Tropical Cyclone Axisymmetric Physics

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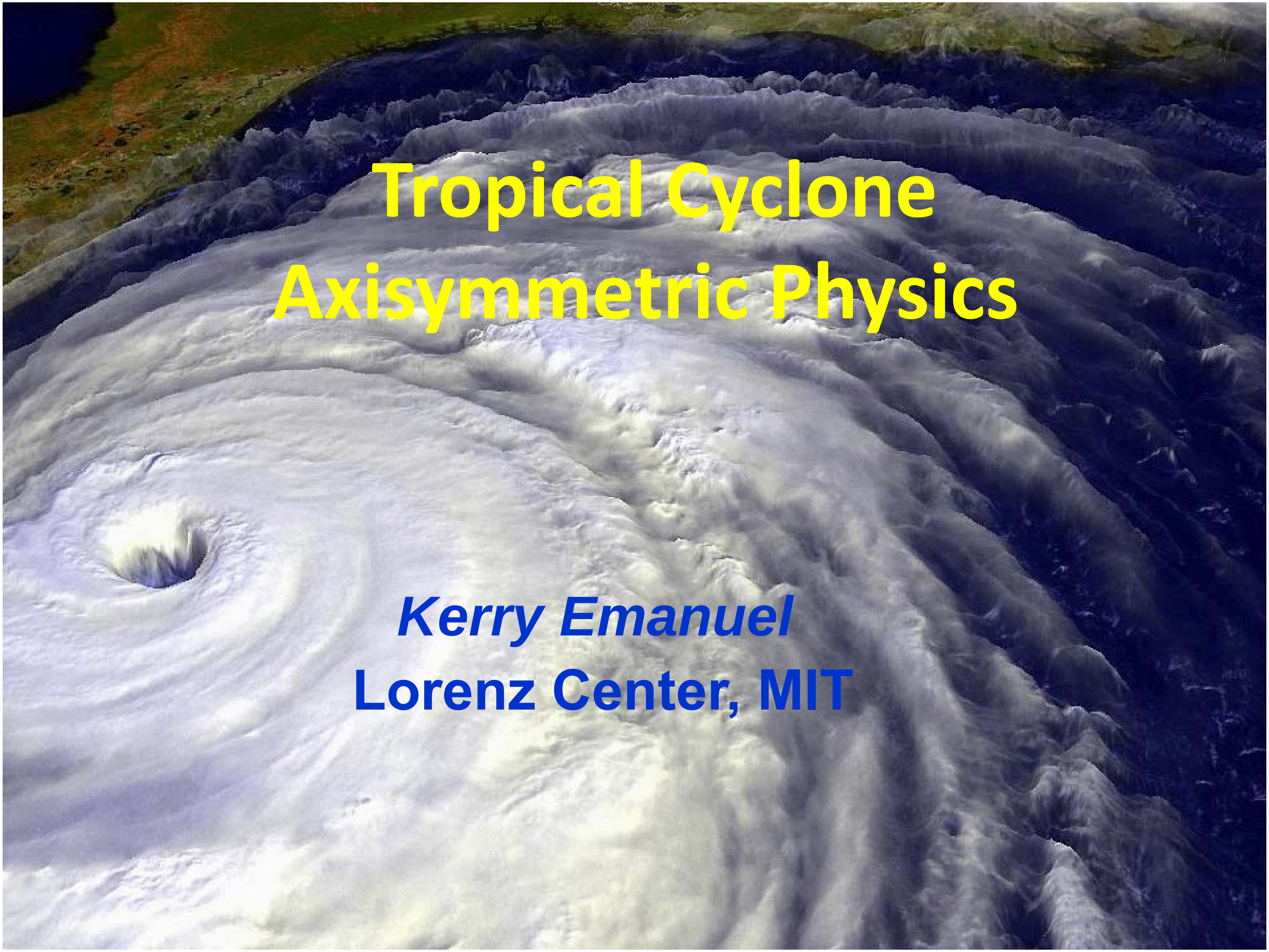


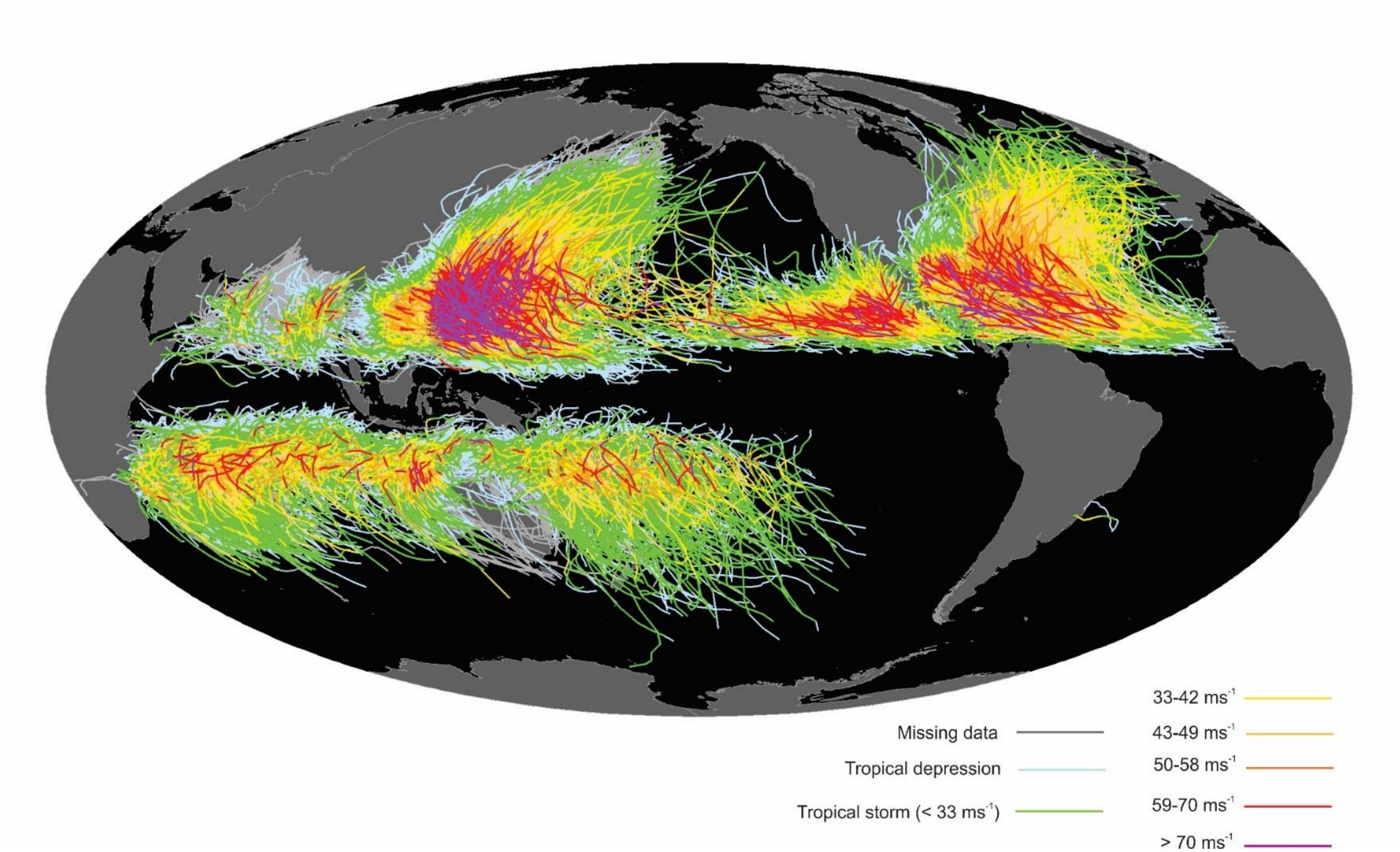
Program

- Brief Overview
- Steady-state energetics and physics
- Structure
- Intensification physics

Overview: What is a Tropical Cyclone?

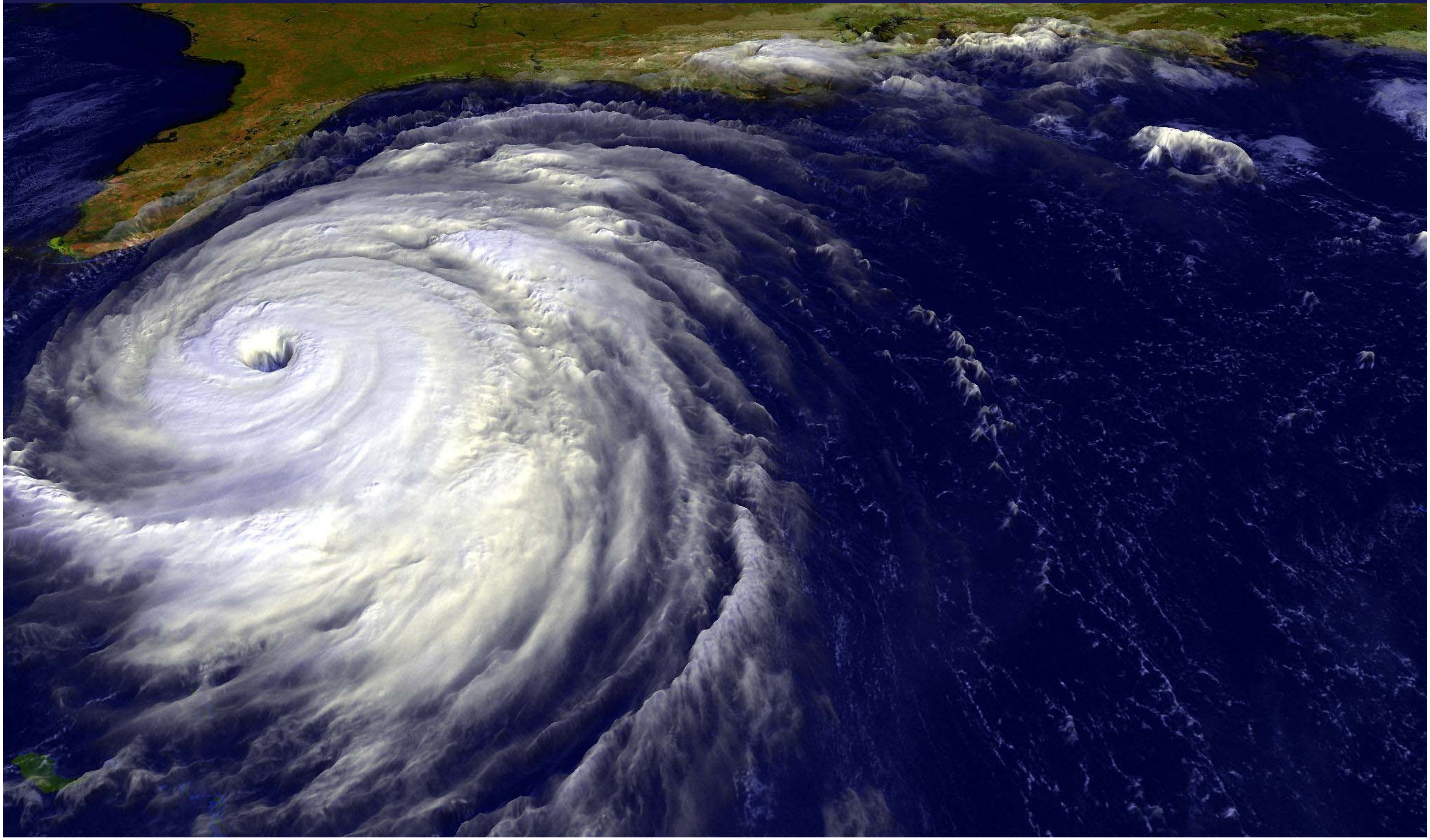
A *tropical cyclone* is a nearly symmetric, warm-core cyclone powered by wind-induced enthalpy fluxes from the sea surface

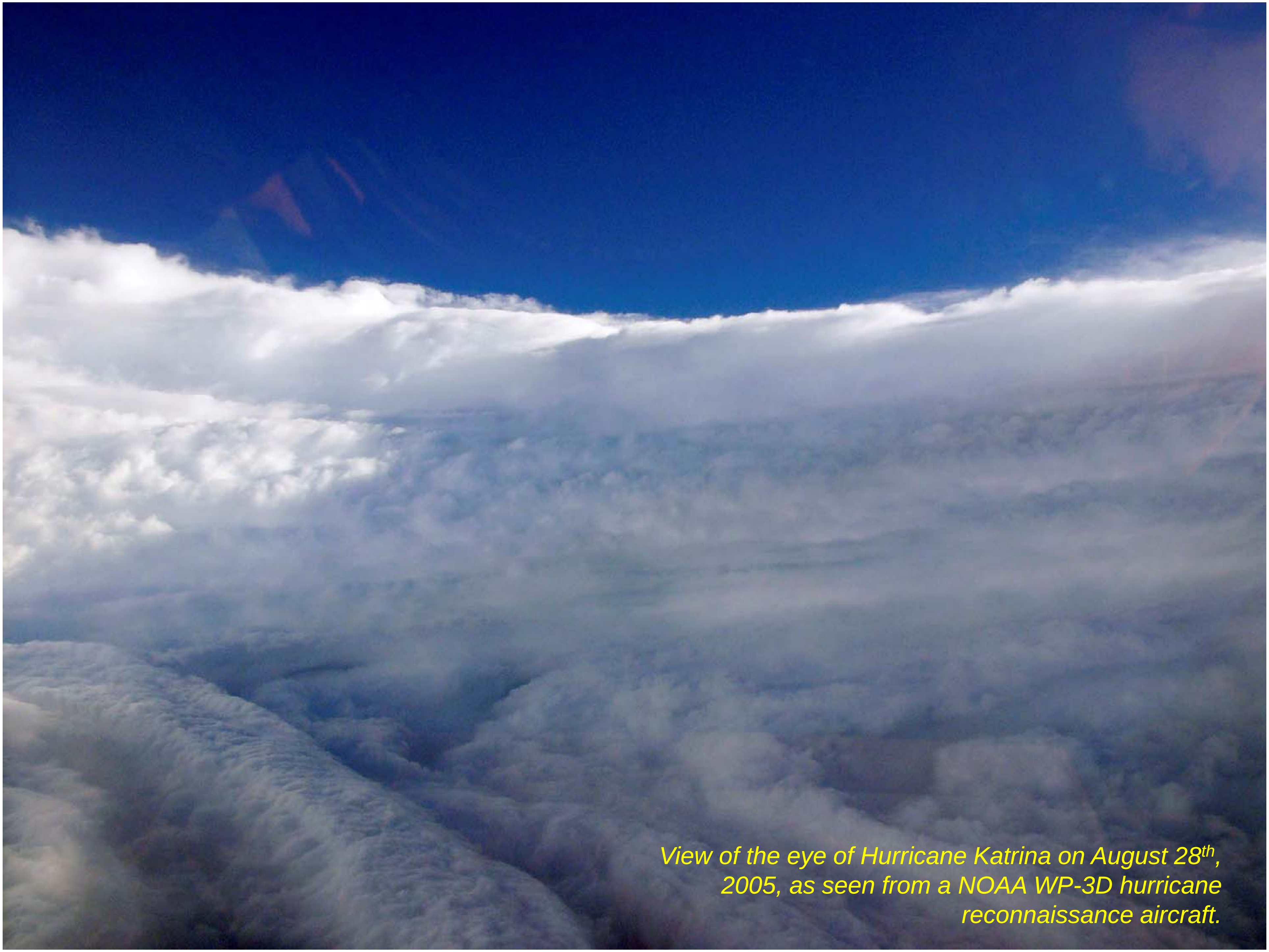
Global Climatology



Tracks of all tropical cyclones in the historical record from 1851 to 2010. The tracks are colored according to the maximum wind at 10 m altitude, on the scale at lower right.

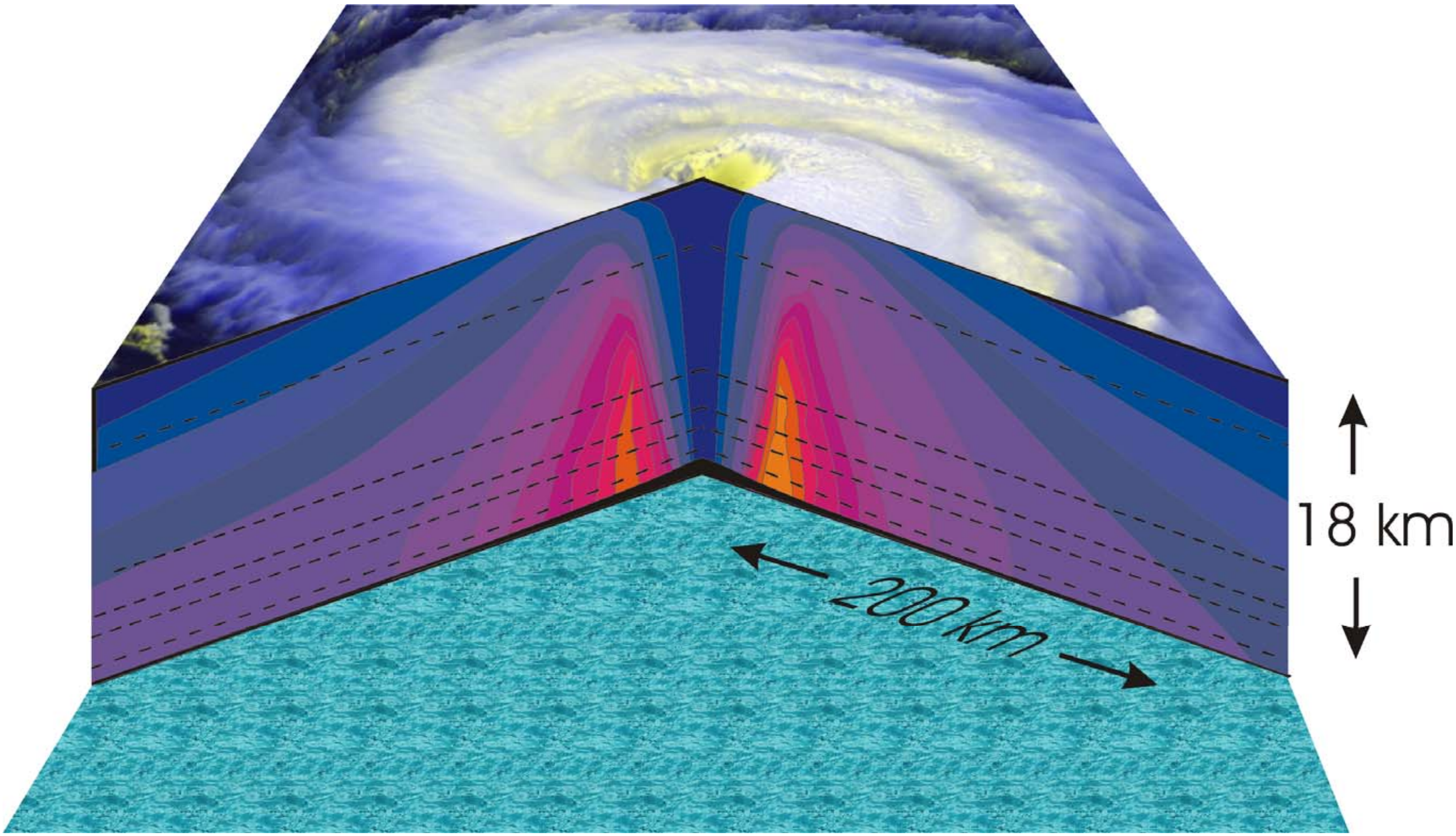
The View from Space





View of the eye of Hurricane Katrina on August 28th, 2005, as seen from a NOAA WP-3D hurricane reconnaissance aircraft.

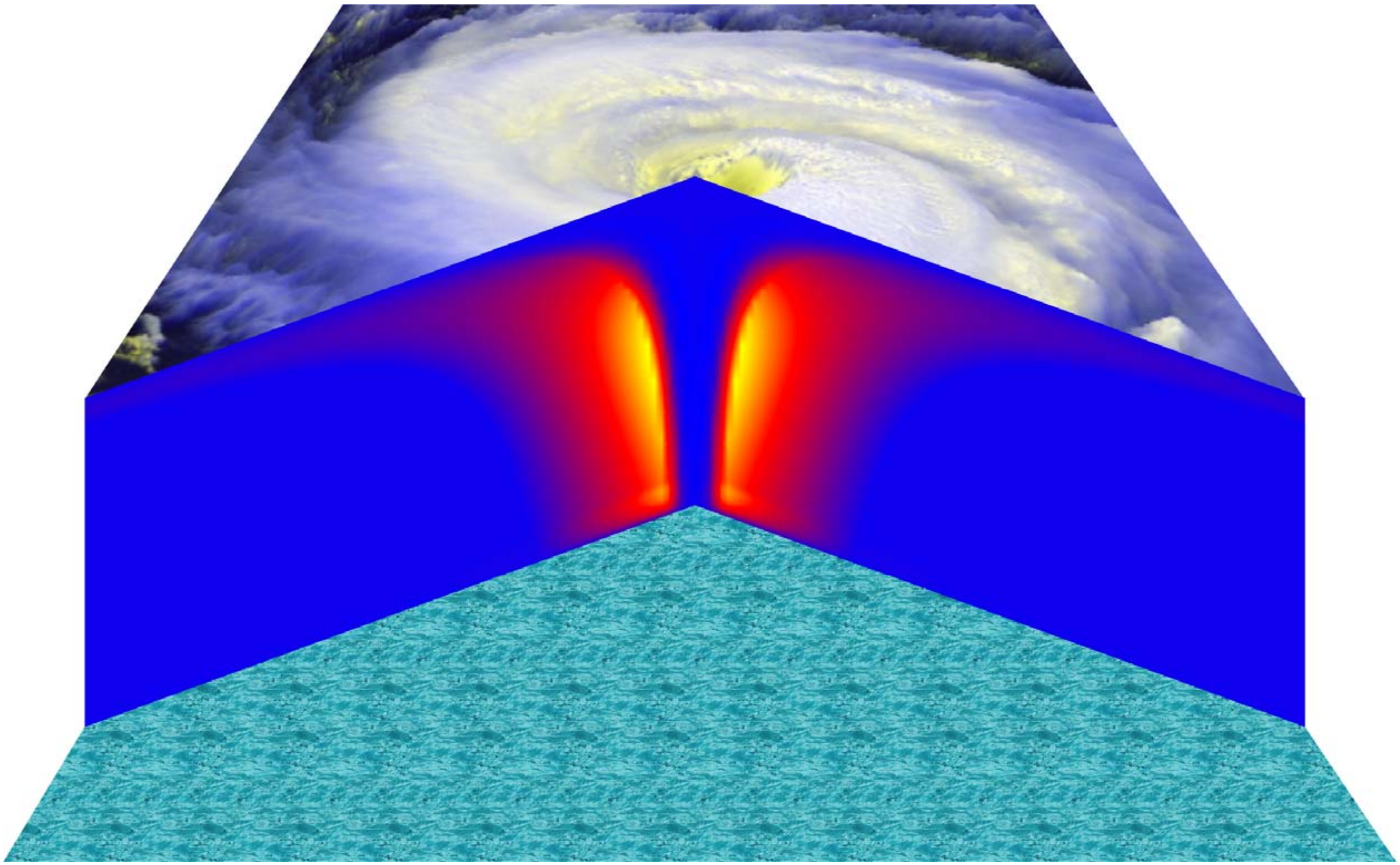
Hurricane Structure: Wind Speed



Azimuthal component of wind

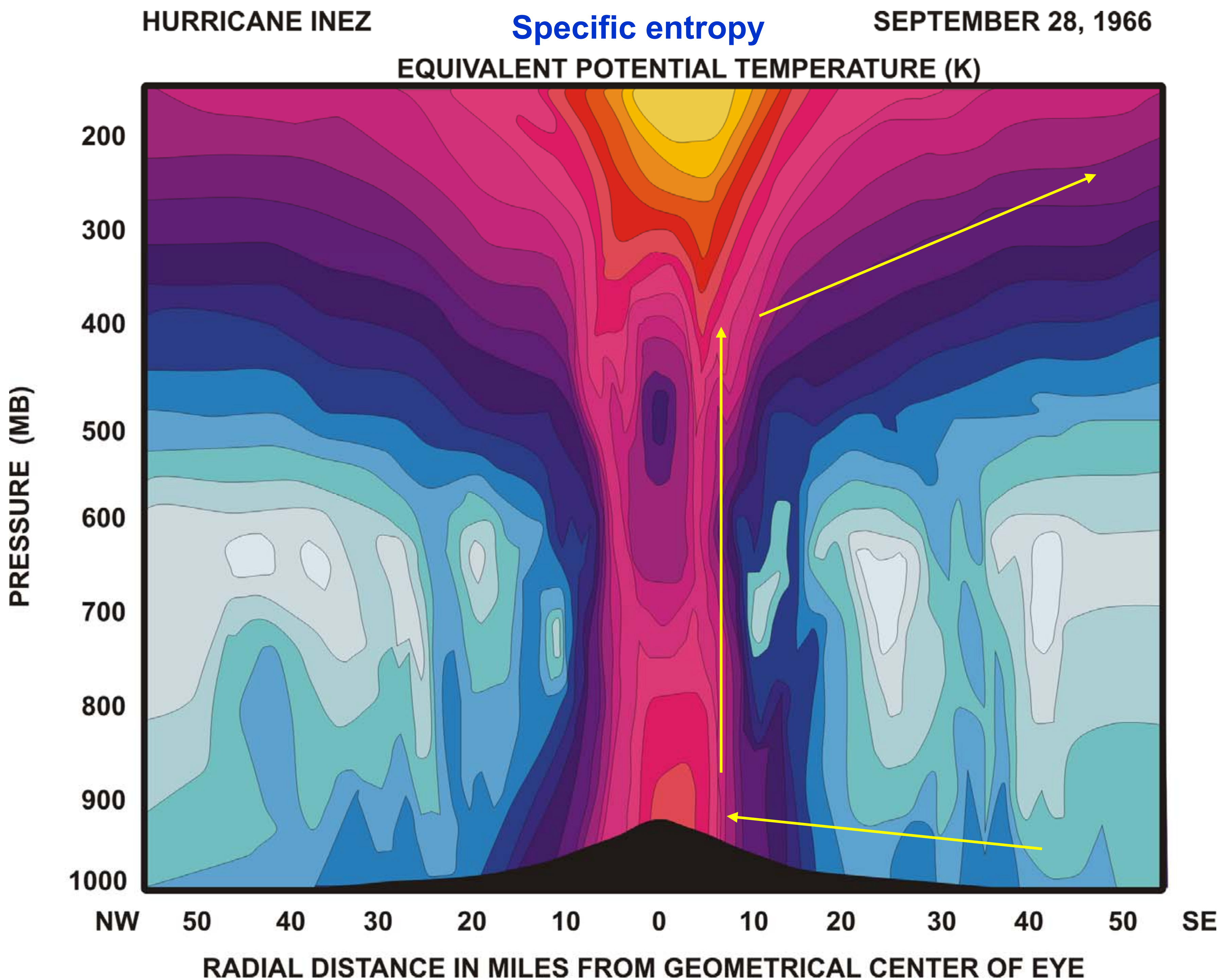
< 11 5 ms^{-1} - > 60 ms^{-1}

Vertical Air Motion



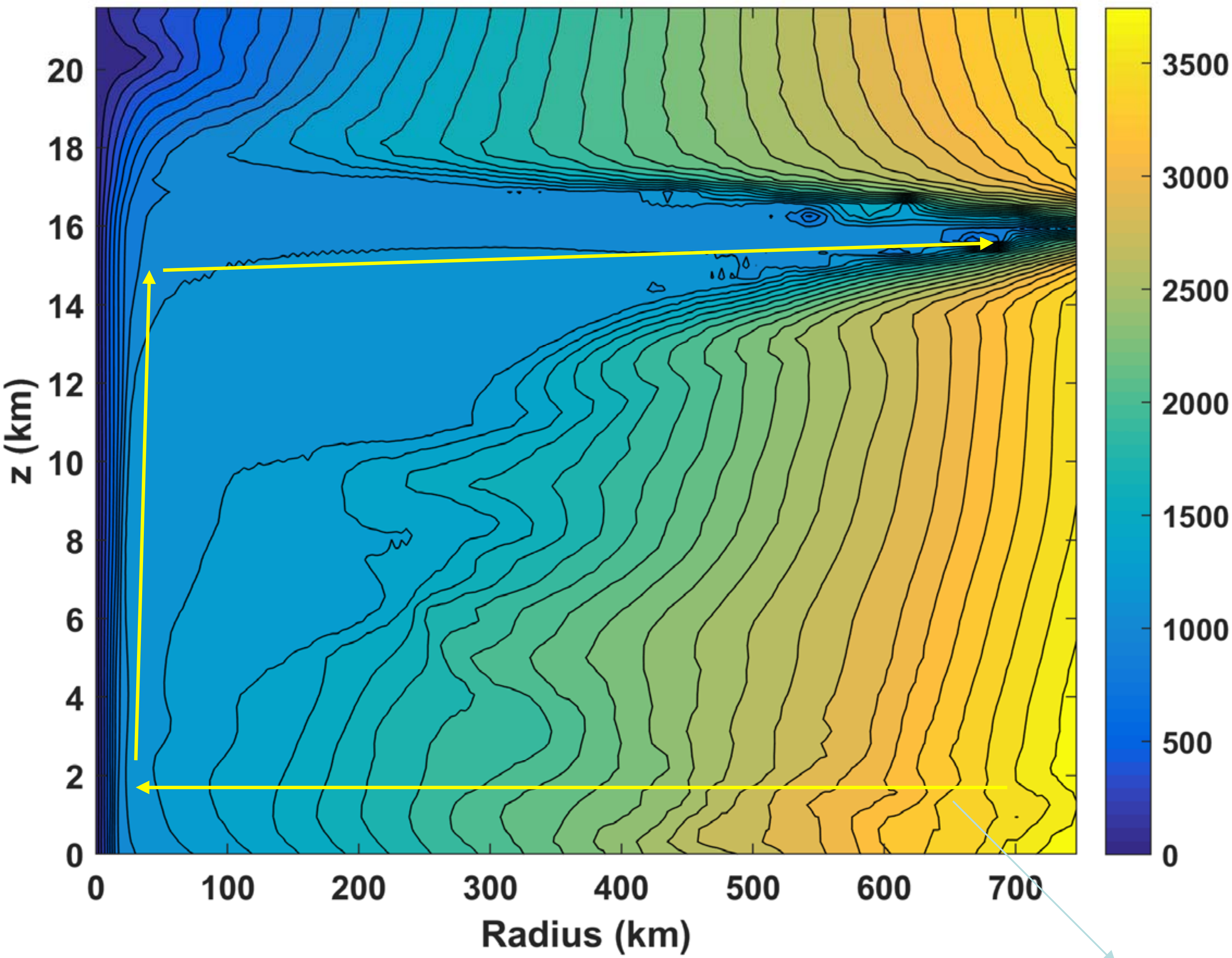
Updraft Speed

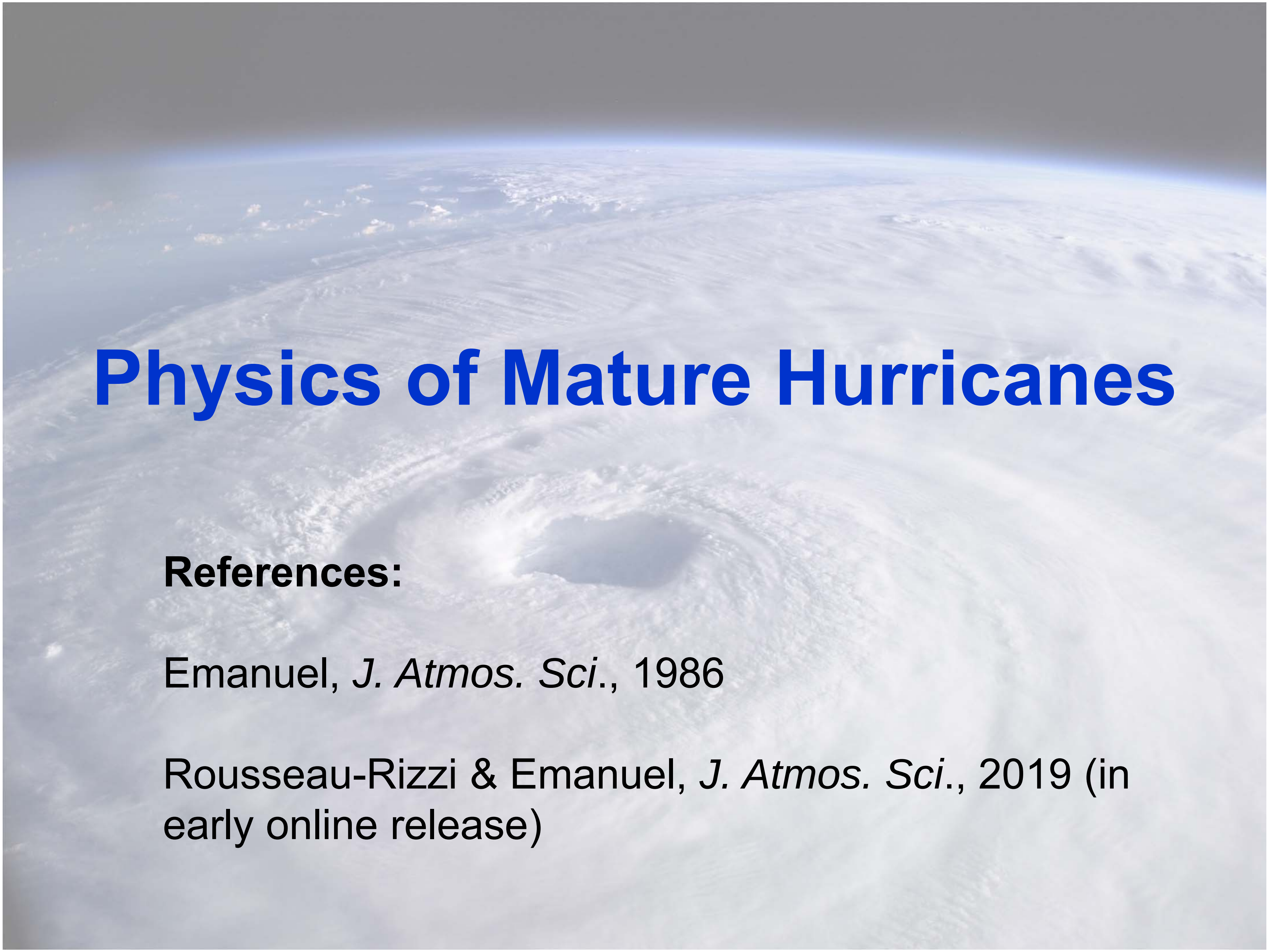
Strong upward motion in the eyewall


HURRICANE INEZ
Specific entropy
SEPTEMBER 28, 1966
EQUIVALENT POTENTIAL TEMPERATURE (K)
PRESSURE (MB)
200
300
400
500
600
700
800
900
1000
NW
50
40
30
20
10
0
10
20
30
40
50
SE
RADIAL DISTANCE IN MILES FROM GEOMETRICAL CENTER OF EYE

Absolute angular momentum per unit mass

$$M = rV + \Omega r^2$$





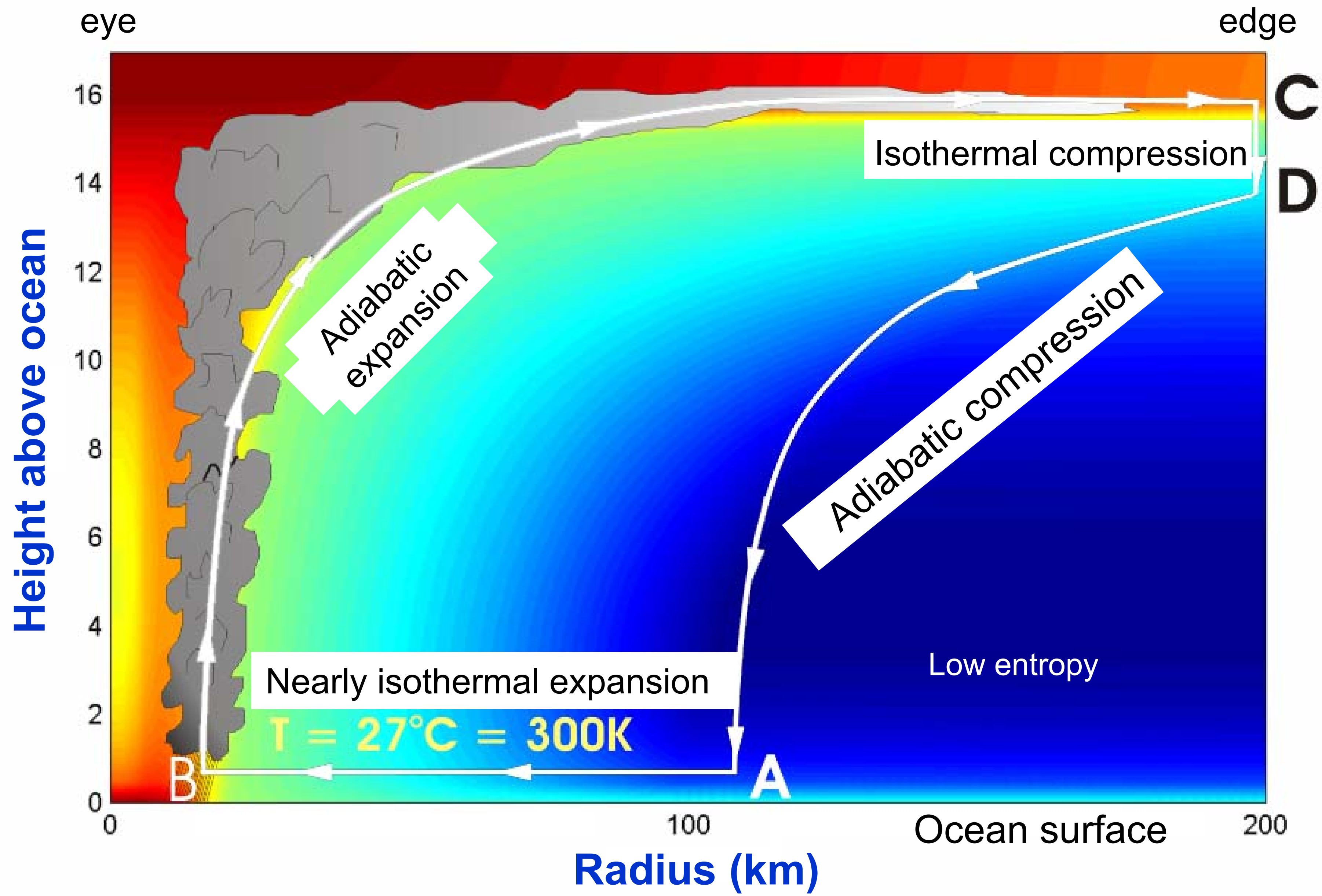
Physics of Mature Hurricanes

References:

Emanuel, *J. Atmos. Sci*., 1986

Rousseau-Rizzi & Emanuel, *J. Atmos. Sci*., 2019 (in early online release)

Cross-section through a Hurricane & Energy Production



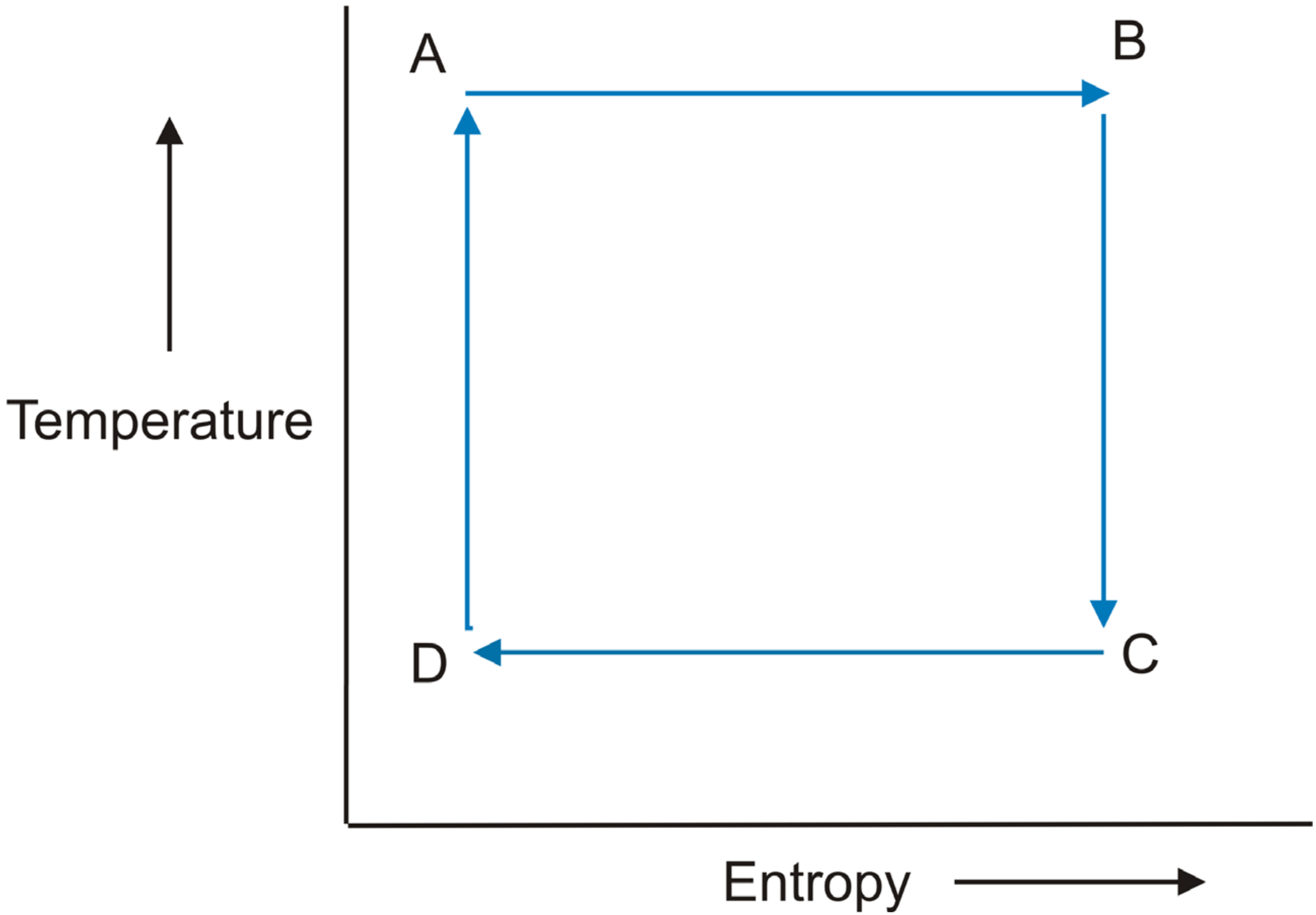
Carnot Theorem: Maximum efficiency results from a particular energy cycle:

- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Note: Last leg is not adiabatic in hurricanes: Air cools radiatively. But since the environmental temperature profile is moist adiabatic, the amount of radiative cooling is the same as if air were saturated and descending moist adiabatically.

Maximum rate of energy production:

$$P = \frac{T_s - T_o}{T_s} \dot{Q}$$


A
B
C
D
Temperature
Entropy

Total rate of heat input to hurricane:

$$\dot{Q} = 2\pi \int_0^{r_0} \rho \left[C_k |\mathbf{V}| \left(k_0^* - k \right) + C_D |\mathbf{V}|^3 \right] r dr$$

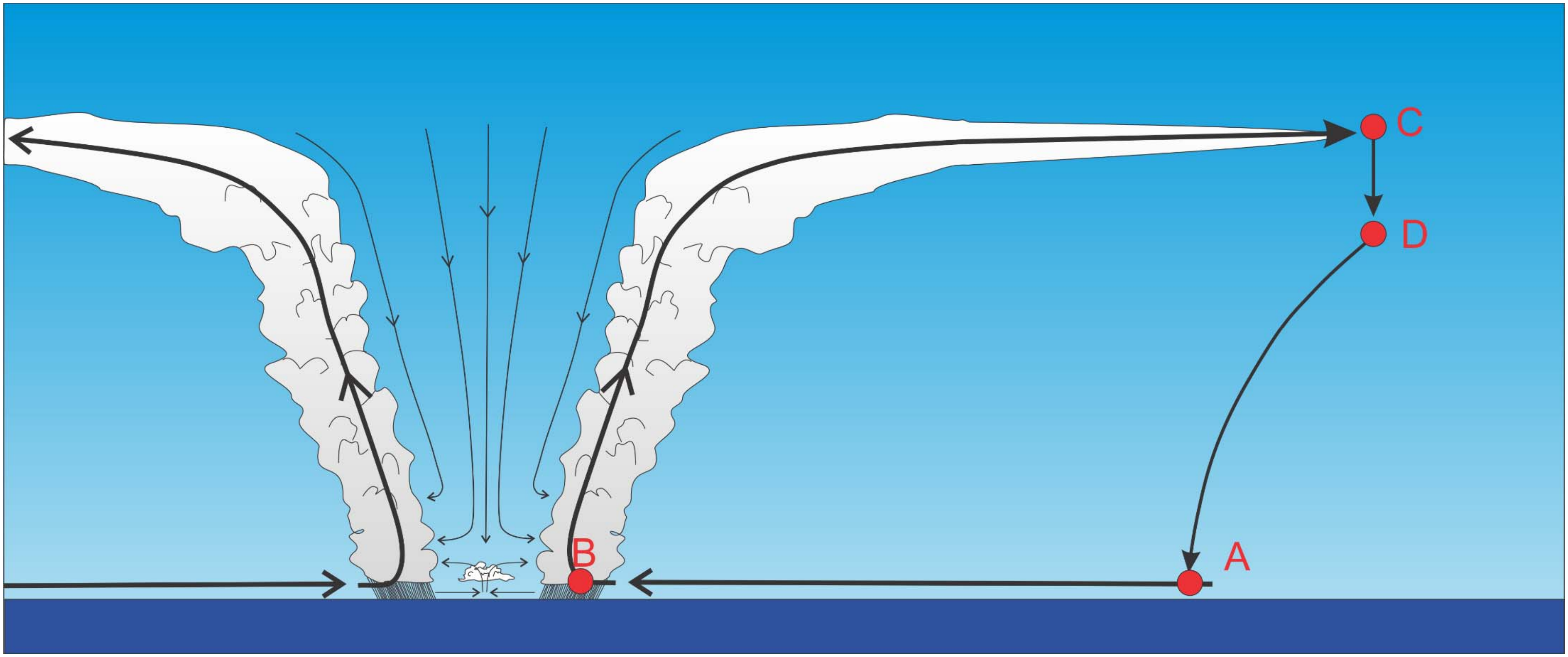
Surface enthalpy flux

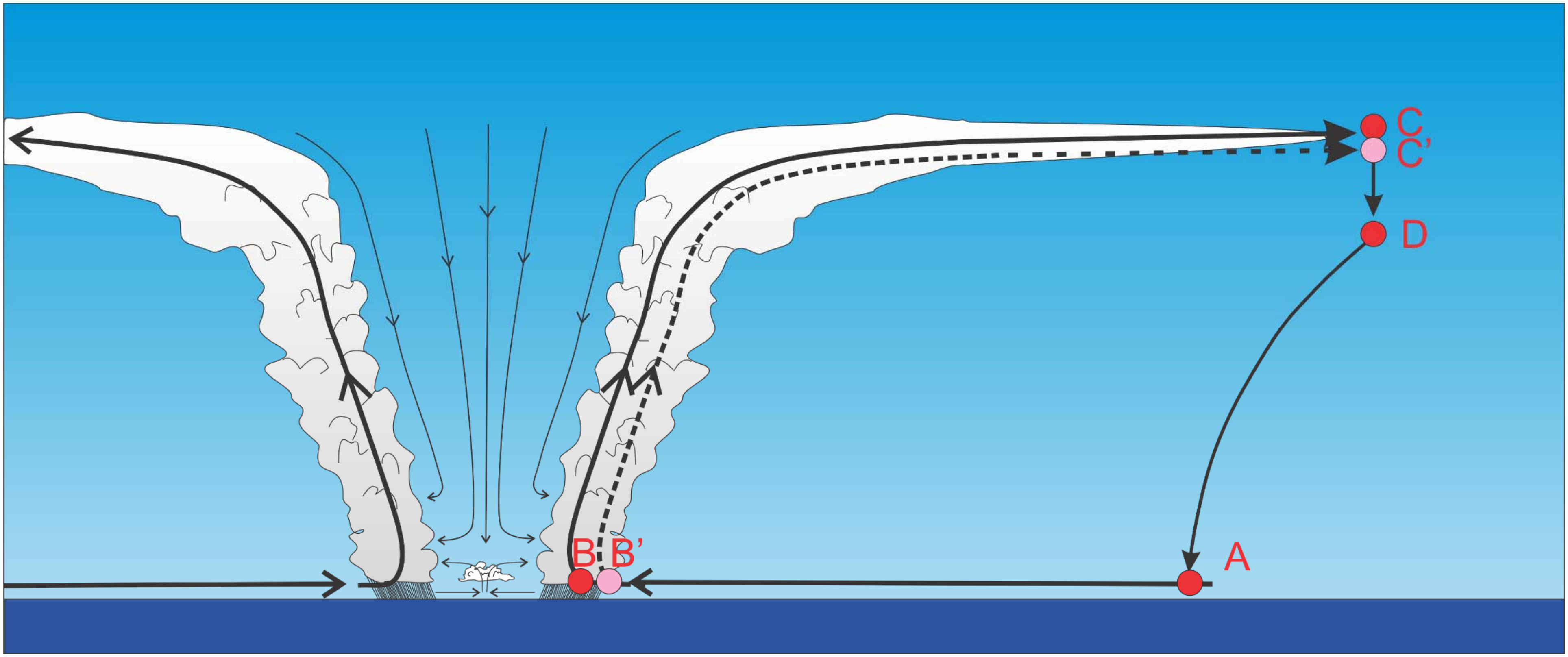
Dissipative heating

In steady state, energy production is used to balance frictional dissipation:

$$D = 2\pi \int_0^{r_0} \rho \left[C_D |\mathbf{V}|^3 \right] r dr$$

Differential Carnot Cycle





C
C’
D
B
B’
A

$$D = \frac{T_s - T_o}{T_s}\dot{Q}$$

$$\rho\left[C_D\,|\,\mathbf{V}_{max}\,|^3\right] = \frac{T_s - T_o}{T_s}\rho\left[C_k\,|\,\mathbf{V}_{max}\,|\left(k_0^* - k\right) + C_D\,|\,\mathbf{V}_{\max}\,|^3\right]$$

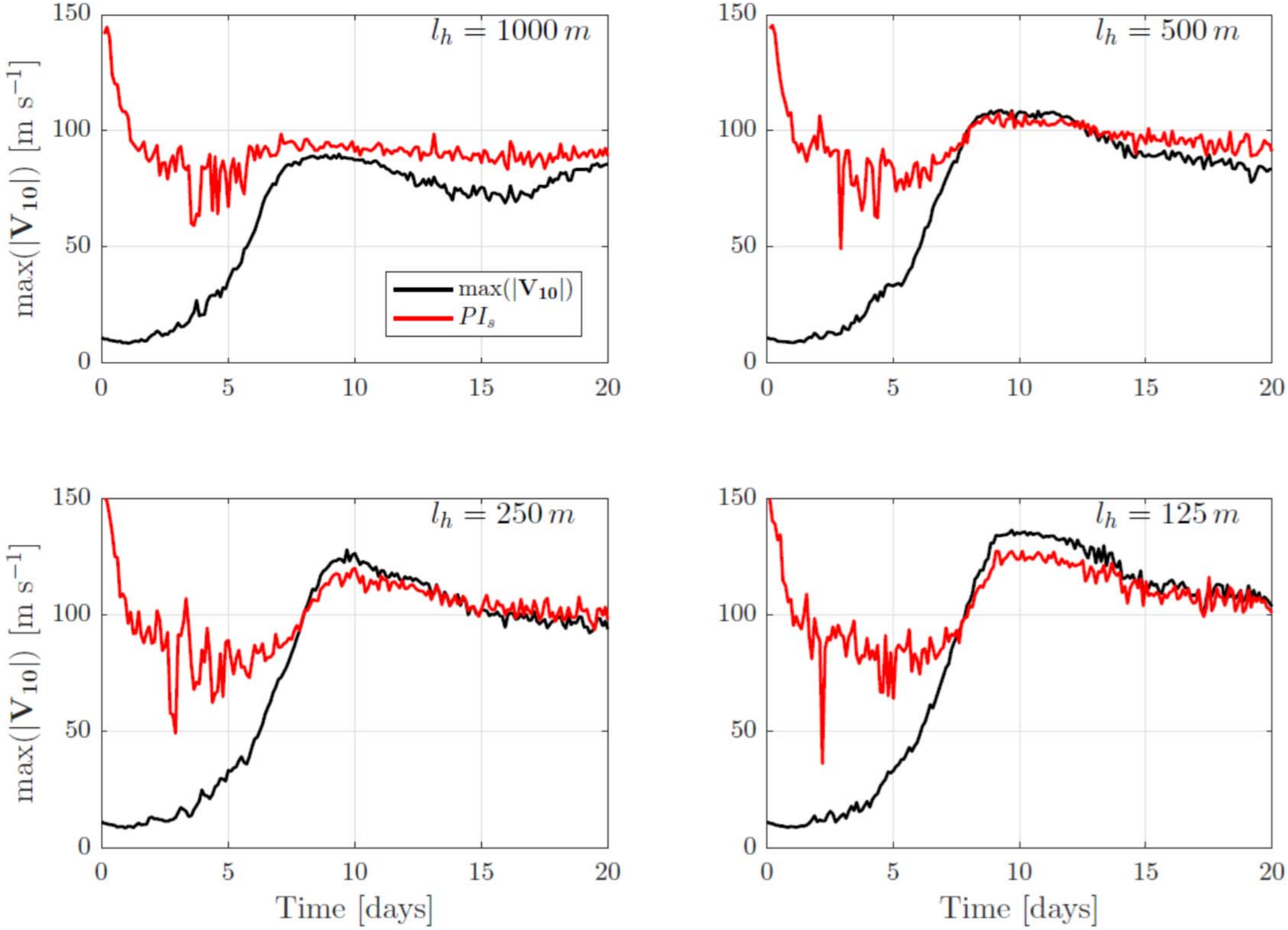
$$\rightarrow \rho\left[C_D\,|\,\mathbf{V}_{max}\,|^3\right] = \frac{T_s - T_o}{T_o}\rho\left[C_k\,|\,\mathbf{V}_{max}\,|\left(k_0^* - k\right)\right]$$

Internally determined

$$\rightarrow \quad |\,V_{max}\,|^2 \cong \frac{C_k}{C_D}\frac{T_s - T_o}{T_o}\left(k_0{}^* - k\right)$$

Note that this is valid between *ANY* two streamlines in the region of ascent

Simulations with cloud-permitting axisymmetric model for different horizontal mixing lengths



Rousseau-Rizzi and Emanuel, *J. Atmos. Sci.*, 2019

Derivation of gradient wind potential intensity from thermal wind balance

$$M_g = rV_g + \frac{1}{2} f r^2$$

$$\frac{\partial \phi}{\partial r} = \frac{V_g^{\ 2}}{r} + fV = \frac{M_g^{\ 2}}{r^3} - \frac{1}{4} f^2 r$$

Gradient balance

$$\frac{\partial \phi}{\partial p} = -\alpha$$

Hydrostatic balance

$$\rightarrow \frac{2M_g}{r^3} \frac{\partial M_g}{\partial p} = -\frac{\partial \alpha}{\partial r} = -\left(\frac{\partial \alpha}{\partial s^*}\right)_p \frac{\partial s^*}{\partial r}$$

Thermal wind

$$\frac{2M_g}{r^3} \frac{\partial M_g}{\partial p} = -\left(\frac{\partial T}{\partial p}\right)_{s^*} \frac{ds^*}{dM_g} \frac{\partial M_g}{\partial r}$$

$$\frac{2M_g}{r^3}\frac{\partial M_g}{\partial p} = -\left(\frac{\partial T}{\partial p}\right)_{s*}\frac{ds*}{dM_g}\frac{\partial M_g}{\partial r}$$

$$\rightarrow \frac{1}{r^3}\left(\frac{\partial r}{\partial p}\right)_{M_g} = \frac{1}{2M_g}\frac{ds*}{dM_g}\left(\frac{\partial T}{\partial p}\right)_{s*}$$

Integrate in pressure:

$$\frac{M_g}{r_b^2} - \frac{M_g}{r_o^2} = -\left(T_b - T_o\right)\frac{ds*}{dM_g}$$

$$\rightarrow \frac{V_{gb}}{r_b} = \frac{V_{go}}{r_o} - \left(T_b - T_o\right)\frac{ds*}{dM_g} \qquad (1)$$

Define outflow to be where $V_o = 0$

$$V_{gb} = -r_b \left(T_b - T_o \right) \frac{ds^*}{dM_g}$$

Convective criticality: $s^* = s_b$

$$\rightarrow V_{gb} = -r_b \left(T_b - T_o \right) \frac{ds_b}{dM_g} \qquad (1)$$

ds_b/dM_g determined by boundary layer processes

Put (1) in differential form:

$$\left(T_b - T_o\right)\frac{ds}{dt} + \frac{M_g}{r^2}\frac{dM_g}{dt} = 0. \tag{2}$$

Integrate entropy equation through depth of boundary layer:

$$h\frac{d\overline{s}}{dt} = \frac{1}{T_s}\left[C_k\,|\mathbf{V}|\left(k_0^* - k\right) + C_D\,|\mathbf{V}|^3\right] \tag{3}$$

Integrate angular momentum equation through depth of boundary layer:

$$h\frac{d\overline{M}}{dt} \cong h\frac{d\overline{M}_g}{dt} = -C_D r V\,|\mathbf{V}| \tag{4}$$

Substitute (3) and (4) into (2) and equate V with $|\mathbf{V}|$:

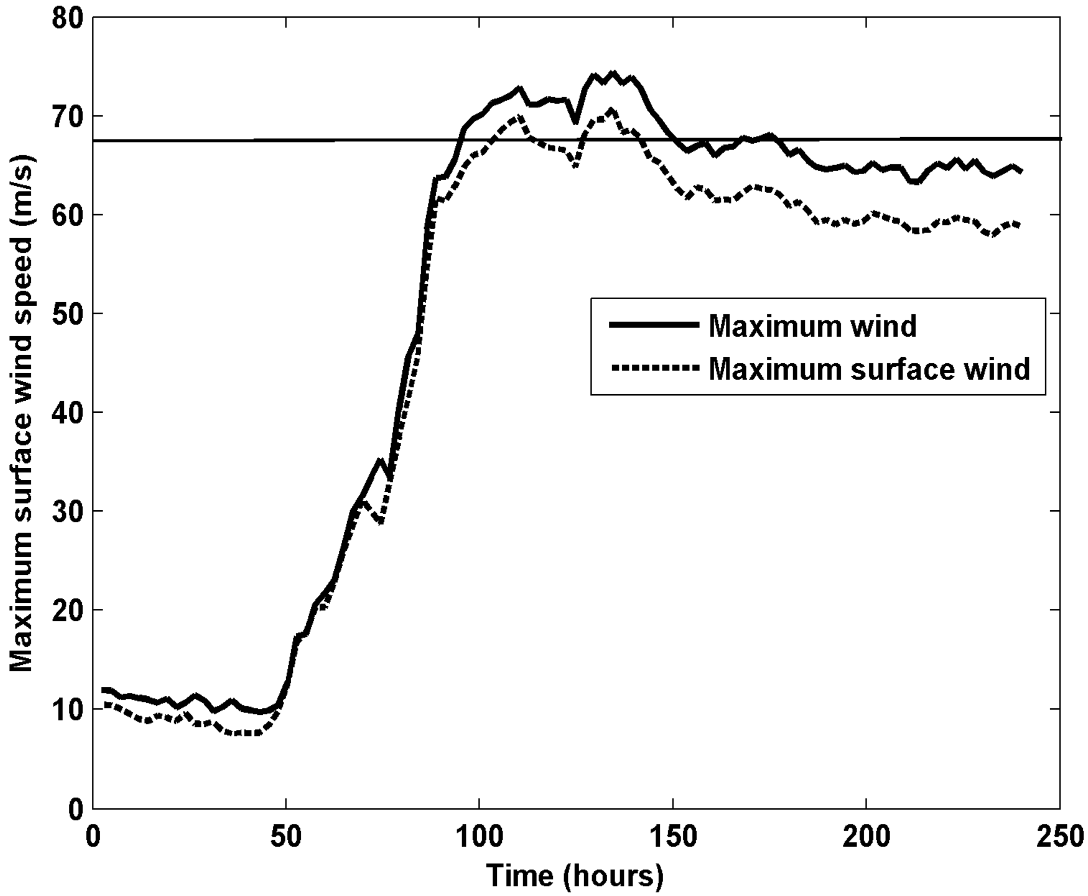
$$\rightarrow \quad |\mathbf{V}|^2 = \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} \left(k_0{}^* - k \right) \qquad (5)$$

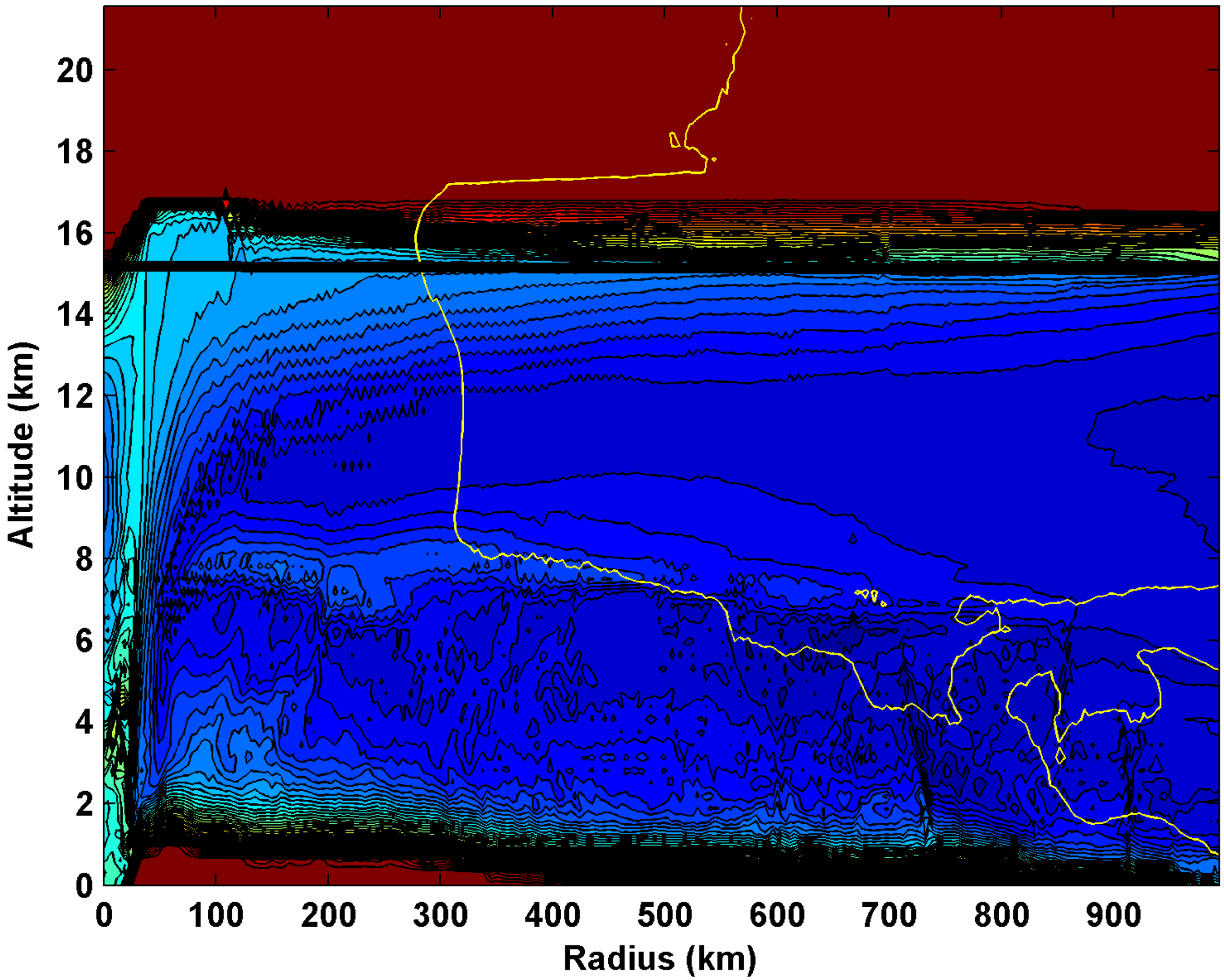
Same answer as from Carnot cycle. This is still not a closed expression, since we have not determined the boundary layer enthalpy, k or the outflow temperature, T_o

What Determines Outflow Temperature?

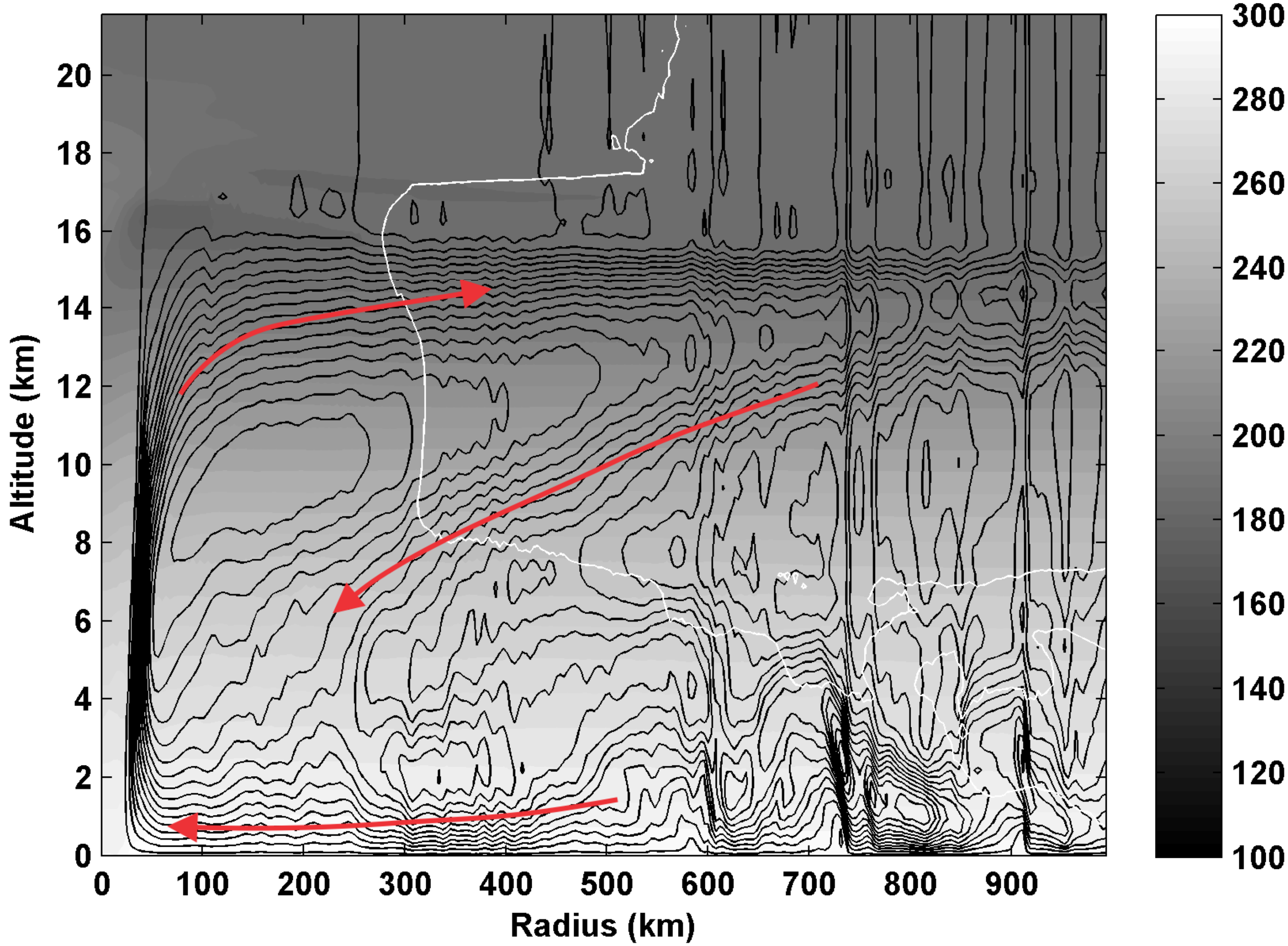
Reference: Emanuel and Rotunno, *J. Atmos. Sci.*, 2011

Simulations with Cloud-Permitting, Axisymmetric Model

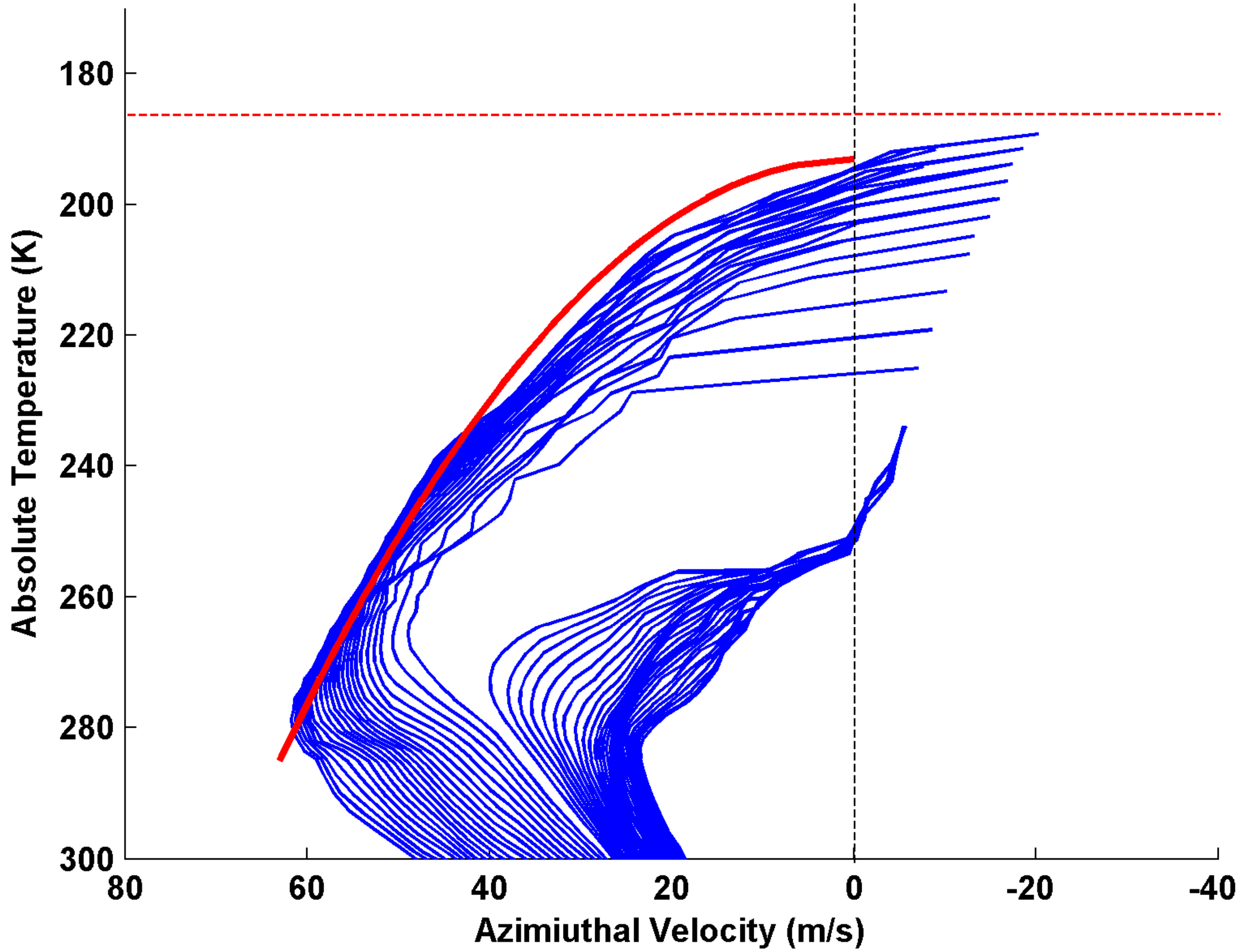




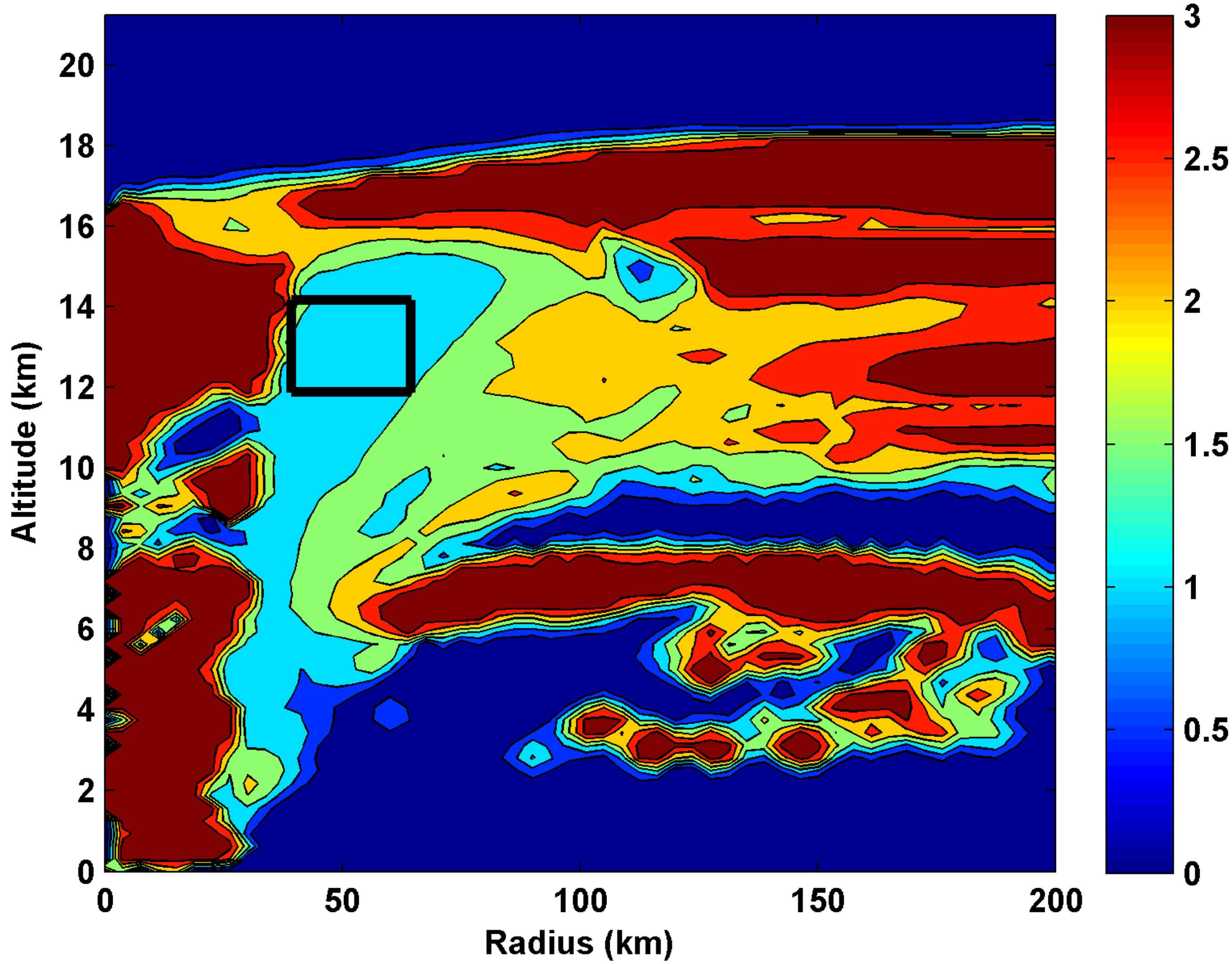
Saturation entropy (contoured) and V=0 line (yellow)



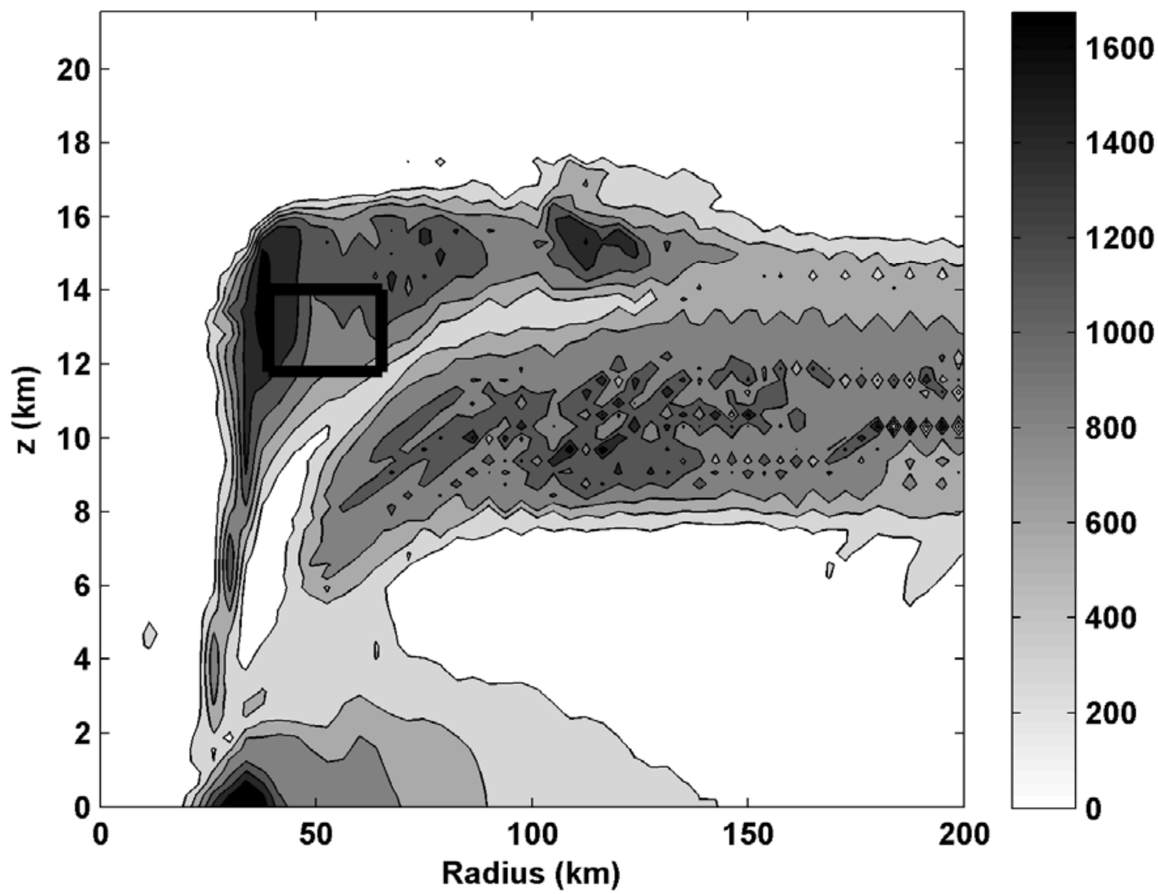
Streamfunction (black contours), absolute temperature (shading) and V=0 contour(white)



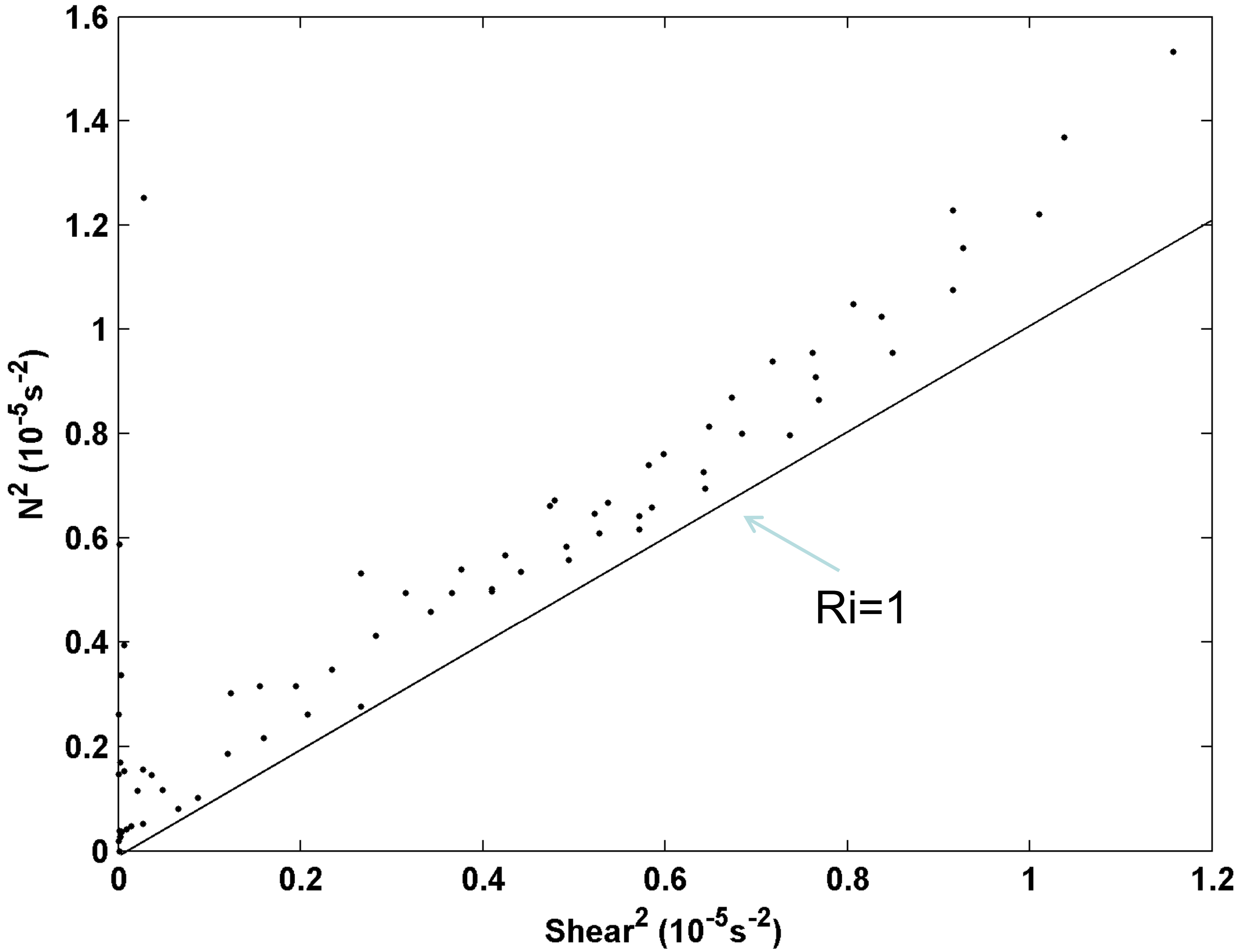
Angular momentum surfaces plotted in the V-T plane. Red curve shows shape of balanced M surface originating at radius of maximum winds. Dashed red line is ambient tropopause temperature.



Richardson Number (capped at 3). Box shows area used for scatter plot.



Vertical Diffusivity (m^2s^{-1})



1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
N^2 ($10^{-5}s^{-2}$)
0
0.2
0.4
0.6
0.8
1
1.2
Shear^2 ($10^{-5}s^{-2}$)
Ri=1

Implications for Outflow Temperature

$$Ri = \frac{\Gamma_d \dfrac{\partial s_d}{\partial z}}{\left(\left|\dfrac{\partial \mathbf{V}}{\partial z}\right|\right)^2} \cong \frac{r^2 \Gamma_m \dfrac{ds^*}{dM}}{\dfrac{\partial M}{\partial z}}.$$

$$\rightarrow \frac{\partial M}{\partial z} \cong \frac{r_t^2 \Gamma_m \dfrac{ds^*}{dM}}{Ri_c}.$$

$$\frac{\partial s^*}{\partial z} = \frac{ds^*}{dM}\frac{\partial M}{\partial z},$$

$$\rightarrow \frac{\partial s^*}{\partial z} \cong \frac{r_t^2 \Gamma_m \left(\frac{ds^*}{dM} \right)^2}{Ri_c}. \tag{5}$$

But the vertical gradient of saturation entropy is related to the vertical gradient of temperature:

$$\frac{\partial T}{\partial s^*} = \left(\frac{\partial T}{\partial s^*} \right)_p + \frac{\left(\frac{\partial T}{\partial p} \right)_{s^*}}{\frac{\partial s^*}{\partial p}}, \tag{6}$$

Use definition of s* and C.-C.:

$$\left(\frac{\partial T}{\partial s^*} \right)_p = \frac{\frac{T}{c_p}}{\left[1 + \frac{L_v^2 q^*}{R_v c_p T^2} \right]} \tag{7}$$

Substitute (16) into (15) and use hydrostatic equation:

$$\frac{\partial T}{\partial s^*} = \frac{\frac{T}{c_p}}{\left[1 + \frac{L_v^2 q^*}{R_v c_p T^2}\right]} - \frac{\Gamma_m}{\frac{\partial s^*}{\partial z}}. \qquad (8)$$

If $V_b^2 << c_p \frac{(T_b - T_o)^2}{T_b} \left(1 + \frac{L_v^2 q^*}{R_v c_p T^2}\right) \left[Ri_c \frac{r_b^2}{r_t^2}\right]$ we can neglect first term on left of (8)

Substitute (5) into (8):

$$\frac{\partial T_o}{\partial s^*} \cong -\frac{Ri_c}{r_t^2} \left(\frac{dM}{ds^*}\right)^2. \qquad (9)$$

Gives dependence of Outflow T on s*

Using $$\frac{\partial T}{\partial s^*} = \frac{\partial T}{\partial M}\frac{dM}{ds^*},$$

We can re-write (9) as $$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right). \qquad (10)$$

We can also re-write (1) as $$M_b = r_b^2\left(\frac{1}{2}f - \left(T_b - T_o\right)\frac{ds^*}{dM}\right) \qquad (11)$$

Boundary layer entropy: $$h\frac{ds_b}{dt} = C_k\,|\,\mathbf{V}\,|\left(s_0^* - s_b\right) + C_d\,\frac{|\,\mathbf{V}\,|^3}{T_b} \qquad (12)$$

Boundary layer angular momentum: $$h\frac{dM}{dt} = -r\,|\,\mathbf{V}\,|V \qquad (13)$$

Combine (12) and (13):

$$\frac{ds_b}{dM} = -\frac{C_k}{C_D}\frac{\left(s_0^* - s_b\right)}{rV} - \frac{|\mathbf{V}|^2}{T_b r V}$$

Let $\quad s_b \simeq s^*, \quad |\mathbf{V}| \simeq V \simeq V_b, \quad r \simeq r_b$

$$\rightarrow \frac{ds^*}{dM} = -\frac{C_k}{C_D}\frac{\left(s_0^* - s^*\right)}{r_b V_b} - \frac{V_b}{T_b r_b} \qquad (14)$$

Balance condition (1):

$$\frac{V_b}{r_b} = -\left(T_b - T_o\right)\frac{ds^*}{dM} \qquad (15)$$

Eliminate V_b between (14) and (15):

$$\left(\frac{ds^*}{dM}\right)^2 = \frac{T_b}{T_o}\frac{C_k}{C_D}\frac{\left(s_0^* - s^*\right)}{r_b^2\left(T_b - T_o\right)} \qquad (16)$$

Eliminate r_b^2 between (11) and (16):

$$\left(\frac{ds^*}{dM}\right)^2 + 2\chi\frac{ds^*}{dM} - \frac{\chi f}{T_b - T_o} = 0, \qquad (17)$$

where

$$\chi \equiv \frac{T_b}{T_o}\frac{C_k}{C_D}\frac{s_0^* - s^*}{2M}$$

Remember that

$$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right) \qquad (10)$$

inward from some outer radius r_o, defined such that

$$V = 0 \quad at \quad r = r_o$$

In general, integrating this system will not yield T_o=T_t at r=r_{max}. Iterate value of r_t until this condition is met.

If $V >> fr$, we ignore dissipative heating, and we neglect pressure dependence of s_0*, then we can derive an approximate closed-form solution.

Assuming that *Ri* is critical in the outflow leads to an equation for T_o that, coupled to the interior balance equation and the slab boundary layer lead (surprisingly!) to a closed form analytic solution for the gradient wind (as represented by angular momentum, *M*, at the top of the boundary layer:

$$\left(\frac{M}{M_m}\right)^{2-\frac{C_k}{C_D}} = \frac{2\left(\frac{r}{r_m}\right)^2}{2-\frac{C_k}{C_D}+\frac{C_k}{C_D}\left(\frac{r}{r_m}\right)^2}, \qquad (18)$$

Evaluate at r_o:

$$\left(\frac{fr_o^2}{2V_m r_m}\right)^{2-\frac{C_k}{C_D}} = \frac{2\left(\frac{r_o}{r_m}\right)^2}{2-\frac{C_k}{C_D}+\frac{C_k}{C_D}\left(\frac{r_o}{r_m}\right)^2}. \qquad (19)$$

$$\text{For } r_o >> r_m: \quad r_m \cong \frac{1}{2} fr_o^2 V_m^{-1}\left(\frac{1}{2}\frac{C_k}{C_D}\right)^{\frac{1}{2-\frac{C_k}{C_D}}} \qquad (20)$$

The maximum wind speed, V_m, found from maximizing the radial dependence of wind speed in the solution (18) on previous slide is given by

$$V_m^{2-\frac{C_k}{C_D}} = V_p^2\left(\frac{2r_m}{fr_o^2}\right)^{\frac{C_k}{C_D}} \qquad (21)$$

Substituting (20) into (21) gives

$$V_m^2 \cong V_p^2 \left(\frac{1}{2} \frac{C_k}{C_D} \right)^{\frac{\frac{C_k}{C_D}}{2-\frac{C_k}{C_D}}} \tag{22}$$

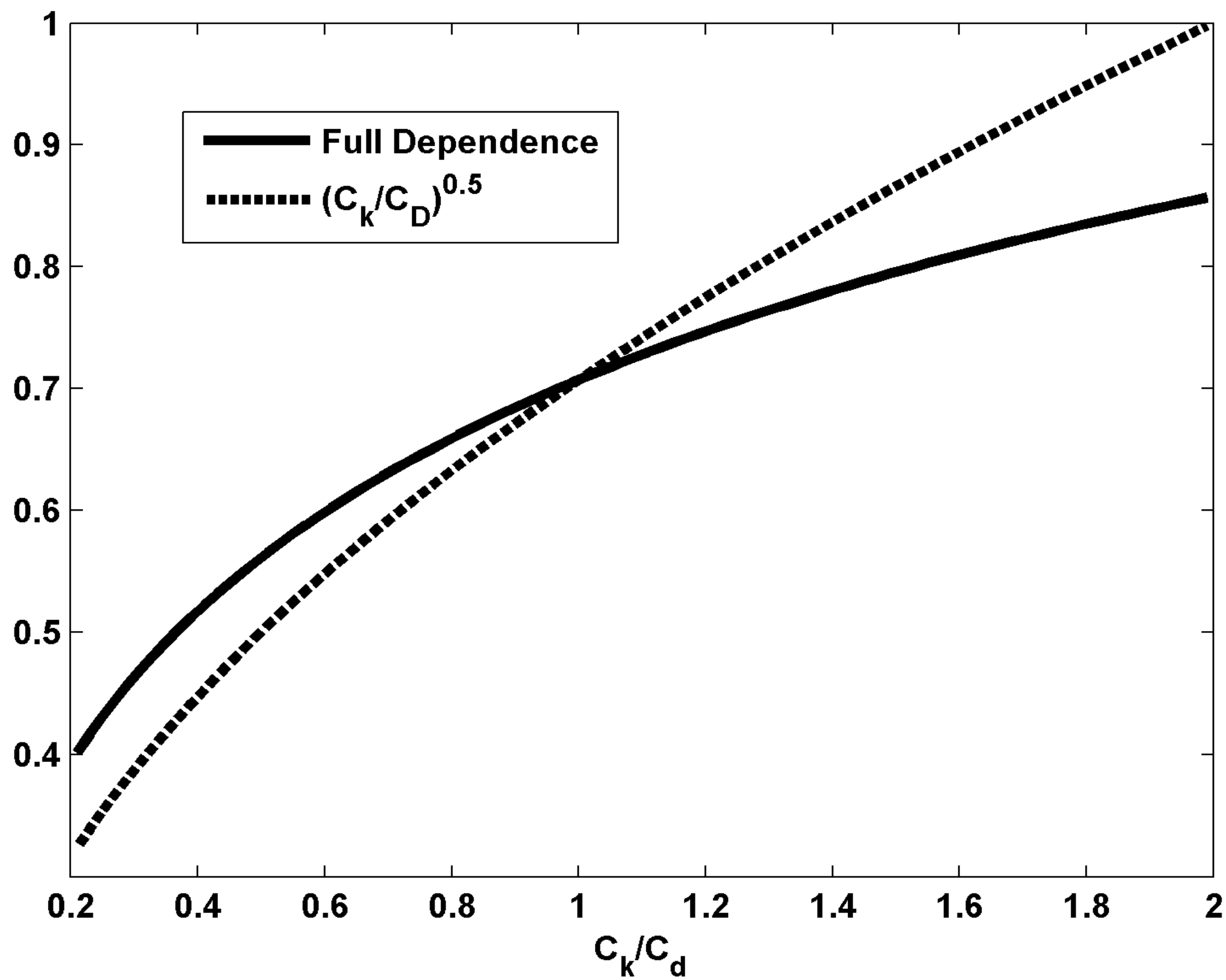
$$\boxed{V_p^2 \equiv \frac{C_k}{C_D} \left(T_b - T_t \right) \left(s_0 {*} - s_e {*} \right)}$$

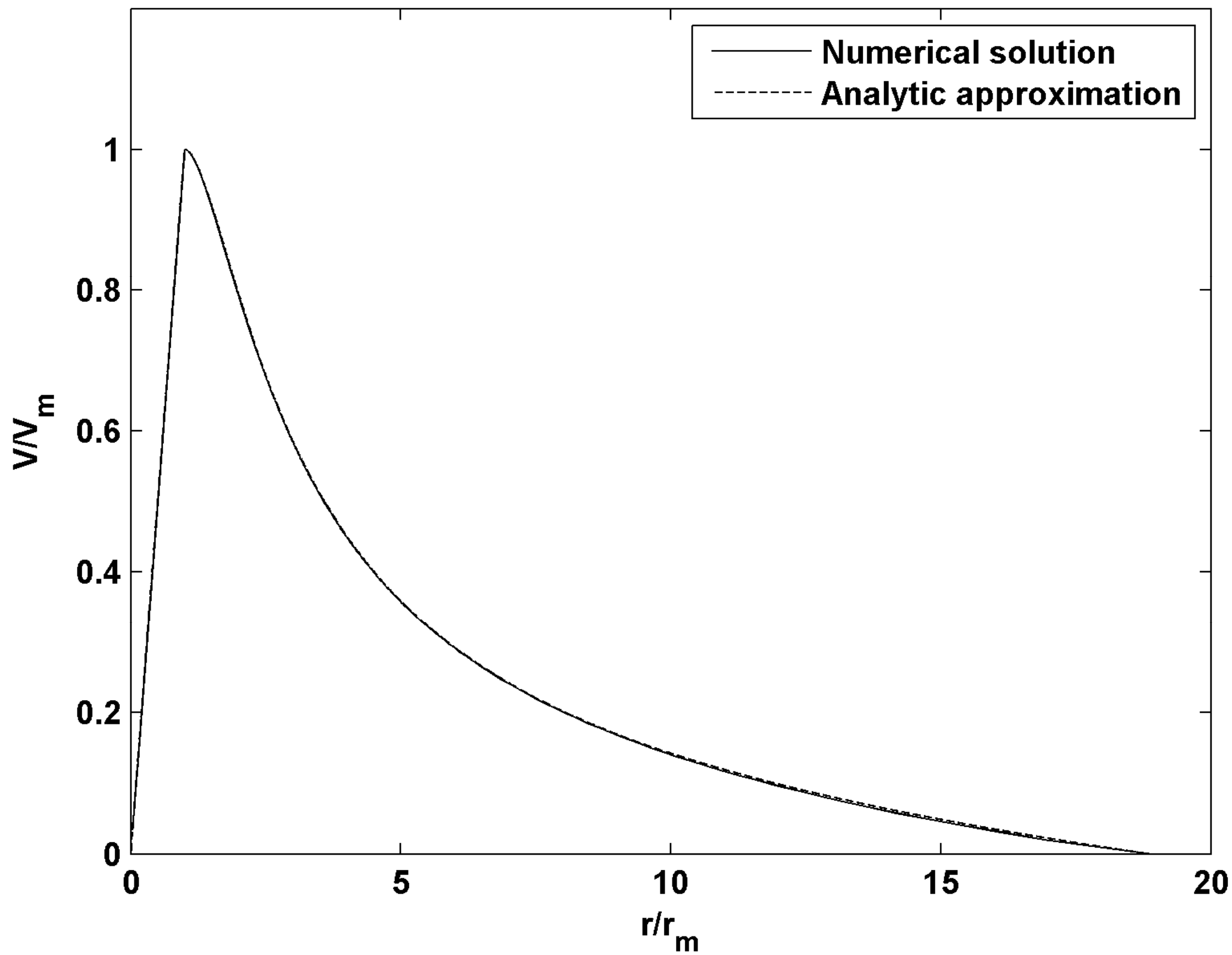
Can be calculated directly from SST and soundings

$$r_m \cong \left(\frac{1}{2}\right)^{\frac{3}{2}} \frac{f r_o^2}{\sqrt{\left(T_b - T_t\right)\left(s_0 * - s_e *\right)}}$$

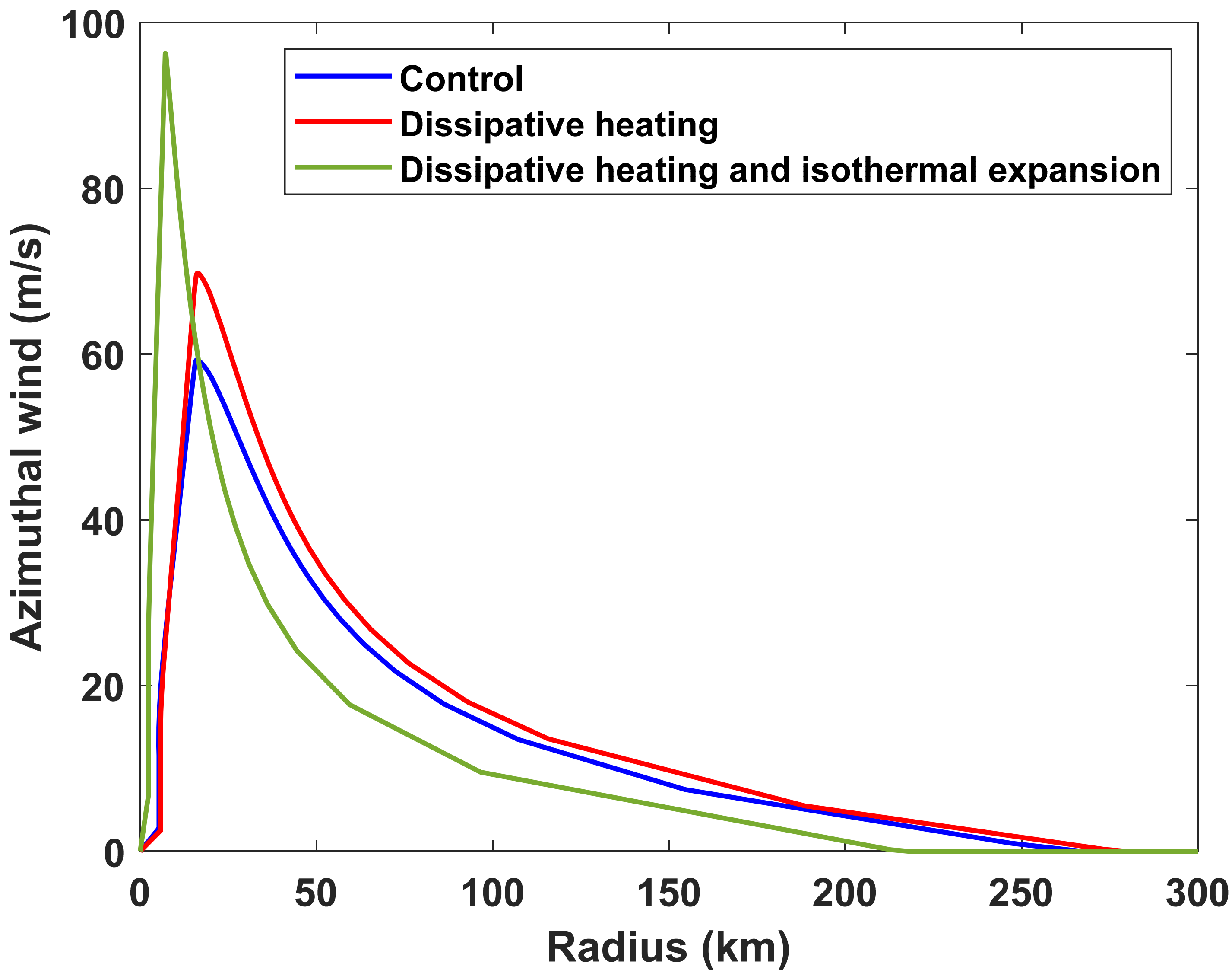
Also,

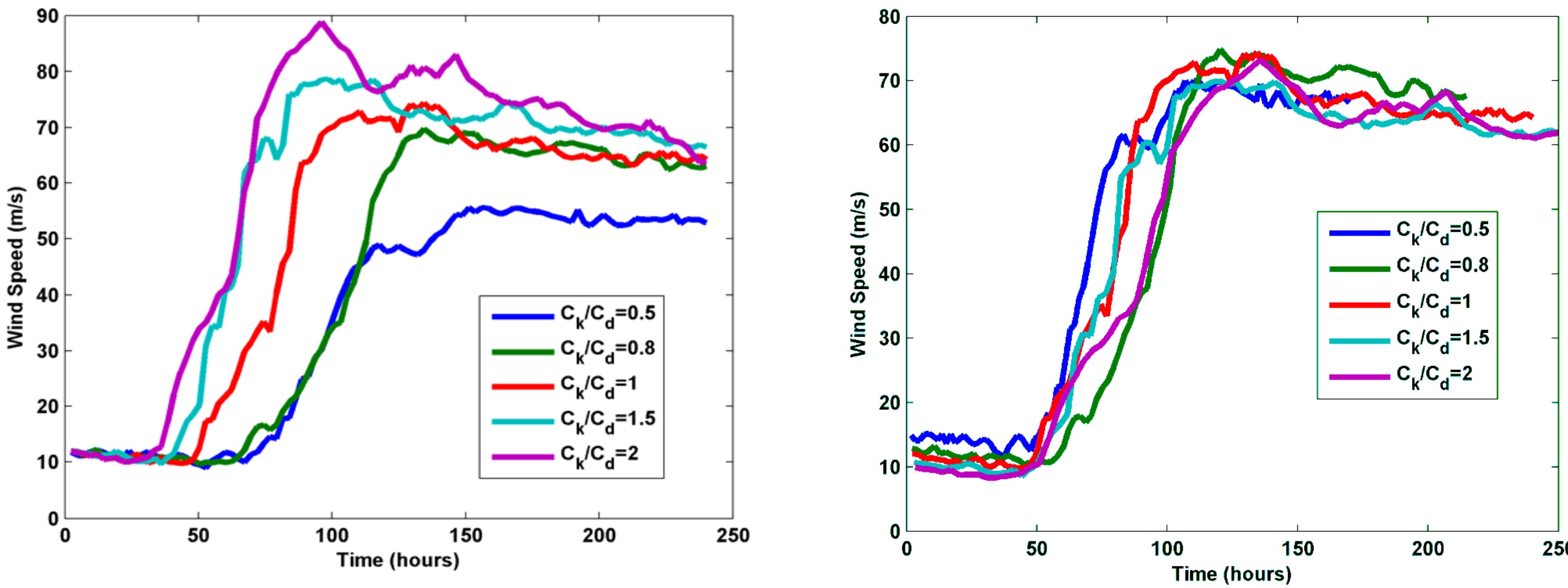
$$r_t^2 = r_m^2 \frac{C_D}{C_k} Ri_c$$


Full Dependence
$(C_k/C_D)^{0.5}$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
C_k/C_d



Numerical solution
Analytic approximation
V/V_m
r/r_m
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20

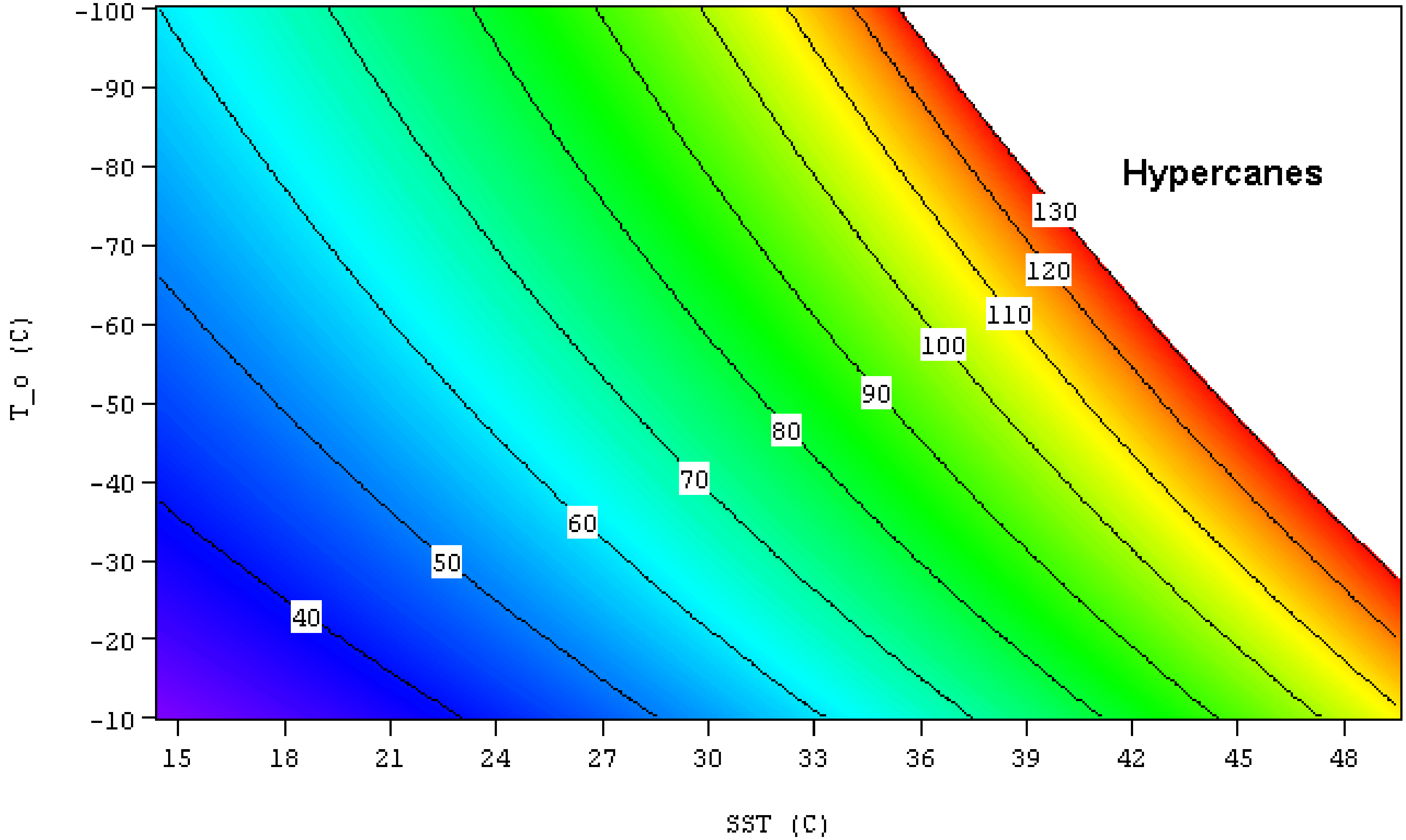

Control
Dissipative heating
Dissipative heating and isothermal expansion
Azimuthal wind (m/s)
Radius (km)
0
20
40
60
80
100
0
50
100
150
200
250
300



Numerical integrations with RE87 model (no dissipative heating, no pressure dependence of k_0*) : Left, regular variables; Right: Velocity scaled by (31) and time scaled by the inverse square-root of the enthalpy exchange coefficient.

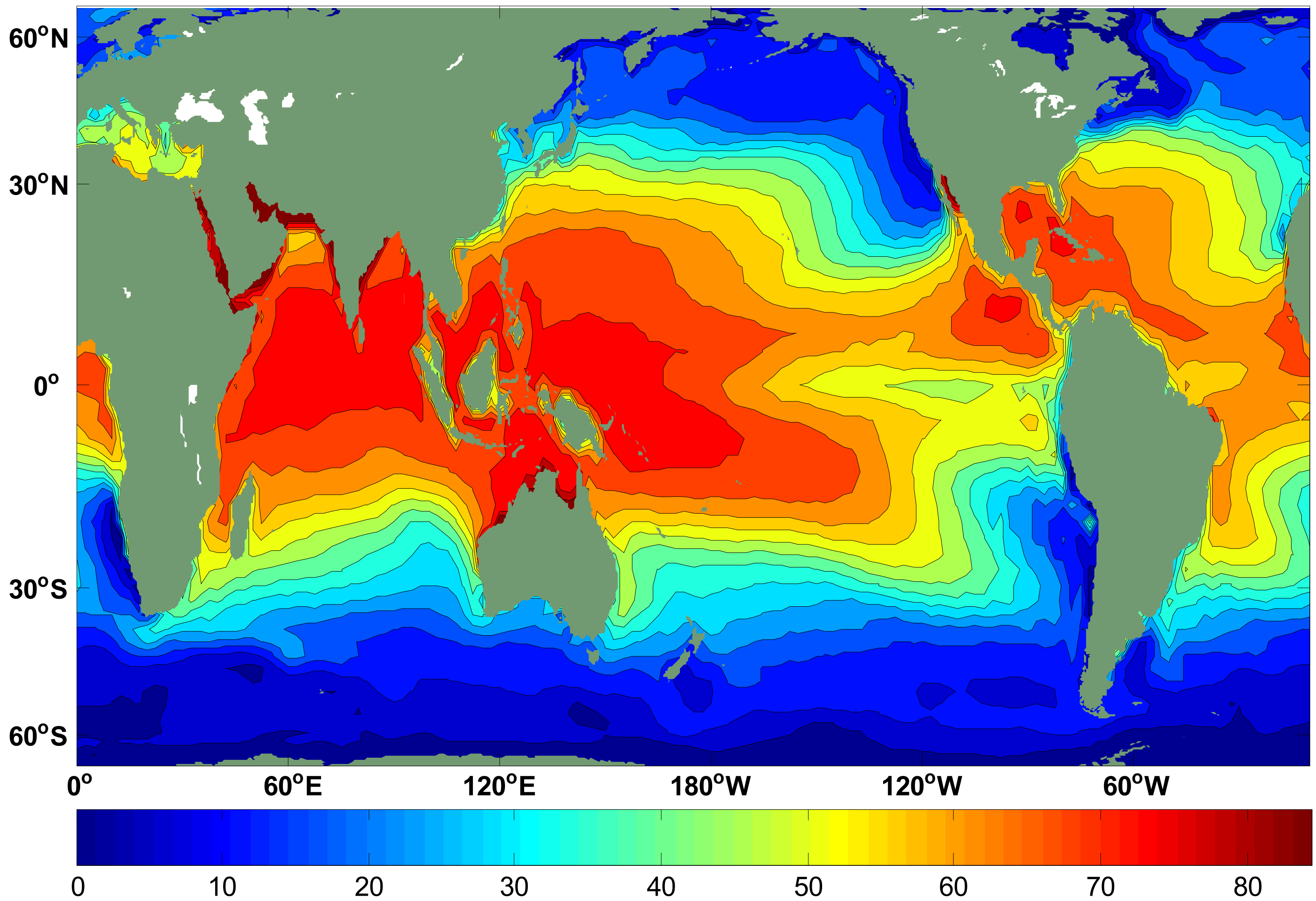
Effects of Pressure-Dependence of Surface Saturation Enthalpy

Maximum Wind Speed (m/s)

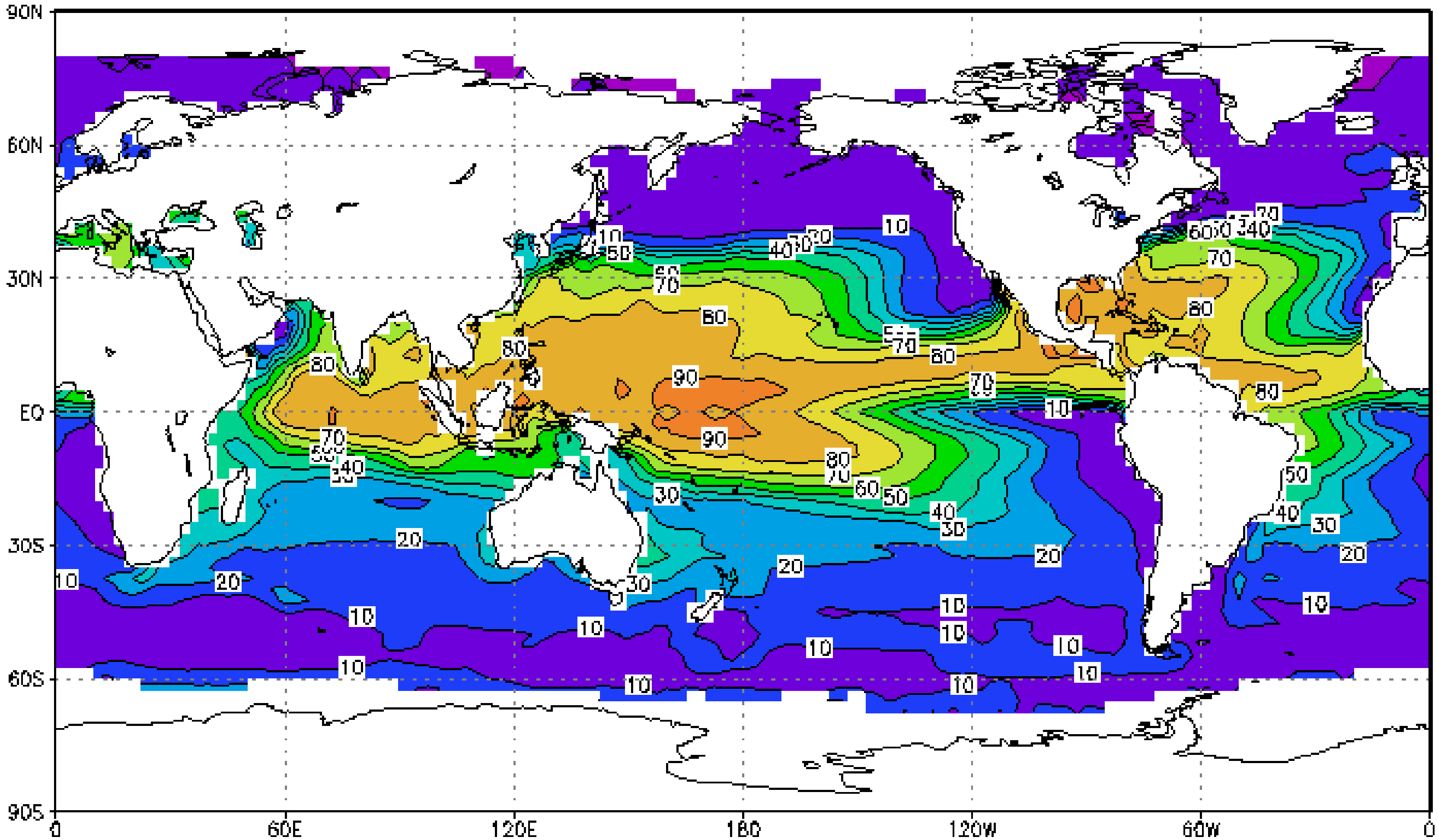


$\mathcal{H} =$ **0.75** **C_k/C_D = 1.2**

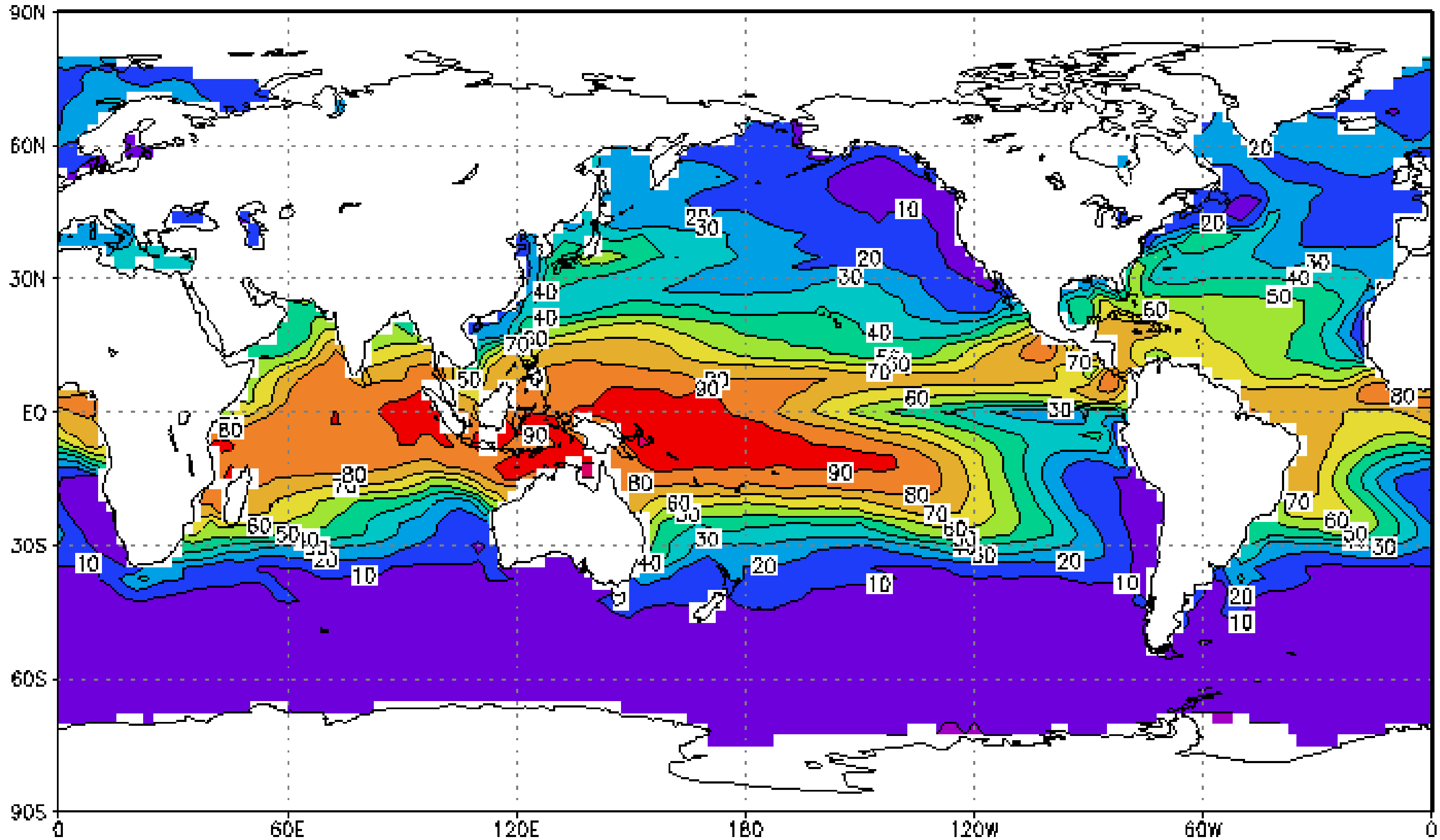
Annual Maximum Potential Intensity (m/s)

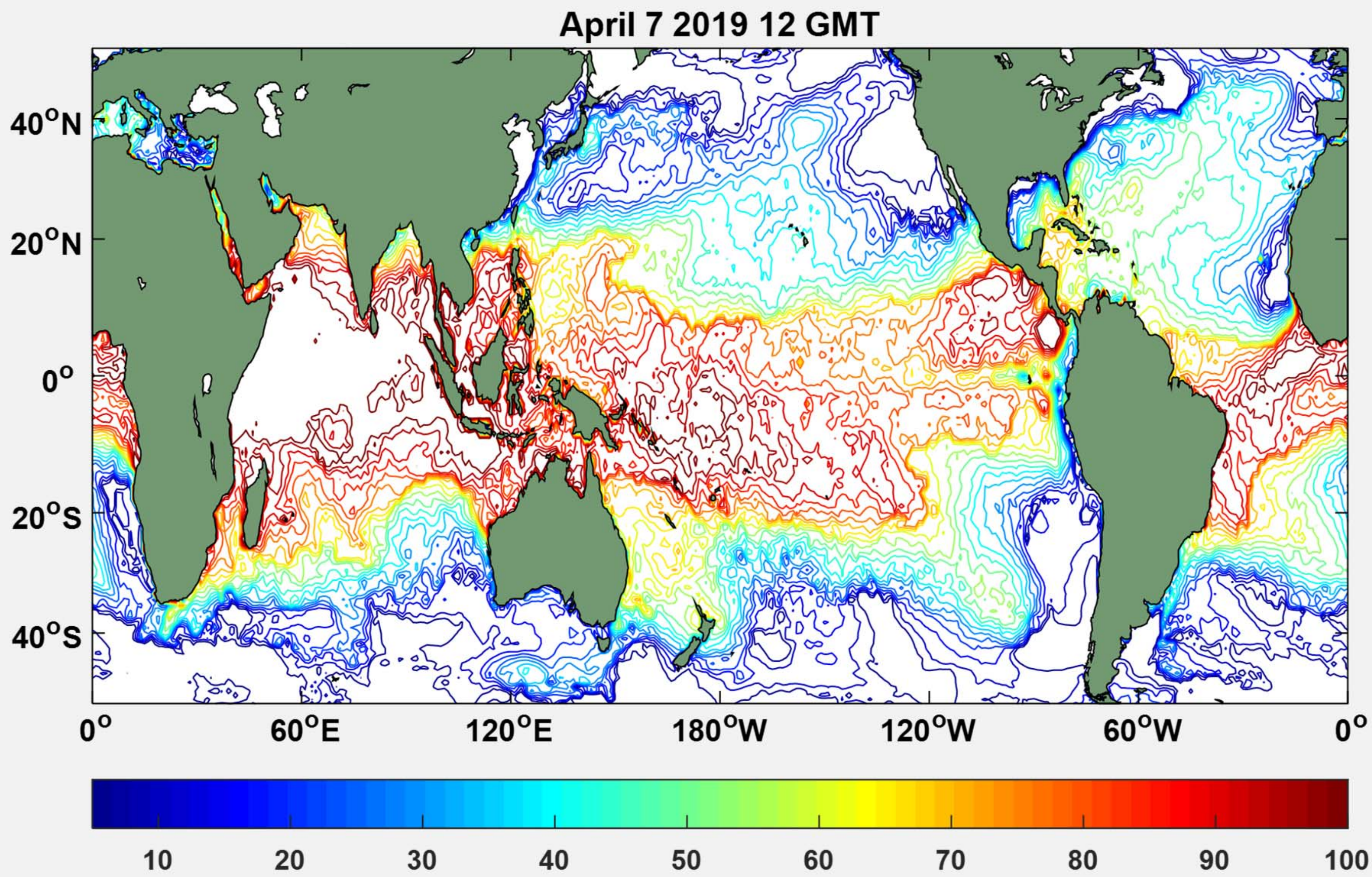


AUGUST MEAN



JANUARY MEAN





April 7 2019 12 GMT
40°N
20°N
0°
20°S
40°S
0°
60°E
120°E
180°W
120°W
60°W
0°
10
20
30
40
50
60
70
80
90
100

Thermodynamic disequilibrium is necessary to maintain ocean heat balance:

Ocean mixed layer Energy Balance (neglecting lateral heat transport):

$$C_k \rho |\mathbf{V_s}| \left(k_0^* - k\right) = F_\downarrow - F_\uparrow - F_{entrain}$$

Ocean mixed layer entrainment

$$\rightarrow$$

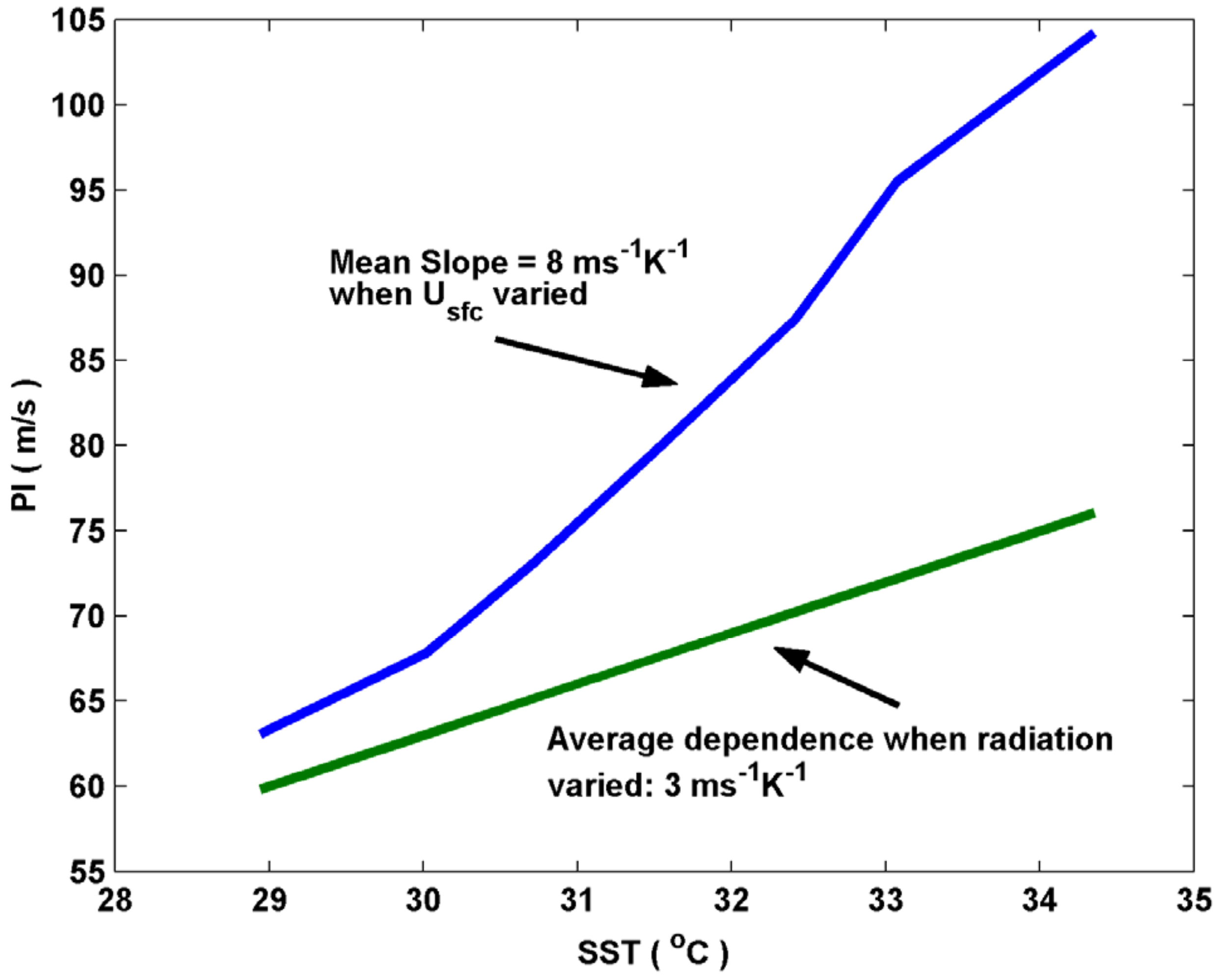
Greenhouse effect decreases this

$$V_p^2 = \frac{T_s - T_o}{T_o} \frac{F_\downarrow - F_\uparrow - F_{entrain}}{C_D \rho |\mathbf{V}_s|}$$

Weak explicit dependence on T_s

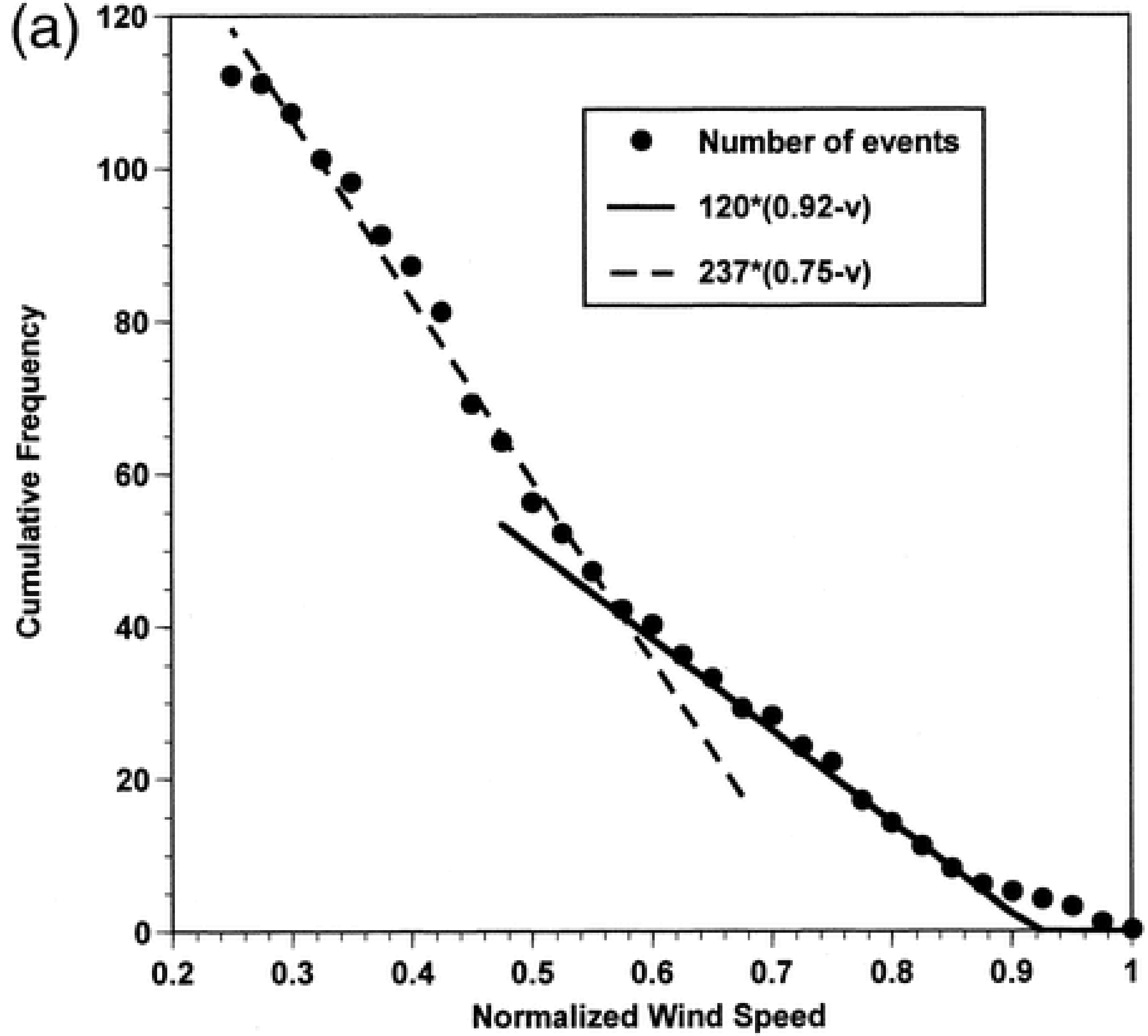
Mean surface wind speed

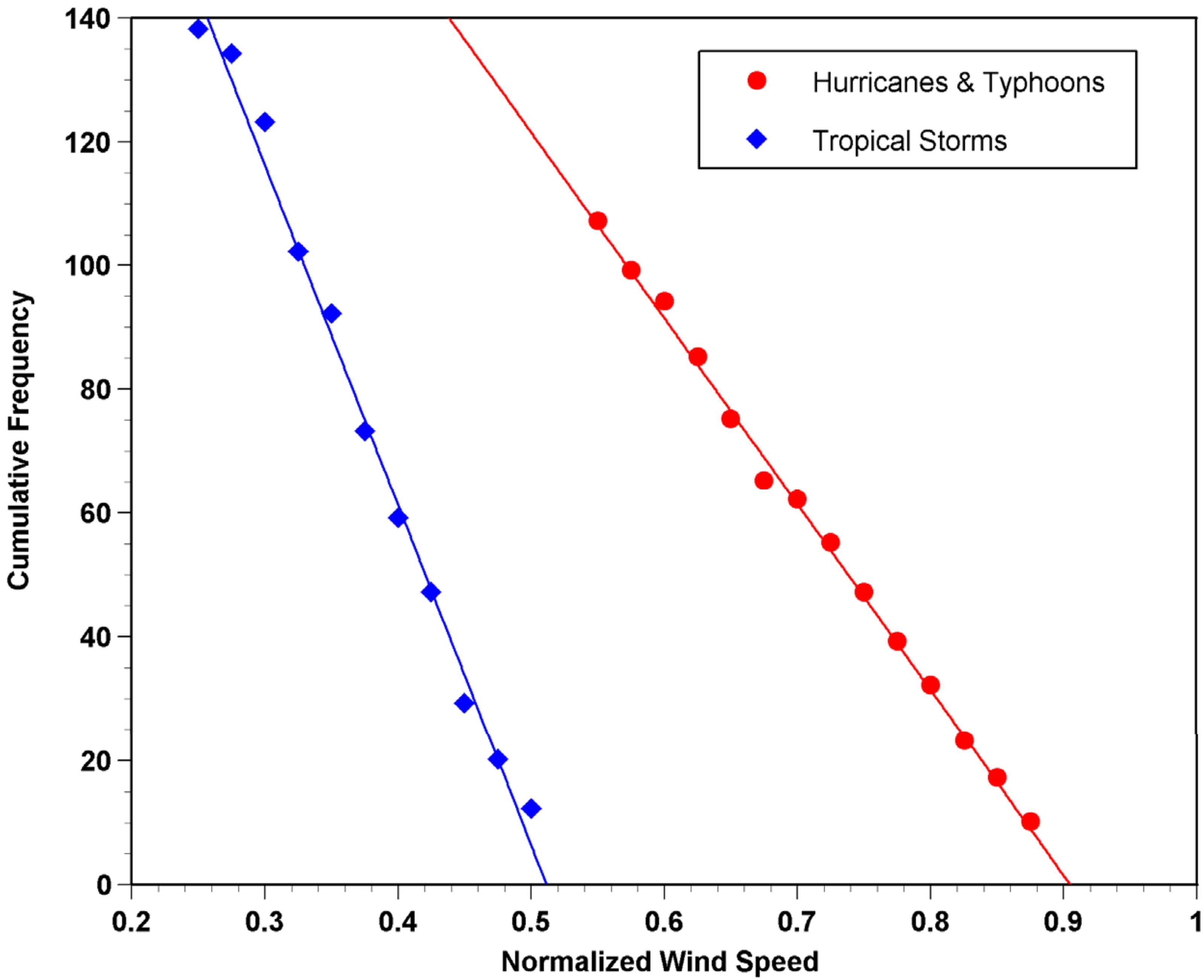
Dependence on Sea Surface Temperature (SST):



Relationship between potential intensity (PI) and intensity of real tropical cyclones

(Following slides from Emanuel, K.A., 2000: A statistical analysis of hurricane intensity. *Mon. Wea. Rev.*, **128**, 1139-1152.)


(a)
Number of events
120*(0.92-v)
237*(0.75-v)
Cumulative Frequency
Normalized Wind Speed
0
20
40
60
80
100
120
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Hurricanes & Typhoons
Tropical Storms
Cumulative Frequency
Normalized Wind Speed
0
20
40
60
80
100
120
140
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1