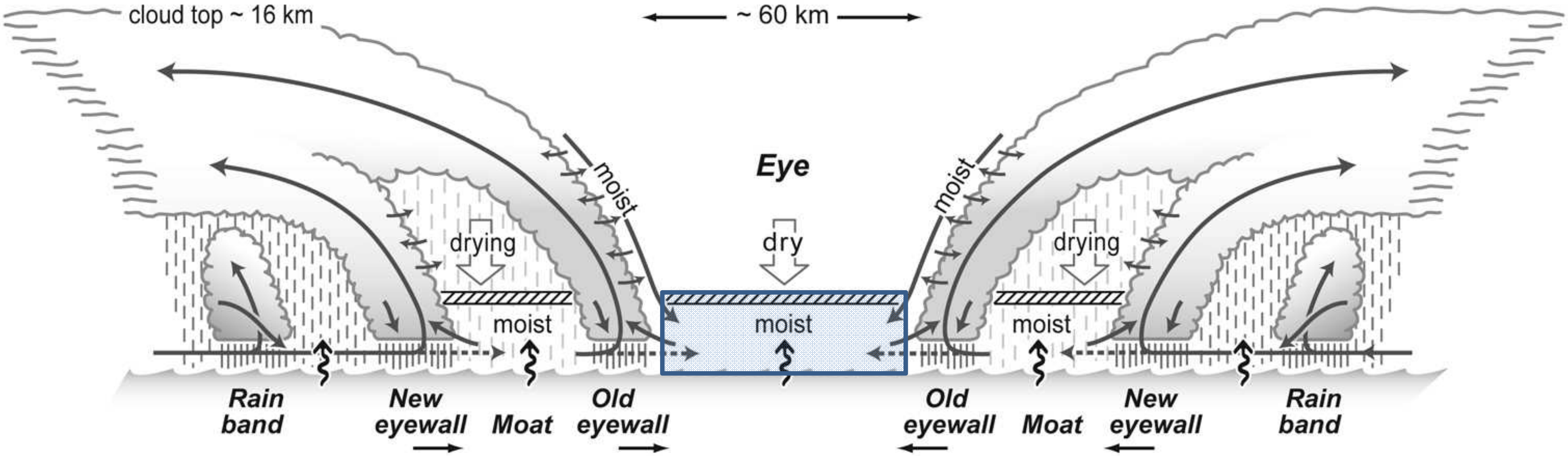
Tropical Cyclone Structure

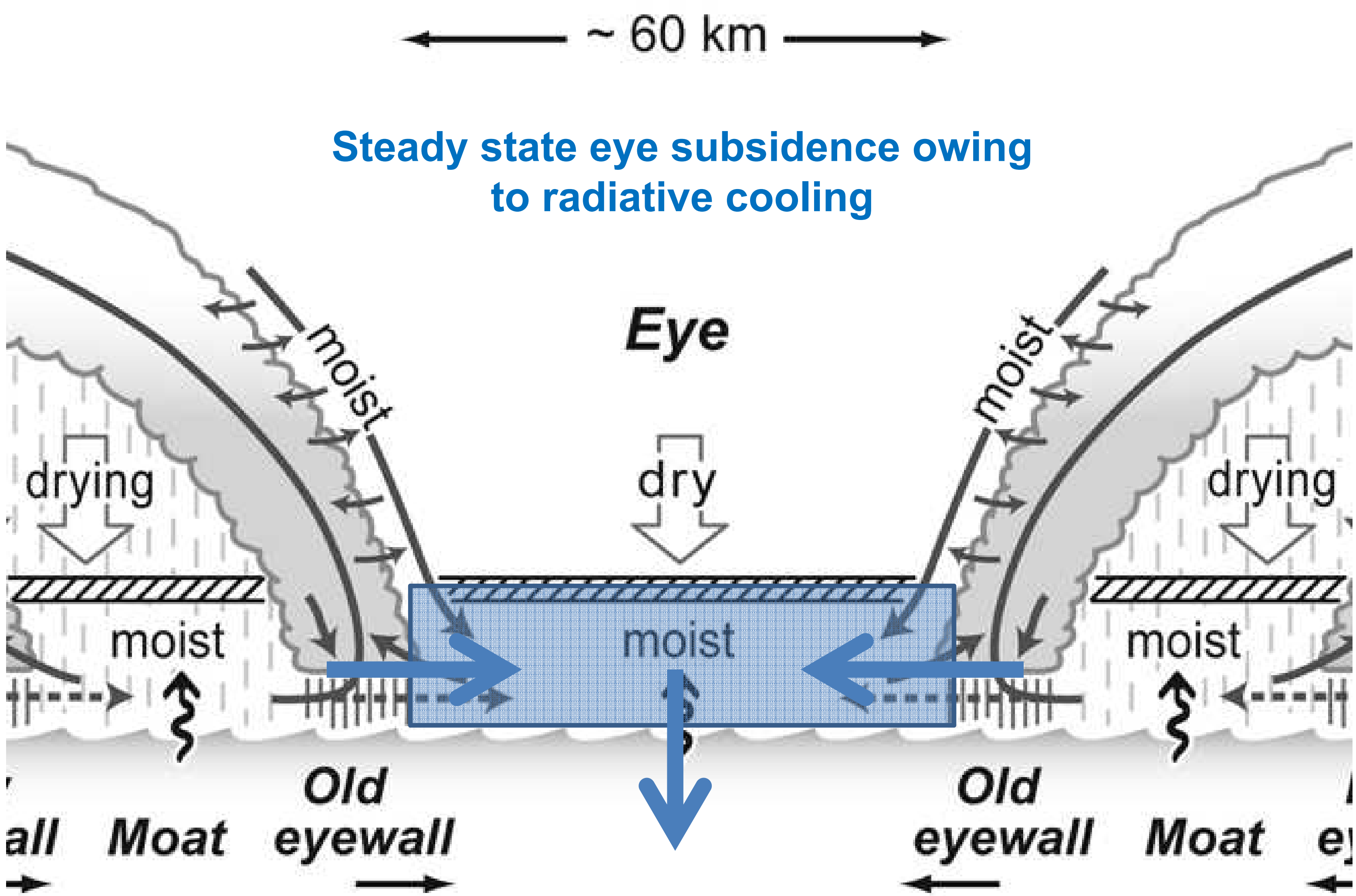
Three Regions

- Eye $(w < 0)$
- Inner Core $(w > 0)$
- Outer region $(w < 0)$

The Eye

Angular Momentum Budget in Eye Boundary Layer





Blue arrows show turbulent angular momentum fluxes. In eye PBL, radial M flux from eyewall must balance oceanic M sink.

Eye Angular Momentum Budget

$$h\frac{\partial M}{\partial t} \approx 0 = -uh\frac{\partial M}{\partial r} - C_D r\,|\mathbf{V}|\,V + \frac{h}{r}\frac{\partial}{\partial r}\left(r^3 \upsilon \frac{\partial}{\partial r}\left(\frac{V}{r}\right)\right)$$

horizontal advection

torque from surface drag

convergence of radial eddy flux of angular momentum

In steady state, azimuthal velocity profile must be concave

$$V \sim r^n,$$

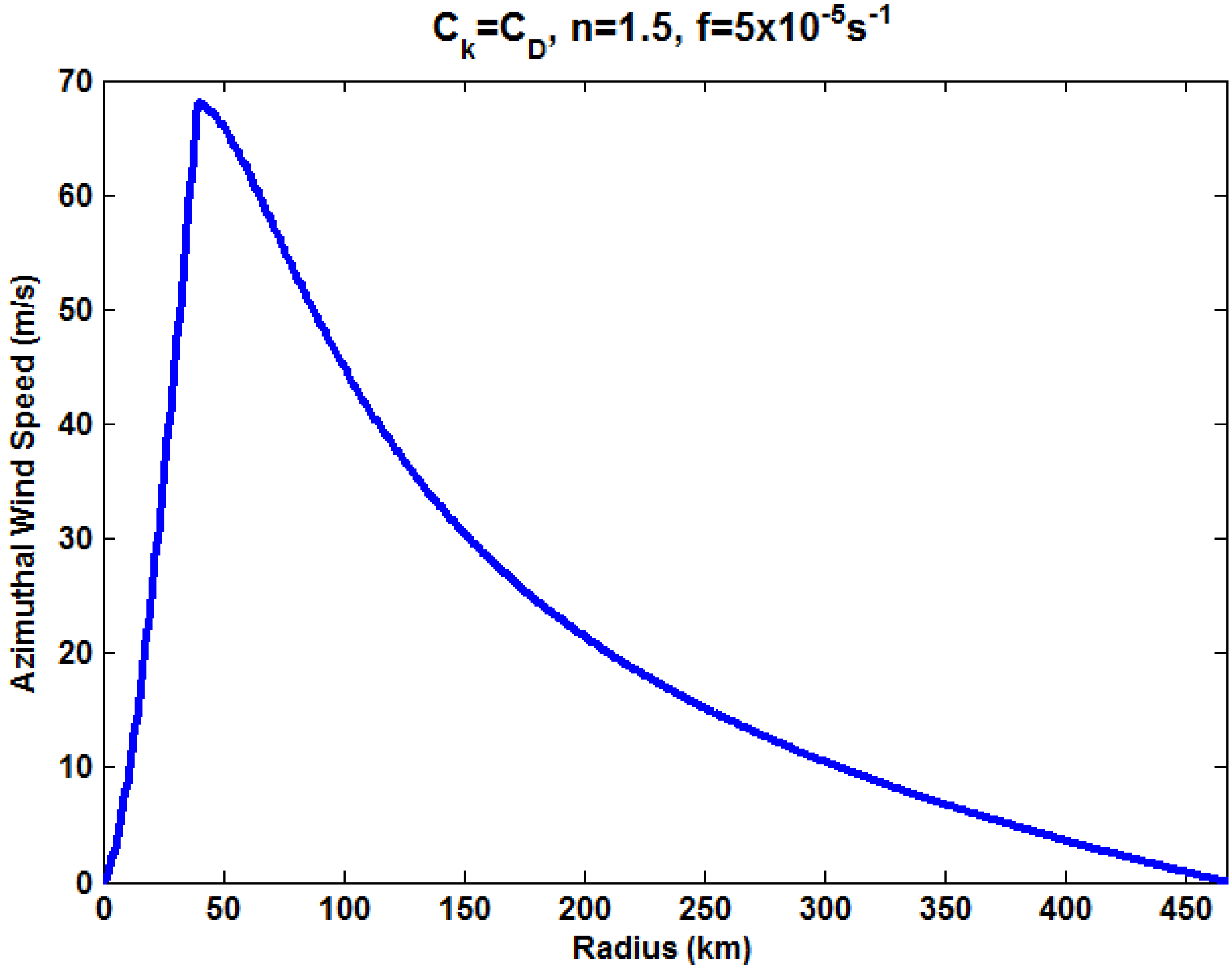
$$n > 1$$

Inner core:

$$\left(\frac{M}{M_m}\right)^{2-\frac{C_k}{C_D}} = \frac{2\left(\frac{r}{r_m}\right)^2}{2-\frac{C_k}{C_D}+\frac{C_k}{C_D}\left(\frac{r}{r_m}\right)^2},$$

$$V = M/r - \frac{1}{2}fr,$$

$$M_m \cong r_m V_m$$



$C_k=C_D$, n=1.5, f=5x10^{-5}s^{-1}
Azimuthal Wind Speed (m/s)
Radius (km)
0
10
20
30
40
50
60
70
0
50
100
150
200
250
300
350
400
450

Approximate PBL Radial Wind:

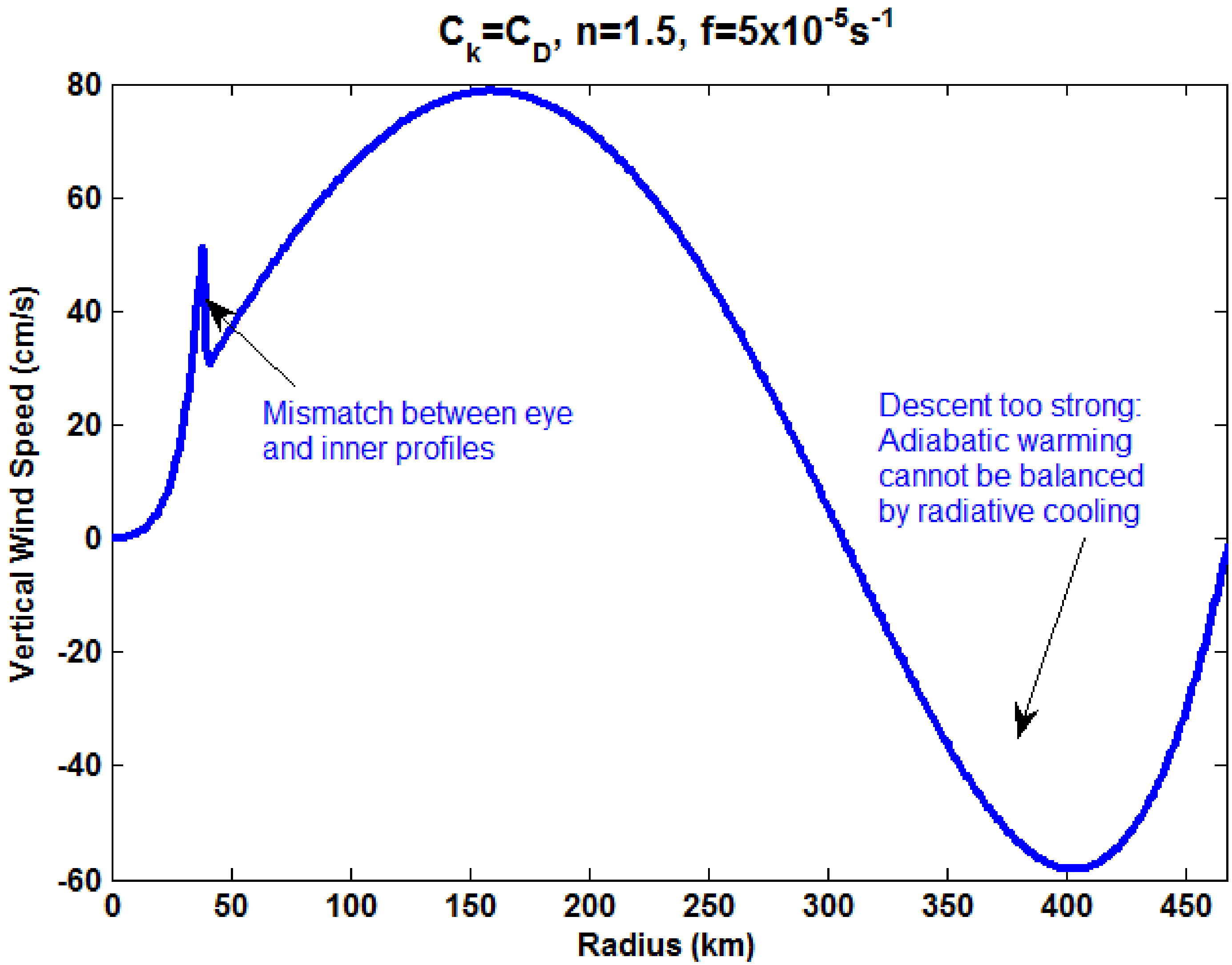
$$h\frac{\partial M}{\partial t} \approx 0 = -uh\frac{\partial M}{\partial r} - C_D r\,|\,\mathbf{V}\,|V$$

$$\rightarrow hu \approx \frac{C_D r\,|\,\mathbf{V}\,|V}{\dfrac{\partial M}{\partial r}}$$

Vertical Velocity:

$$w \simeq -\frac{1}{r}\frac{\partial}{\partial r}\left(rhu\right)$$

Vertical Velocity



Outer Region

Assume zero moist convection, so subsidence warming balances radiative cooling:

$$w = -w_{cool} \equiv \frac{-\dot{Q}_{rad}}{c_p \frac{T}{\theta} \frac{\partial \theta}{\partial z}}$$

Assume w_{cool} constant:

Integrate

$$w \simeq -\frac{1}{r}\frac{\partial}{\partial r}(rhu)$$

$$\rightarrow uh = -\frac{1}{2} w_{cool} \frac{\left(r_o^2 - r^2\right)}{r}$$

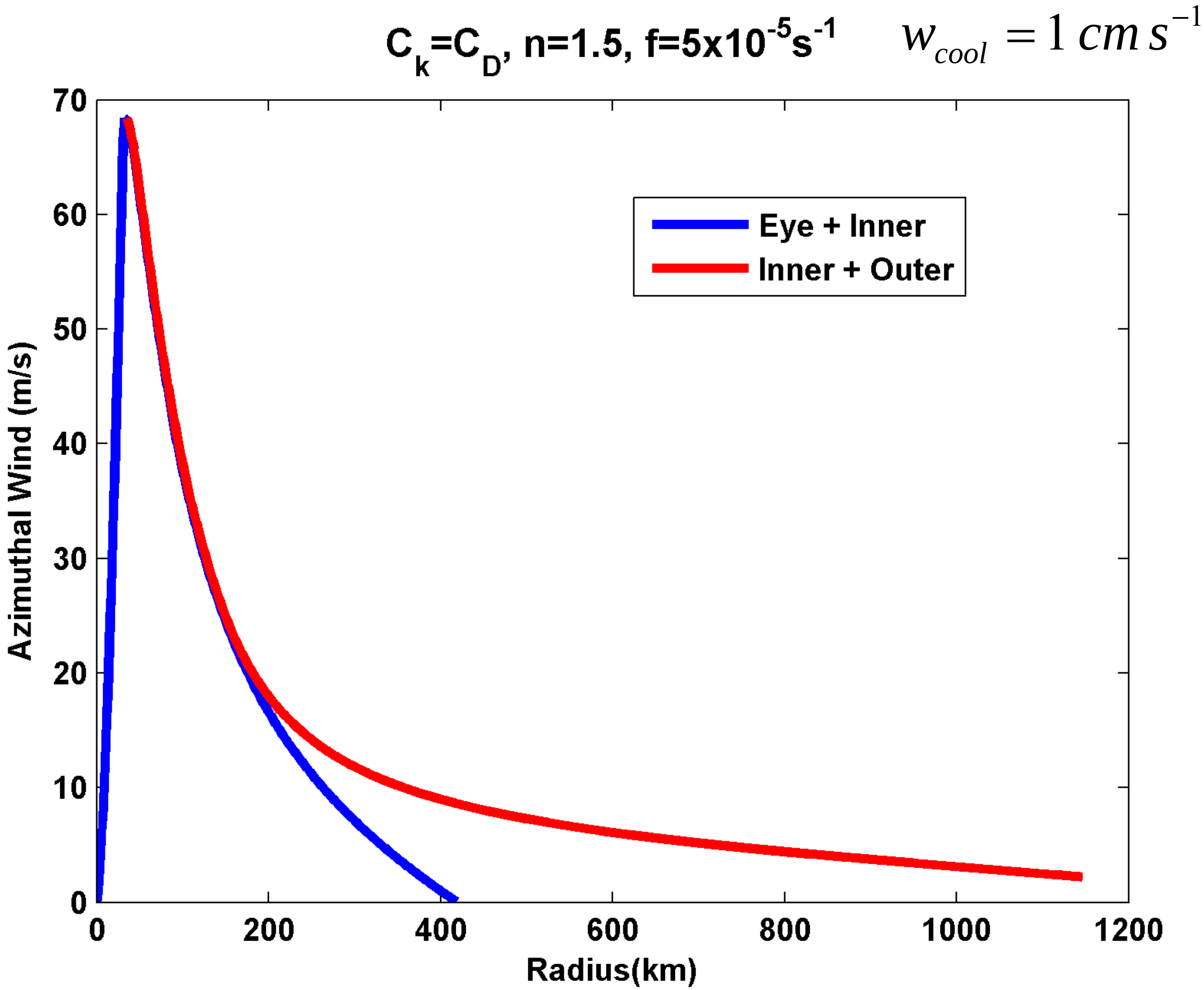
Now using $$h\frac{\partial M}{\partial t} \approx 0 = -uh\frac{\partial M}{\partial r} - C_D r\,|\,\mathbf{V}\,|\,V$$

and taking $$|\,\mathbf{V}\,| \simeq V$$

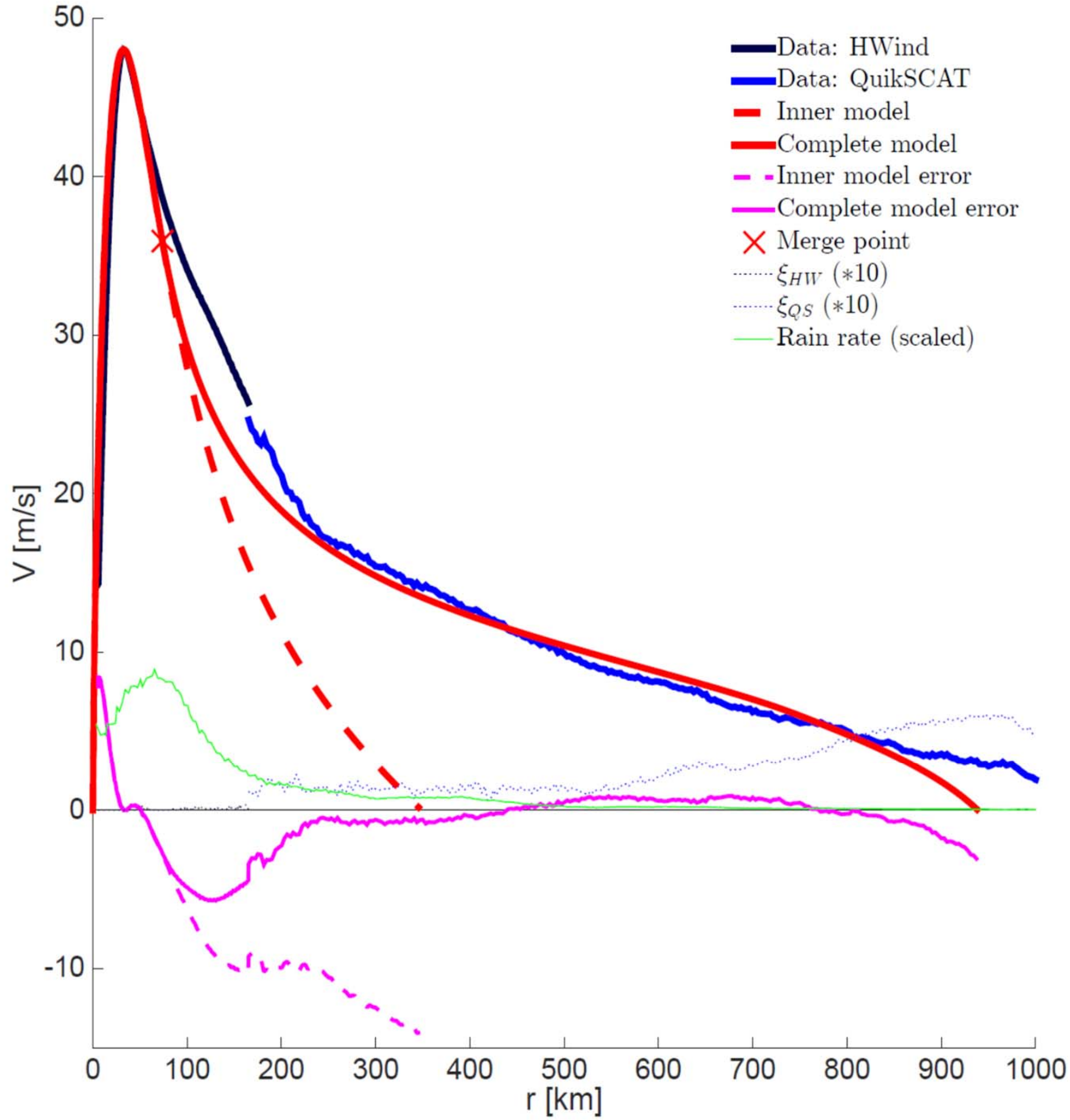
results in $$\frac{dm}{dr} = \frac{2C_D}{w_{cool}}\frac{m^2}{r_o^2 - r^2} - fr,$$

$$where \quad m \equiv rV$$

Integrate inward from $r = r_o$. No exact analytic solution.



$C_k=C_D$, n=1.5, $f=5x10^{-5}s^{-1}$ $w_{cool}=1\,cm\,s^{-1}$
Eye + Inner
Inner + Outer
Azimuthal Wind (m/s)
Radius(km)
0
10
20
30
40
50
60
70
0
200
400
600
800
1000
1200



Chavas et al., *J. Atmos*., 2015

Tropical Cyclone Inner Core Dynamics

Main Assumptions

- Axisymmetric flow
- Gradient and hydrostatic balance above PBL
- Troposphere neutral to slantwise moist convection outside eye

Assume that interior flow is always close to gradient and hydrostatic balance:

$$\frac{M}{r_b^2} = \frac{M}{r_o^2} - \left(T_b - T_o\right)\frac{ds^*}{dM}, \qquad (1)$$

where r_o is the radius of the angular momentum surface where its absolute temperature is T_o. Define that point as the point along the angular momentum surface at which the azimuthal velocity changes sign. At that point, by definition,

$$M = \frac{1}{2} f r_o^2$$

so (1) becomes:

$$r_b^2 = \frac{M}{\frac{1}{2} f - \left(T_b - T_o\right)\frac{ds^*}{dM}}. \qquad (2)$$

Also assume Richardson Number criticality of outflow:

$$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right) \qquad (3)$$

Conservation of entropy in PBL (in *M* coordinates):

$$\frac{\partial s_b}{\partial \tau} + \dot{M}\frac{\partial s_b}{\partial M} = g\frac{\partial F}{\partial P} + D, \qquad (4)$$

where $\dot{M}$ is the total time derivative of angular momentum, *F* is the vertical turbulent flux of entropy, and *D* represents the irreversible entropy sources of owing to kinetic energy dissipation, non-equilibrium evaporation of liquid water, and diffusion of water vapor.

$$\dot{M} = gr\frac{\partial \tau_\theta}{\partial P}$$

Substitute into (4) and integrate over depth of PBL:

$$\Delta p_b \frac{\partial s_b}{\partial \tau} + gr\tau_{\theta s}\frac{\partial s_b}{\partial M} = gF_s + \bar{D} \qquad (5)$$

$$F_s = \frac{C_k \rho \,|\,\mathbf{V}\,|\left(k_0^* - k_b\right)}{T_s} \cong C_k \rho \,|\,\mathbf{V}\,|\left(s_0^* - s_b\right)$$

$$\tau_{\theta s} = -C_D \rho \,|\,\mathbf{V}\,|\, V$$

$$\bar{D} \cong g\rho \frac{C_D \,|\,\mathbf{V}\,|^3}{T_s} \qquad \text{(Dissipative heating)}$$

(5) becomes

$$h\frac{\partial s_b}{\partial \tau} - C_D r\,|\mathbf{V}|\,V\frac{\partial s_b}{\partial M} = C_k\,|\mathbf{V}|\left(s_0^* - s_b\right) + C_D\frac{|\mathbf{V}|^3}{T_s}, \qquad (6)$$

$$h \equiv \frac{\Delta p_b}{\rho g}$$

Dealing with Eye Dynamics

Begin with equation for thermal wind balance:

$$V_{gb} = -r_b\left(T_b - T_o\right)\frac{ds^*}{dM_g}$$

Assume solid body rotation inside $r = r_{max}$:

$$V_{gb} = V_{max}\left(\frac{r_b}{r_{max}}\right)$$

Solve for temperature of eye:

$$\frac{ds^*}{dM_g} = \frac{-1}{T_s - T_t}\frac{V_{max}}{r_{max}} \qquad (7)$$

Note: Continue to solve (6) for s_b in eye, but this is less than (and decoupled from) s*

Complete System

$$h\frac{\partial s_b}{\partial \tau} - C_D r\,|\mathbf{V}|\,V\frac{\partial s_b}{\partial M} = C_k\,|\mathbf{V}|\left(s_0^* - s_b\right) + C_D\frac{|\mathbf{V}|^3}{T_s} \quad (8)$$

$$r_b^2 = \frac{M}{\frac{1}{2}f - \left(T_b - T_o\right)\frac{ds^*}{dM}} \quad (9)$$

$$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right) \quad (10)$$

$$V = M\big/ r_b - \frac{1}{2} f r_b \quad (11)$$

$$Also,\ T_o = T_t\ at\ r = r_m, \quad s_b = s^*\ except\ in\ eye, where\ (7)\ applies.$$

MATLAB code of model (very fast!)

ftp://texmex.mit.edu/pub/emanuel/scripts/smodel_public.m

Approximate System

- Neglect pressure dependence of s_0^*
- V~M/r   (inner core)
- Neglect dissipative heating
- $|\mathbf{V}| \sim V$
- *h=constant*

$$V^2 = -(T_b - T_o) M \frac{\partial s^*}{\partial M}, \tag{12}$$

$$\frac{\partial T_o}{\partial M} = -\frac{Ri_c}{r_t^2}\left(\frac{\partial s^*}{\partial M}\right)^{-1}, \tag{13}$$

$$h\frac{\partial s^*}{\partial \tau} - C_D V M \frac{\partial s^*}{\partial M} = C_k V\left(s_0 - s^*\right). \tag{14}$$

Differentiate (14) with respect to *M*, and substitute from (12) and (13):

$$\frac{(T_b - T_o)h}{V} \frac{\partial}{\partial \tau}\left(\frac{V^2}{T_b - T_o}\right) = \frac{M}{V}\frac{\partial V}{\partial M}\left[3C_D V^2 - C_k (T_b - T_o)(s_0 - s^*)\right]$$

$$+ C_D \frac{Ri_c}{r_t^2} M^2 - C_k V^2. \qquad (15)$$

Suppose that maximum winds always occur on the same M surface. Then, using

$$M_{max} \simeq r_{max} V_{max} \; and \; r_{max}^2 = r_t^2 \frac{C_k}{C_D} Ri_c^{-1},$$

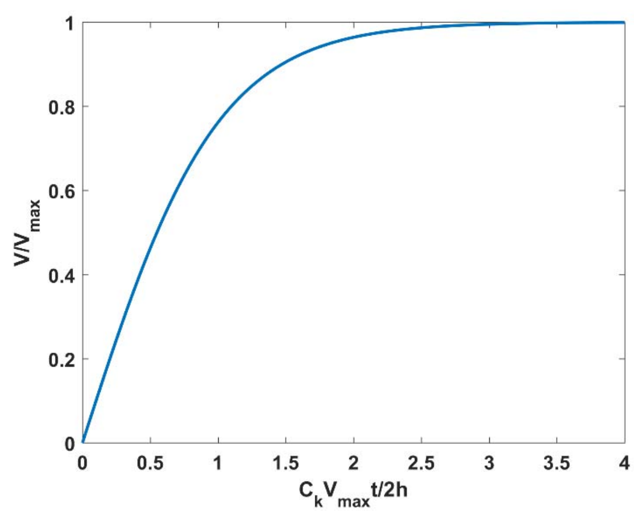
$$\frac{\partial V_m}{\partial \tau} \cong \frac{C_k}{2h}\left(V_{max}^2 - V_m^2\right) \qquad (16)$$

From previous lecture,

$$V_{max}^2 = \frac{C_k}{C_D}\left(\frac{1}{2}\frac{C_k}{C_D}\right)^{\frac{\frac{C_k}{C_D}}{2-\frac{C_k}{C_D}}}\left(T_b - T_t\right)\left(s_0 - s_e^*\right) \qquad (17)$$

If $V = 0$ at $t = 0$, the integration of (16) gives

$$V_m(\tau) = V_{max}\tanh\left(\frac{C_k V_{max}}{2h}\tau\right) \qquad (18)$$



Note that first on right of (15) term steepens *V* gradient when

$$V^2 > \frac{1}{3}\frac{C_k}{C_D}\left(T_b - T_o\right)\left(s_0 - s^*\right)$$

V gradient cannot steepen indefinitely:

$$\varsigma = \frac{V}{r} + \frac{\partial V}{\partial r},$$

$$\frac{\partial}{\partial r} = \frac{\partial M}{\partial r}\frac{\partial}{\partial M} = r\left(f + \frac{V}{r} + \frac{\partial V}{\partial r}\right)\frac{\partial}{\partial M} = r\left(f + \varsigma\right)\frac{\partial}{\partial M}$$

$$\rightarrow \varsigma = \frac{V}{r} + r\left(f + \varsigma\right)\frac{\partial V}{\partial M}$$

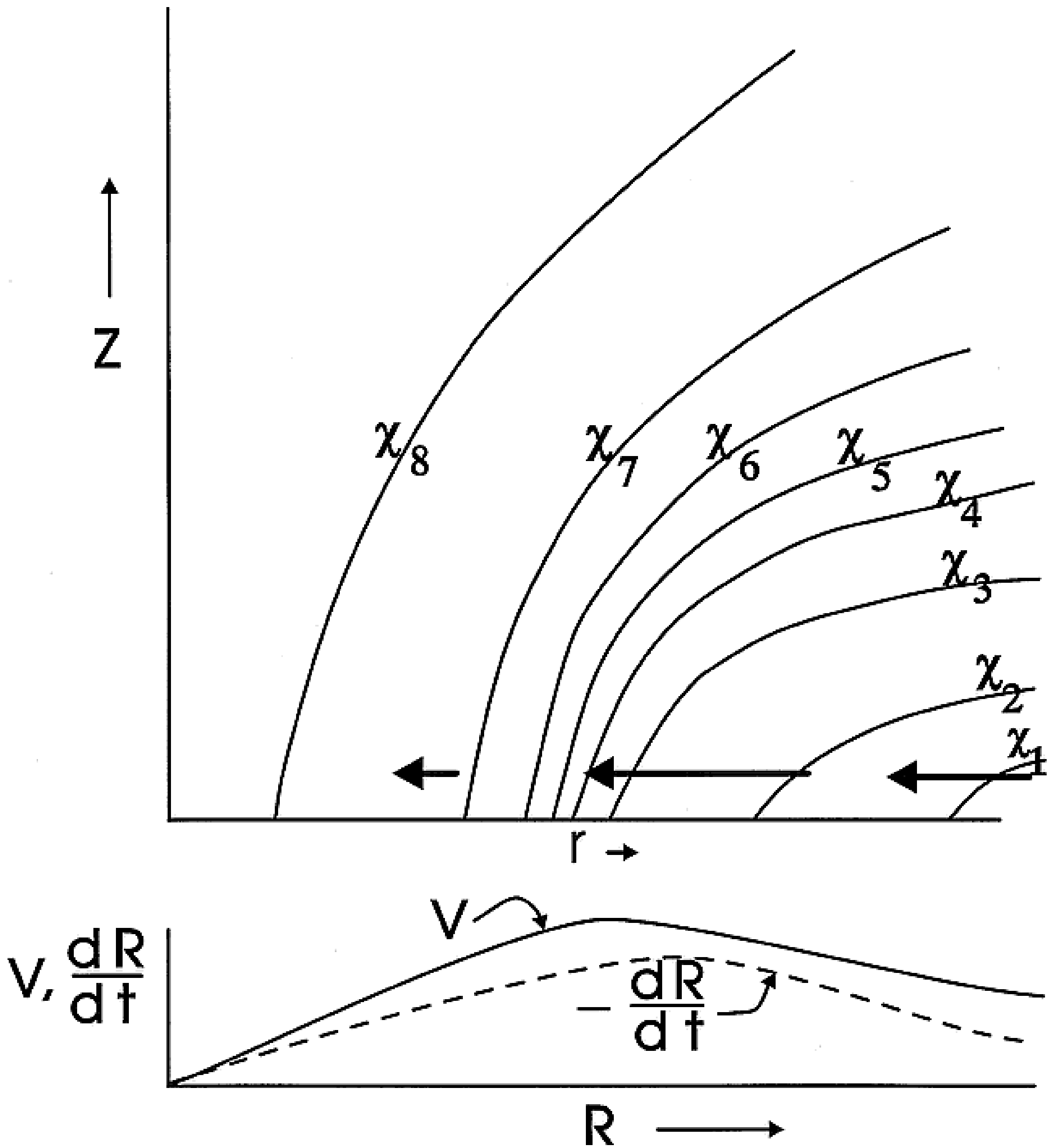
$$\varsigma = \frac{\frac{V}{r} + fr\frac{\partial V}{\partial M}}{1 - r\frac{\partial V}{\partial M}}$$

$$\varsigma \rightarrow \infty \quad when \quad \frac{\partial V}{\partial M} \rightarrow \frac{1}{r} \cong \frac{V}{M}$$

Eyewall undergoes frontal collapse!

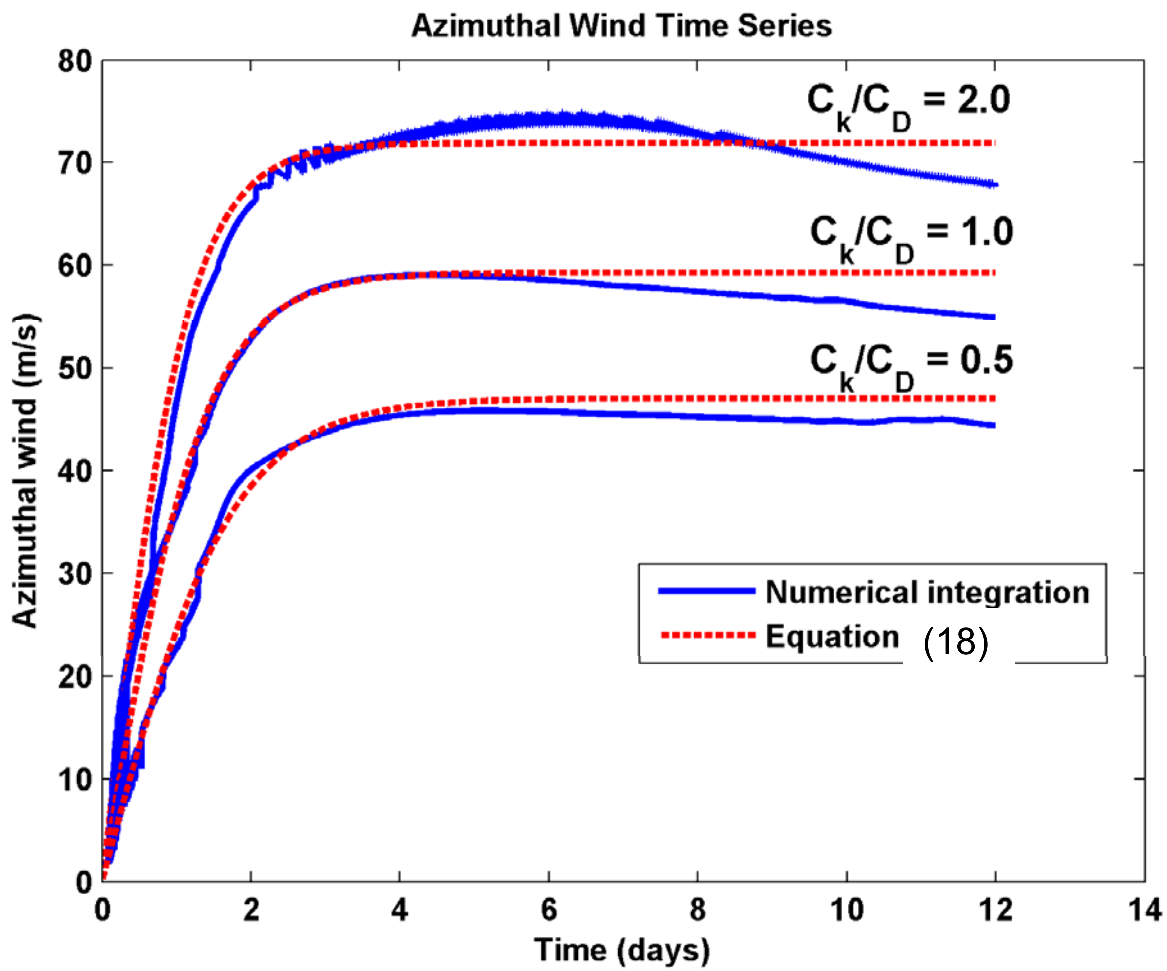
This can only be prevented by 3-D eddies*. In the present model, we prevent frontal collapse by insisting on solid body rotation everywhere inside* r_{max}.

$$\chi = s^*$$

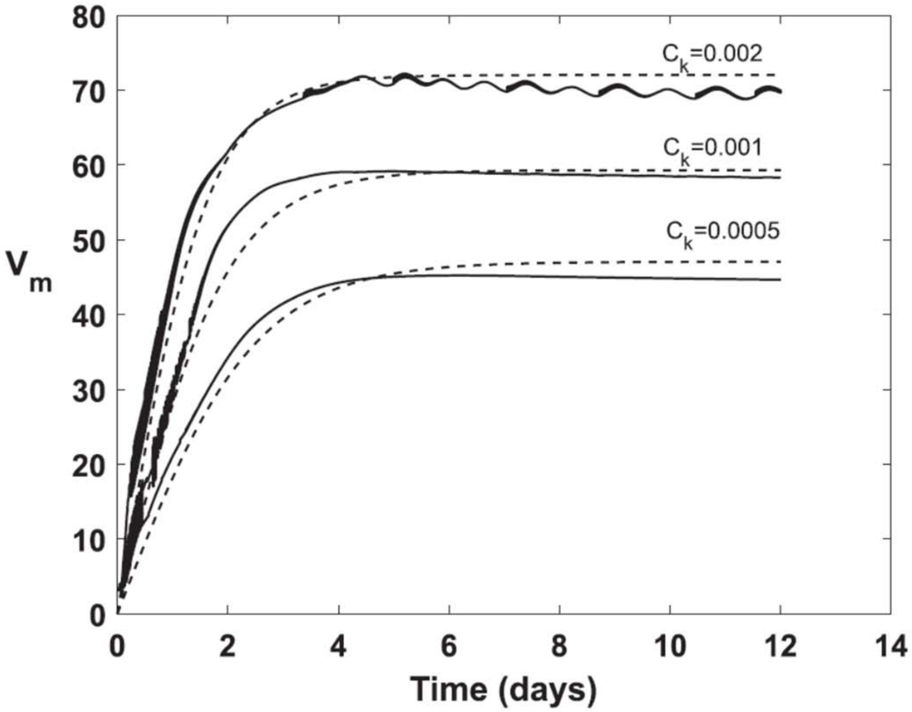


$$\dot{M} \simeq -MV$$

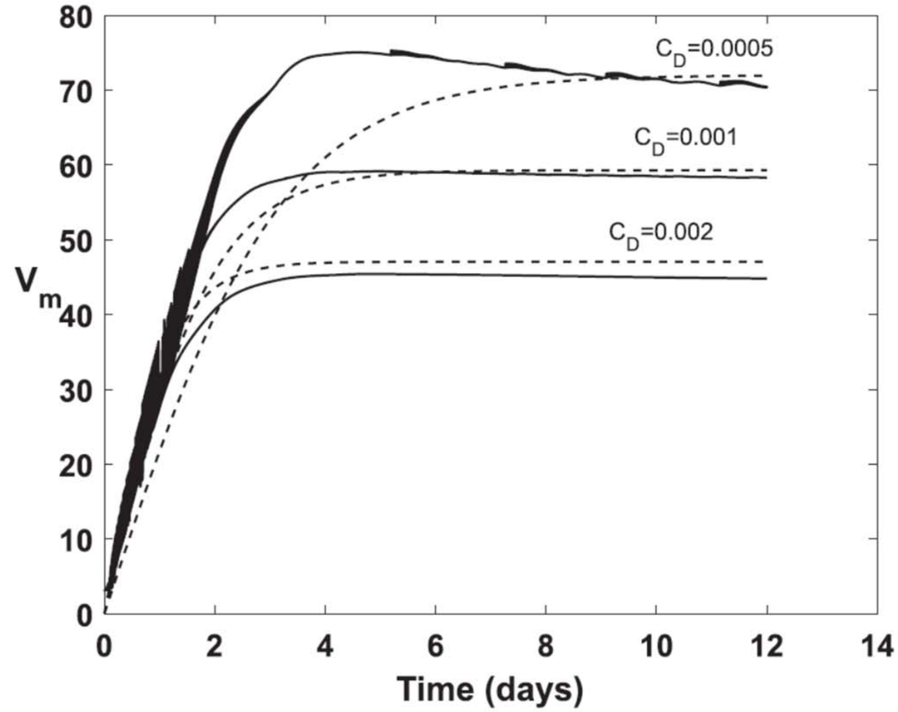
Comparison with numerical solution of (7) – (11)

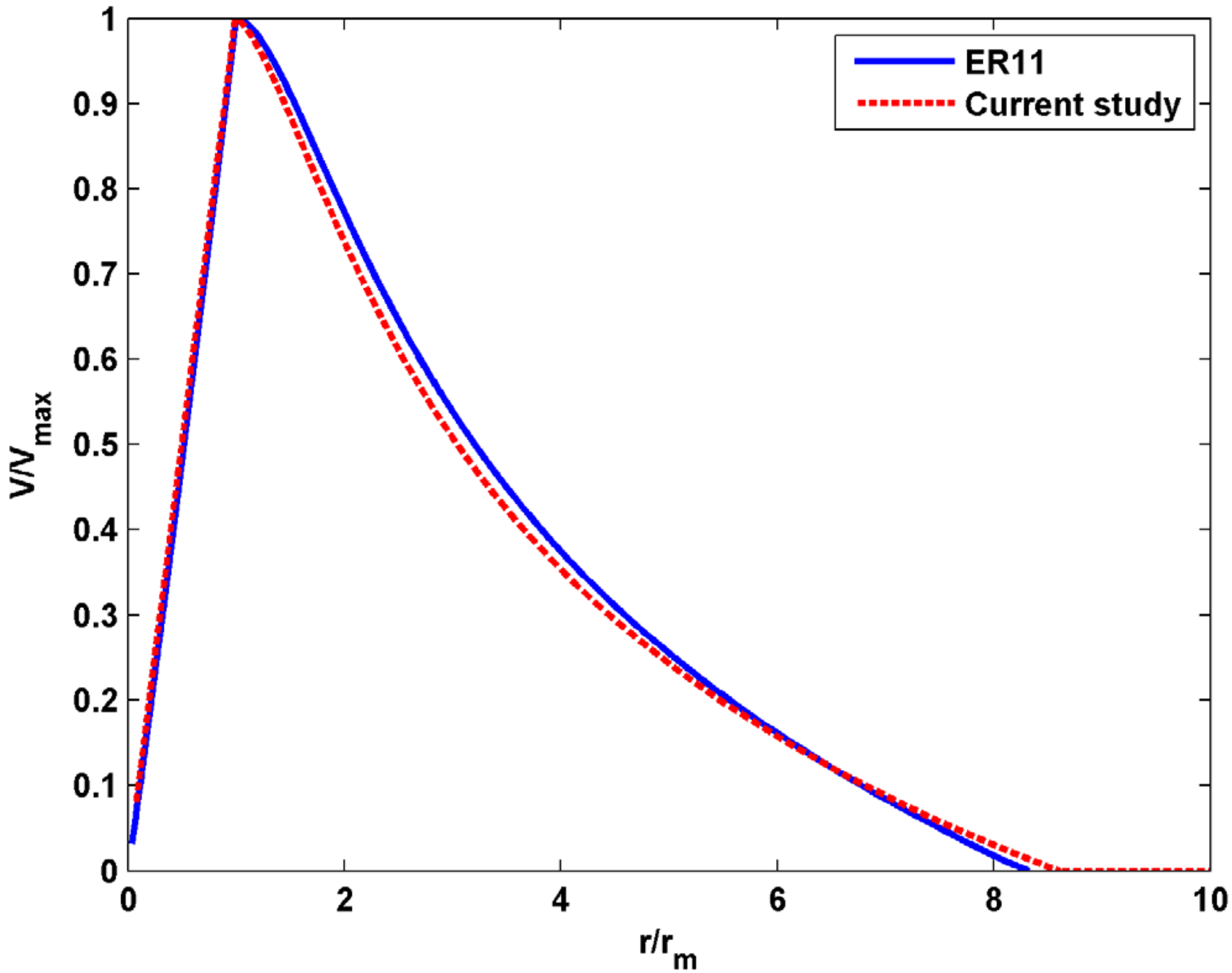


Varying C_k



Varying C_D





ER11
Current study
V/V$_{max}$
r/r$_m$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
2
4
6
8
10

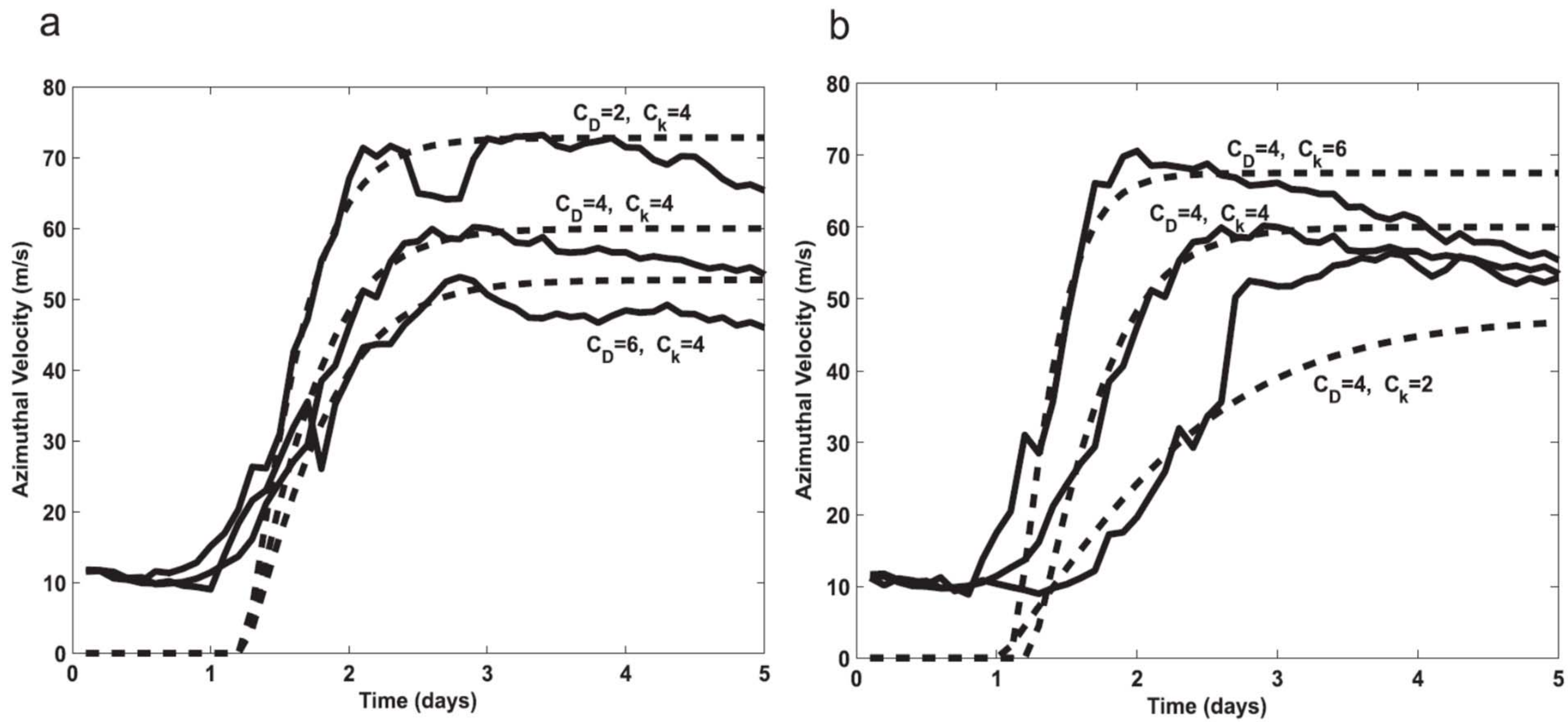


FIG. 3. Evolution with time of the domain maximum azimuthal wind in simulations using the RE87 model (solid), compared with solutions of (19) (dashed), varying (a) the drag coefficient and (b) the enthalpy exchange coefficient. Curves are labeled with the values of the exchange coefficients in units of 10^{-3}.