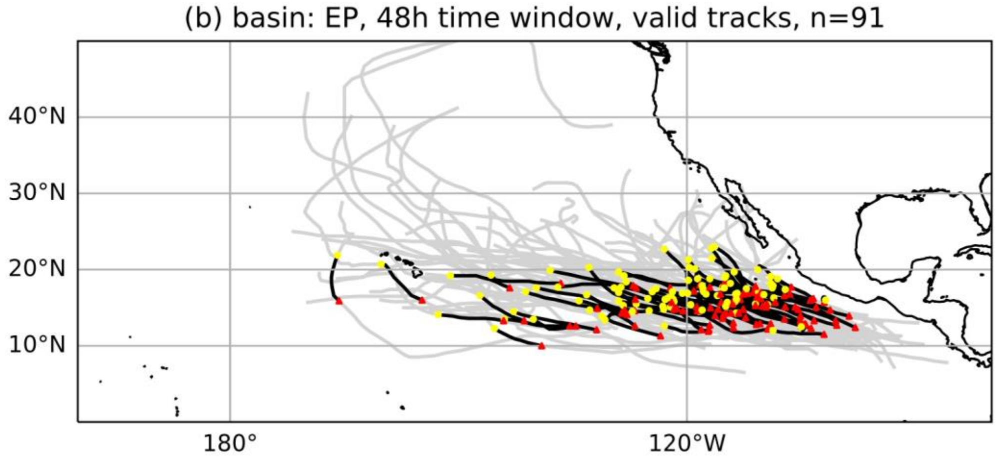


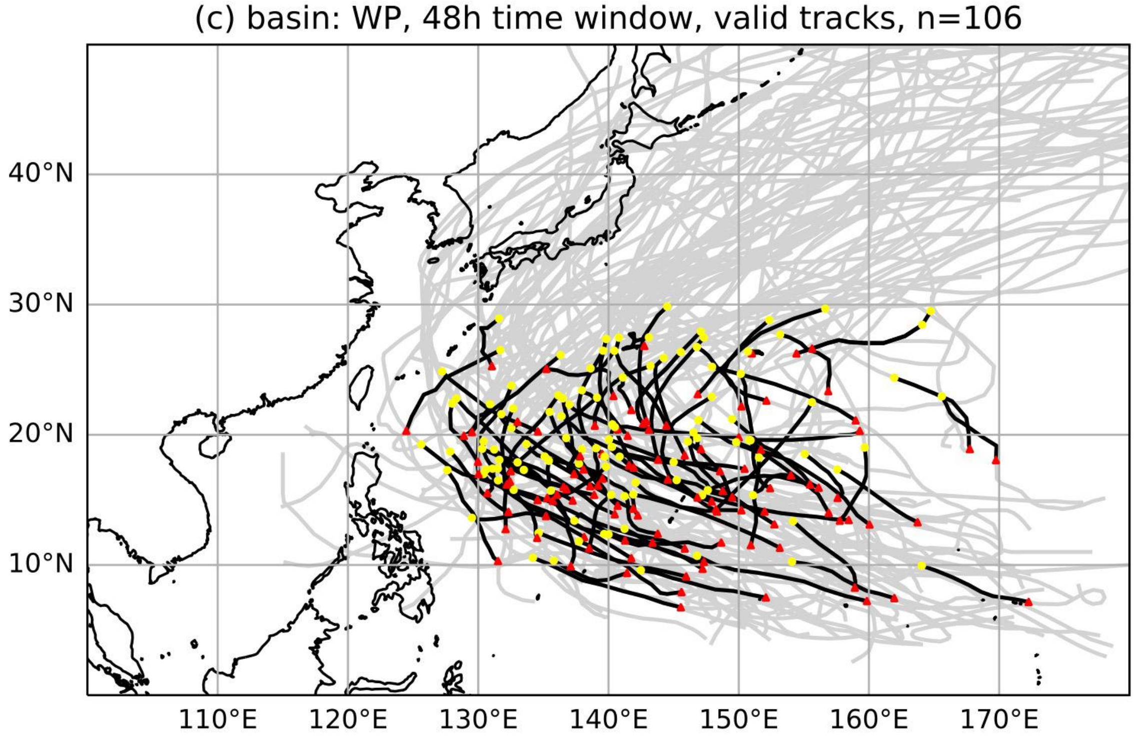

LMI of all TCs analysed for expansion
BT-r8
BT-r17
SST=300
SST=301
0.30
0.25
0.20
0.15
0.10
0.05
0.00
probability
0
20
40
60
80
100
lifetime (within valid periods) maximum intensity (m/s)

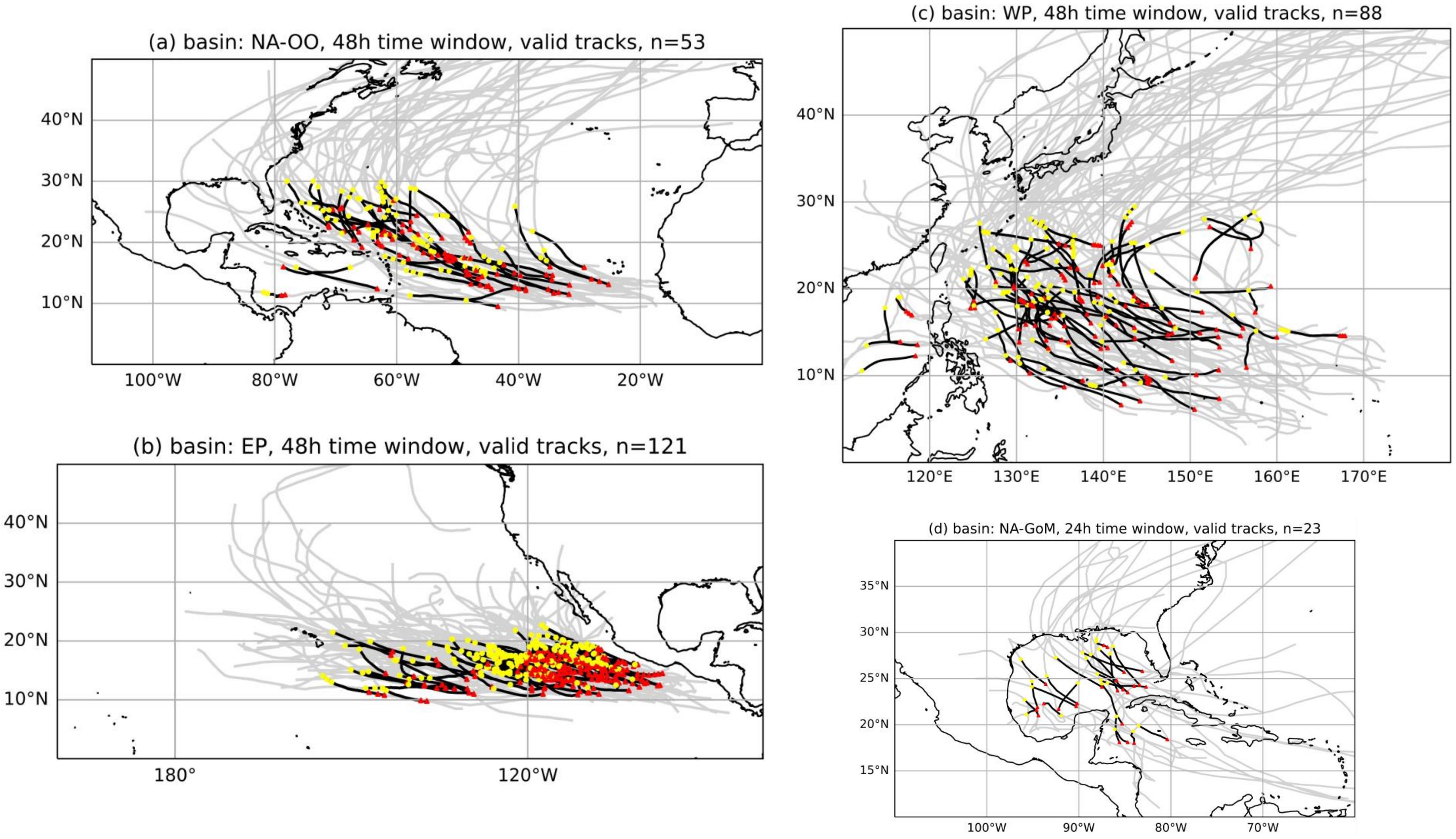


sample size, 48h, lat=0-40
duration ≥ 5 day
sample size
SST (K)
0
50
100
150
200
250
296
298
300
302
304

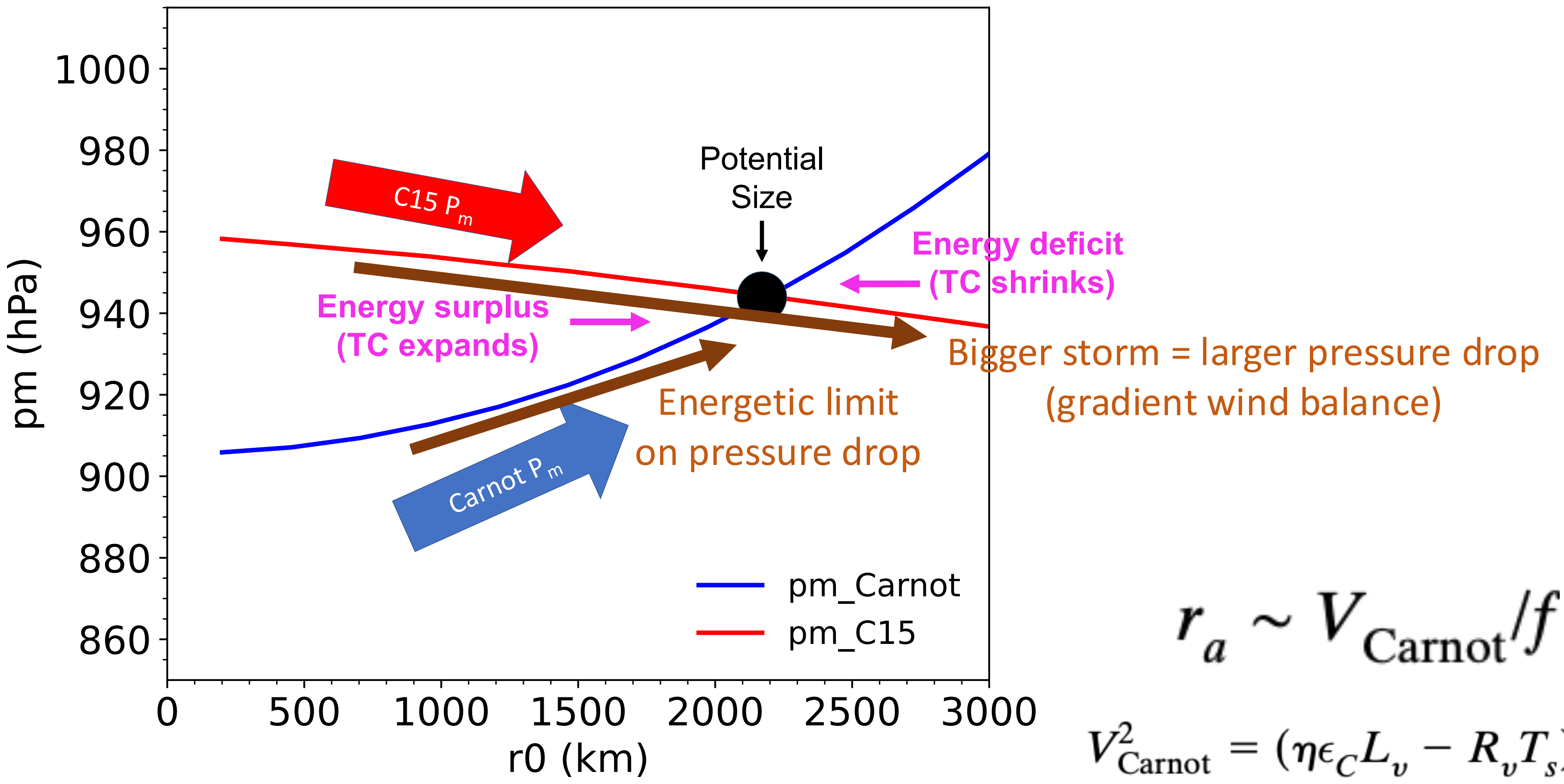

(a) basin: NA-OO, 48h time window, valid tracks, n=26
40°N
30°N
20°N
10°N
100°W
80°W
60°W
40°W
20°W


(b) basin: EP, 48h time window, valid tracks, n=91
40°N
30°N
20°N
10°N
180°
120°W


(c) basin: WP, 48h time window, valid tracks, n=106
40°N
30°N
20°N
10°N
110°E
120°E
130°E
140°E
150°E
160°E
170°E


(a) basin: NA-OO, 48h time window, valid tracks, n=53
40°N
30°N
20°N
10°N
100°W
80°W
60°W
40°W
20°W
(b) basin: EP, 48h time window, valid tracks, n=121
40°N
30°N
20°N
10°N
180°
120°W
(c) basin: WP, 48h time window, valid tracks, n=88
40°N
30°N
20°N
10°N
120°E
130°E
140°E
150°E
160°E
170°E
(d) basin: NA-GoM, 24h time window, valid tracks, n=23
35°N
30°N
25°N
20°N
15°N
100°W
90°W
80°W
70°W

Tropical cyclone potential size set by energetic and dynamic constraints



$$r_a \sim V_{\mathrm{Carnot}}/f$$

$$V^2_{\mathrm{Carnot}} = (\eta \epsilon_C L_v - R_v T_s) q^*_{va}$$

Wang et al (2022). Tropical cyclone potential size, J. Atmos. Sci.