

Understanding Extreme Updrafts and Wind Gusts Using Dropsonde Observations and Large- Eddy Simulations

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Outline of Topics

1. Overview of how we observe TCs
2. Dropsonde observations of extreme updrafts and wind gusts
3. Using Large-Eddy Simulations (LES) to understand the structure and dynamics of updrafts, wind gusts, and associated vortices.

Strong hurricanes can cause tremendous wind damage, and this damage can be extremely localized.

Hurricane Andrew (1992): Category 5



Hurricane Andrew (1992): Category 5



Hurricane Wilma (2005): Category 1



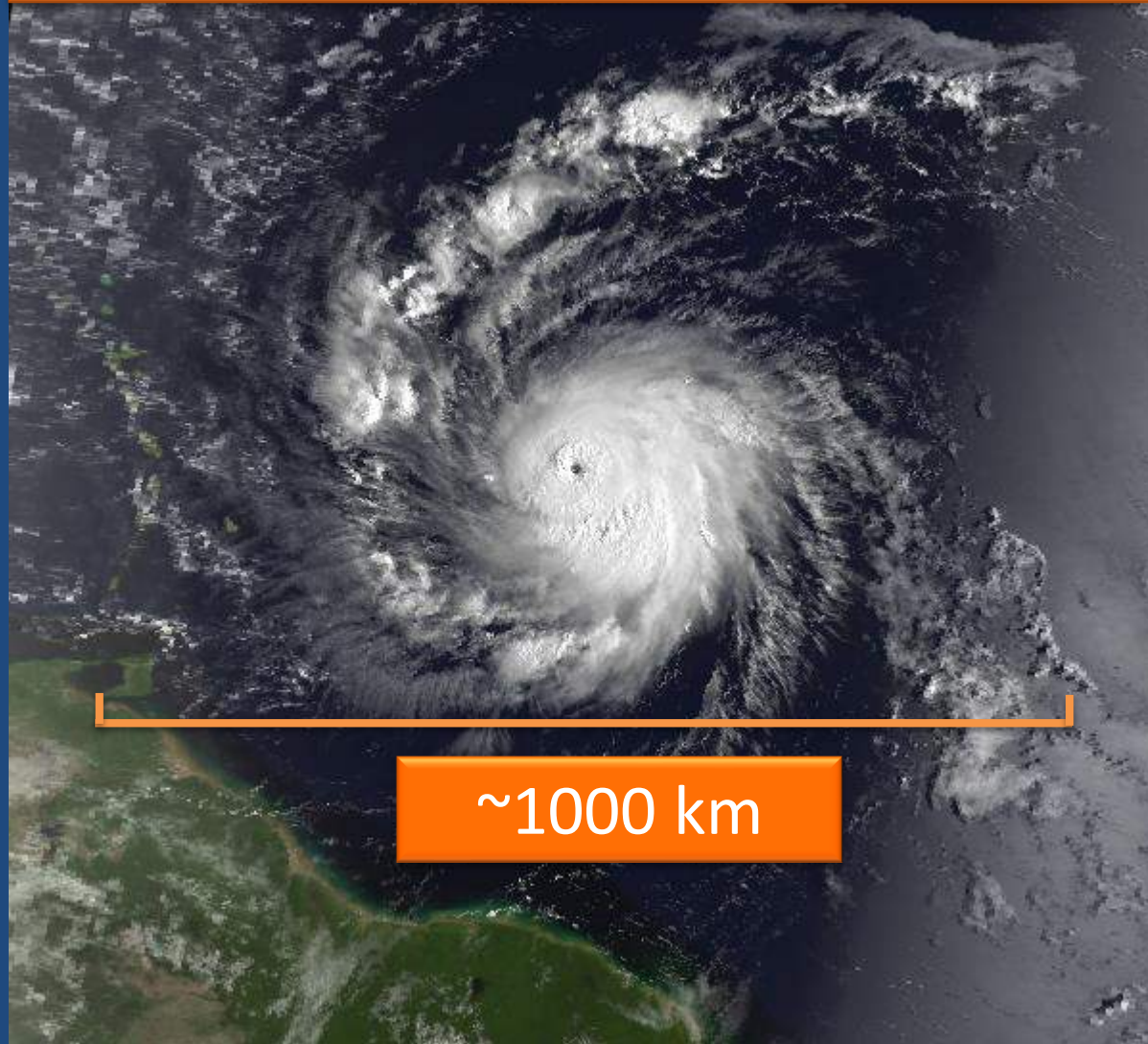
Hurricane Wilma (2005): Category 1

My Car!!!



How Do We Observe Tropical Cyclones?

Hurricane Hugo (1989)

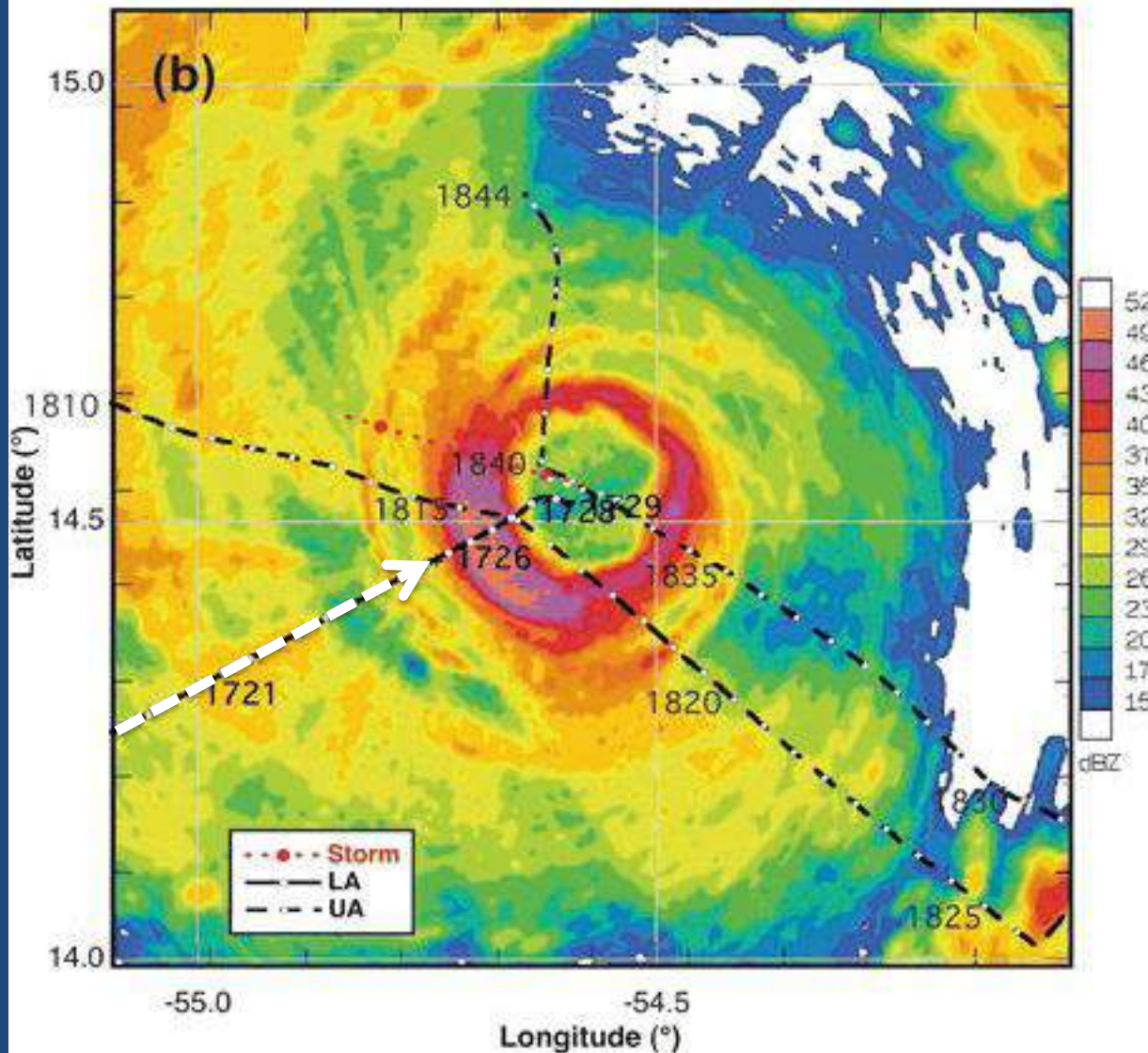


~1000 km

NOAA P3 Hurricane Hunters

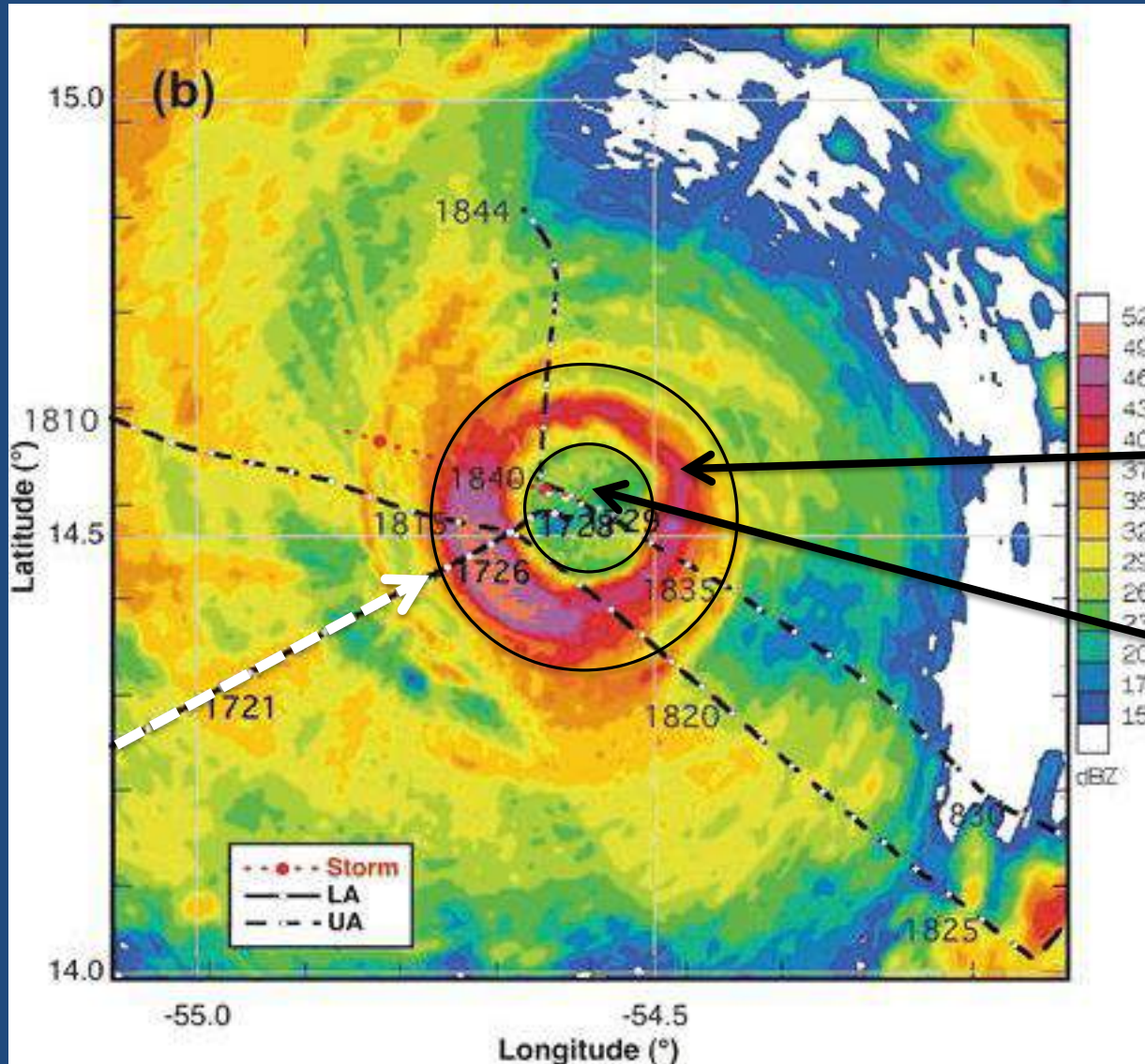


120 km



Marks et al. (2008)

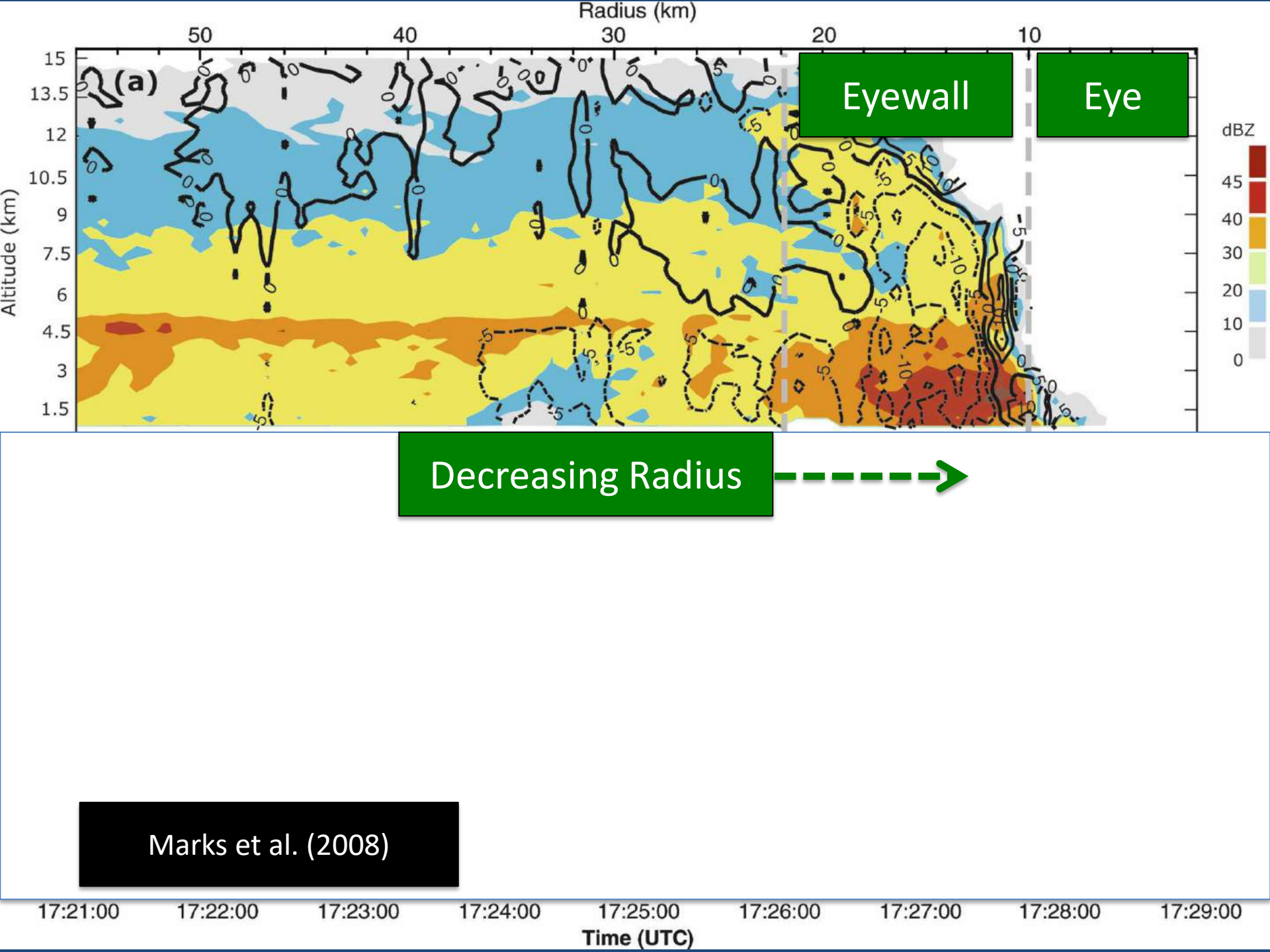
120 km

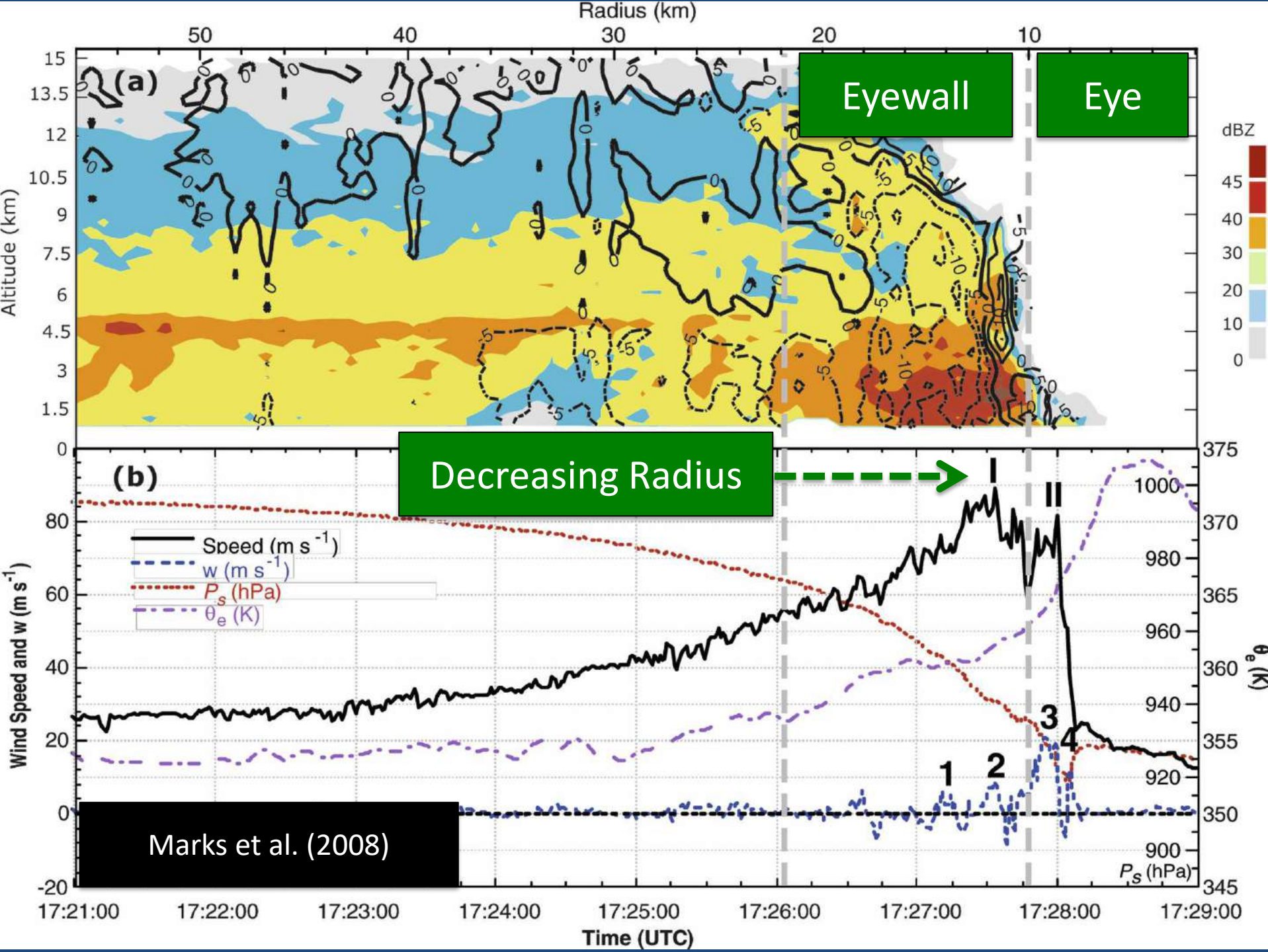


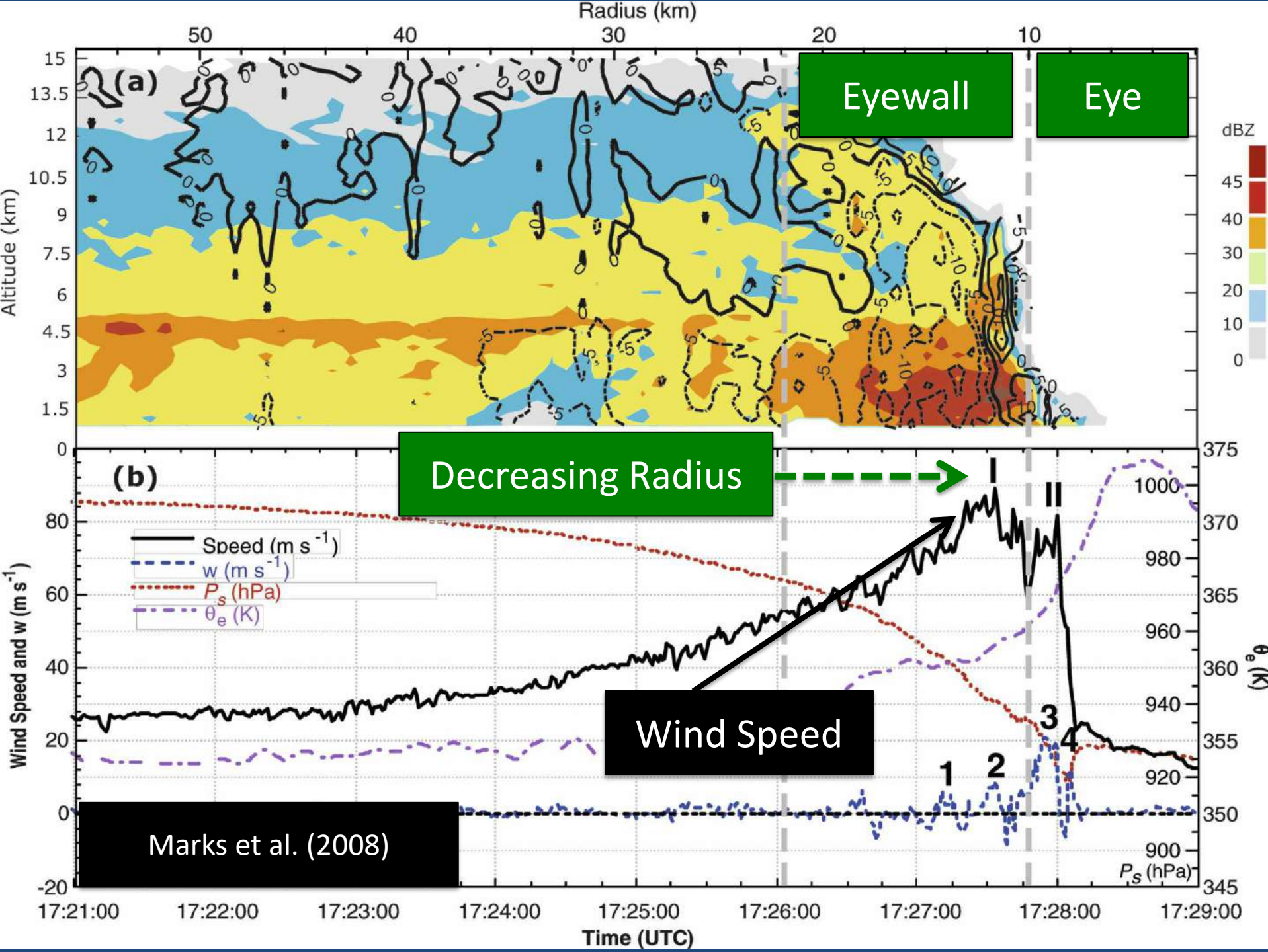
Eyewall

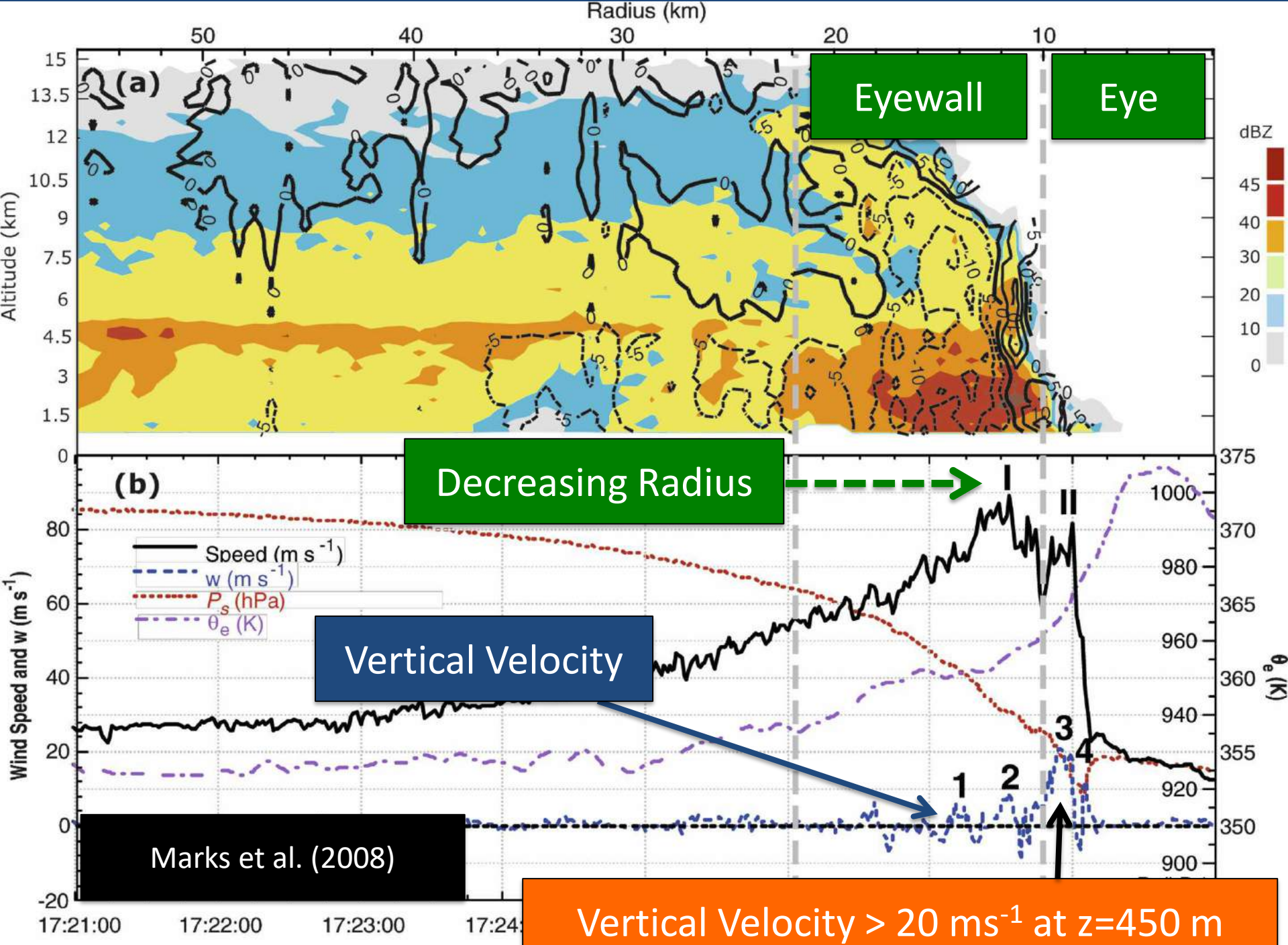
Eye

Marks et al. (2008)







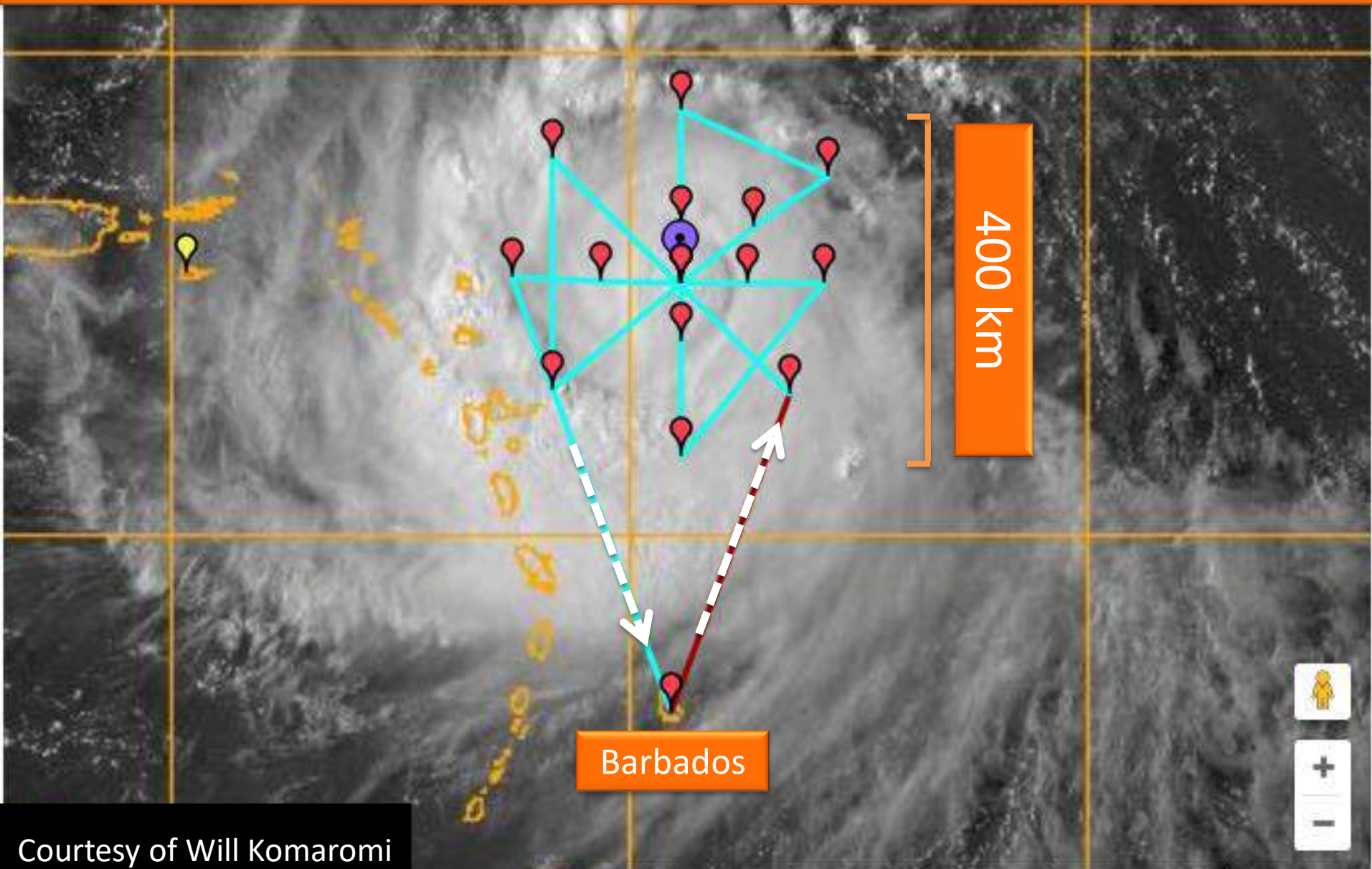


The GPS Dropsonde



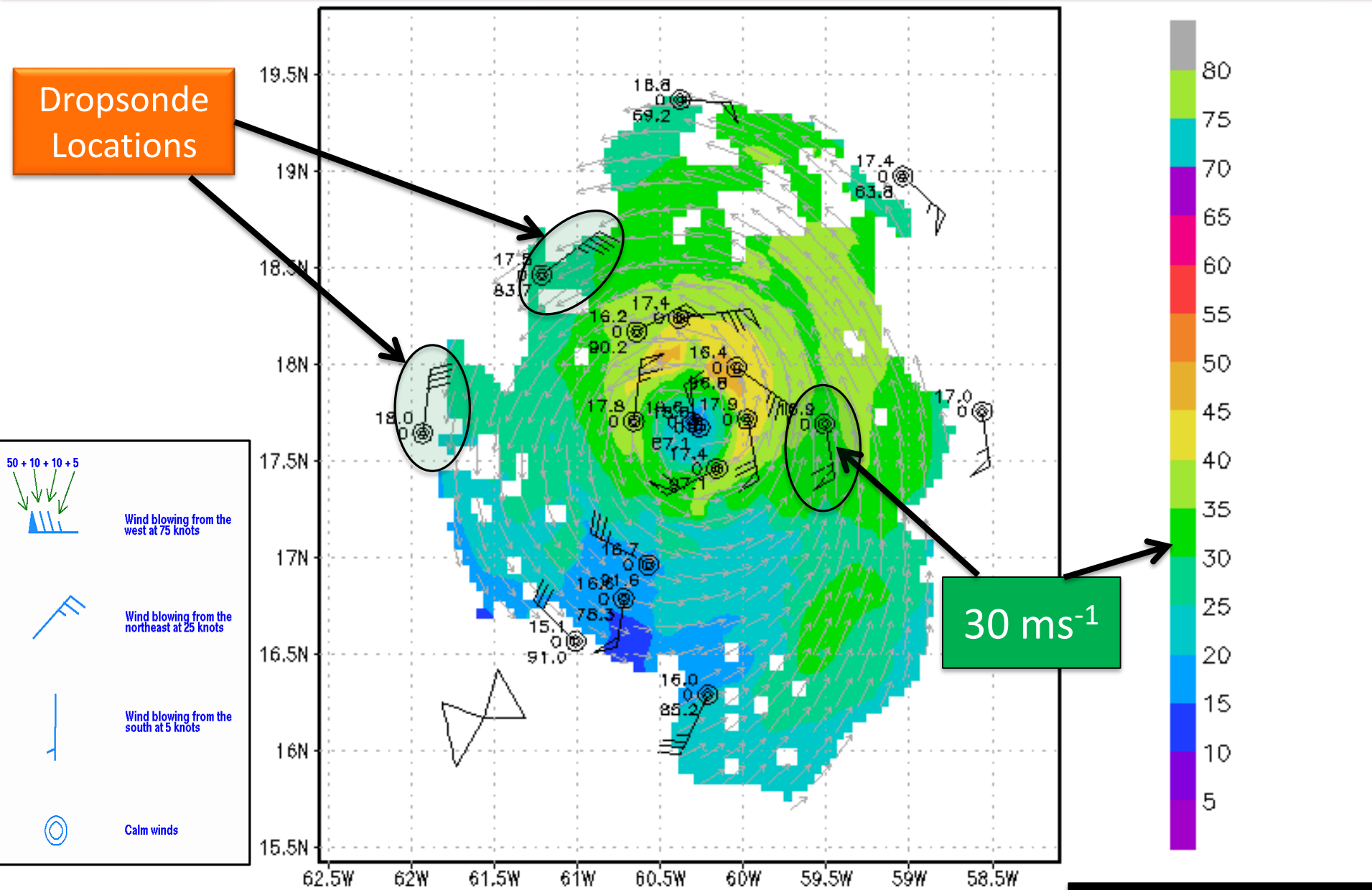
Hurricane Earl (2010)

Hypothetical Flight Plan and Dropsonde Locations



Courtesy of Will Komaromi

Hurricane Earl, Wind Speed (ms^{-1}) at $z=2$ km



Dropsonde Locations

30 ms^{-1}

50 + 10 + 10 + 5

Wind blowing from the west at 75 knots

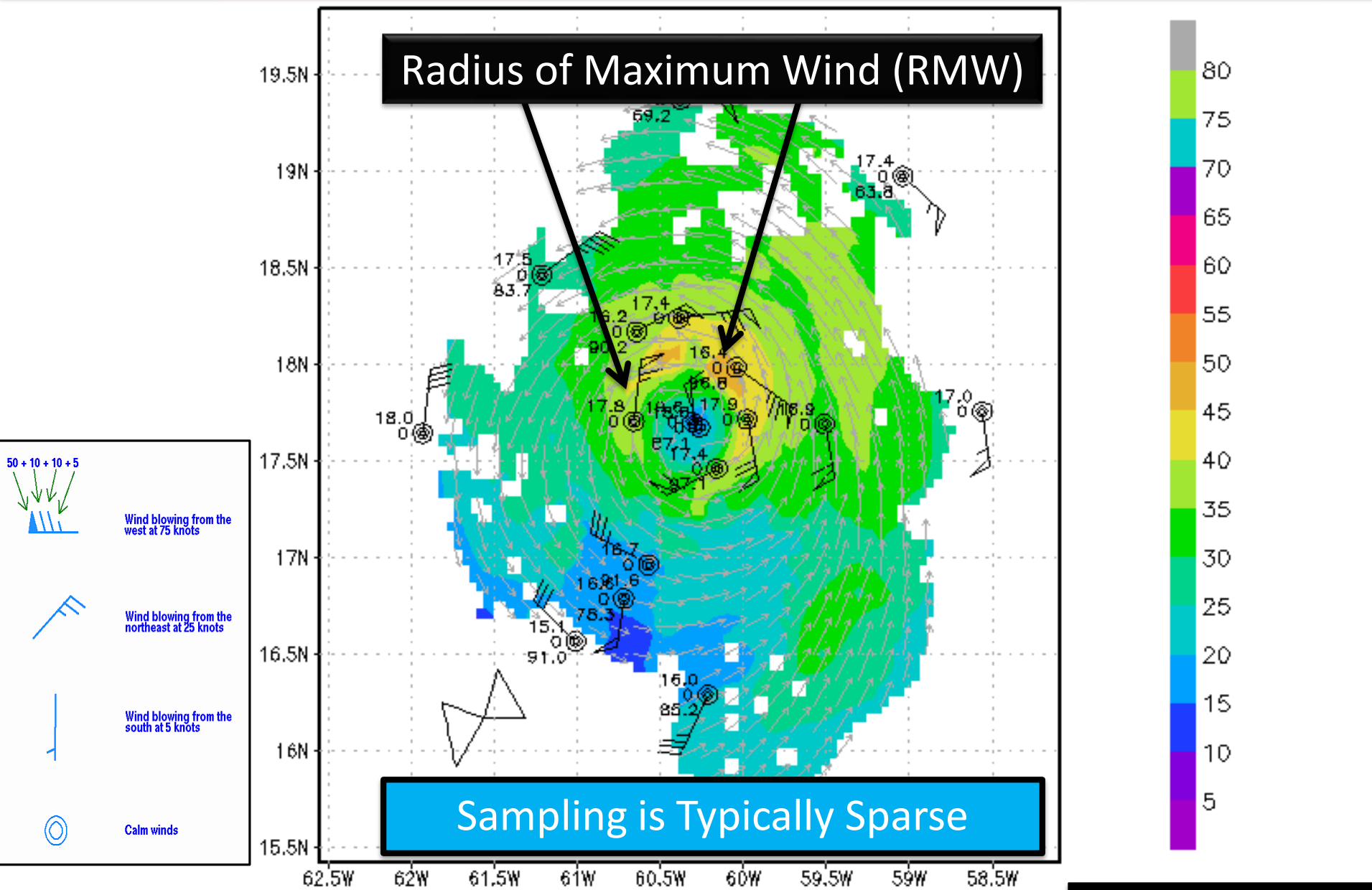
Wind blowing from the northeast at 25 knots

Wind blowing from the south at 5 knots

Calm winds

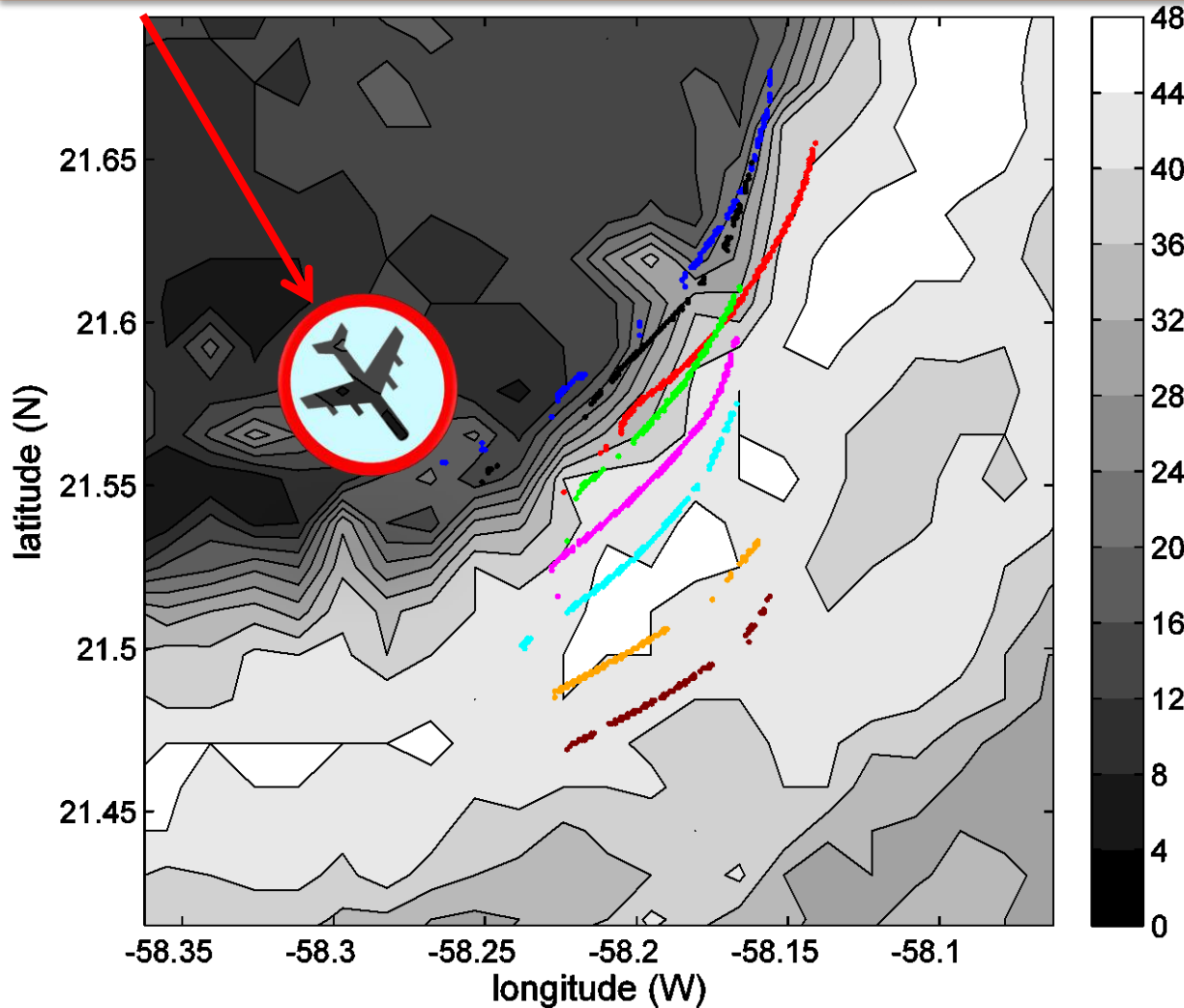
Source: NOAA/HRD

Hurricane Earl, Wind Speed (ms^{-1}) at $z=2$ km

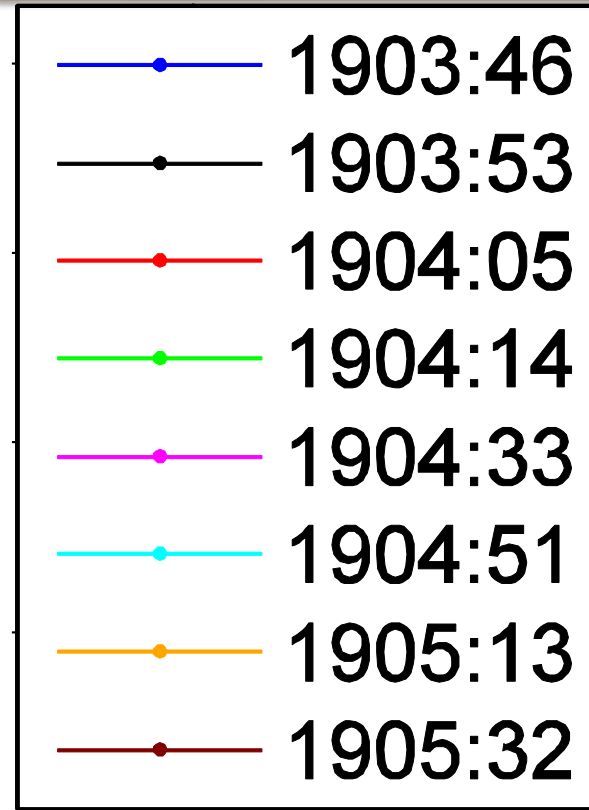


Source: NOAA/HRD

Hurricane Isabel, Radar Reflectivity



Time (UTC)

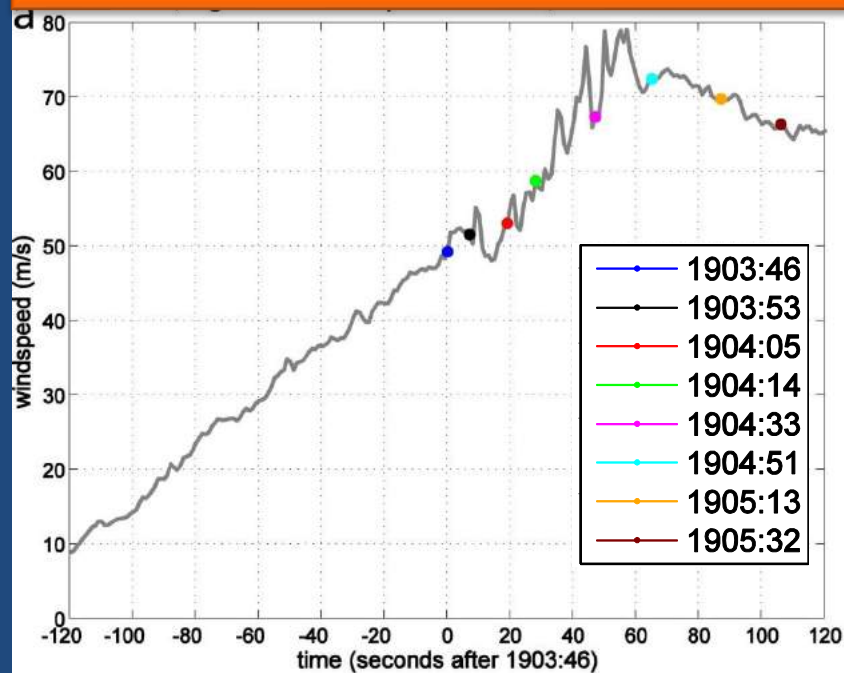


~30 km

Eyewall Sampling of Hurricane Isabel

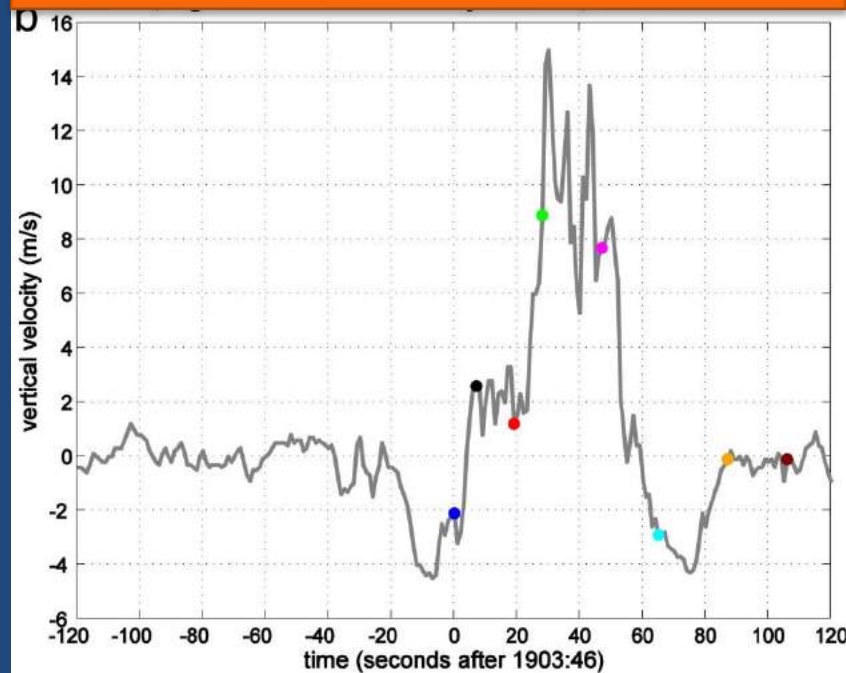
Time Series and Dropsonde Locations

Flight-Level Wind Speed



~30 km

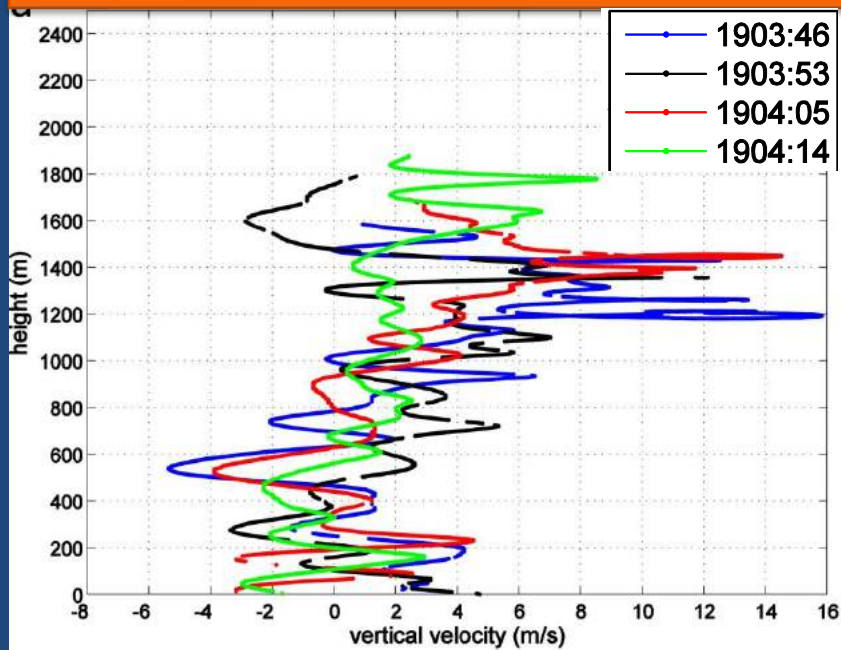
Flight-Level Vertical Velocity



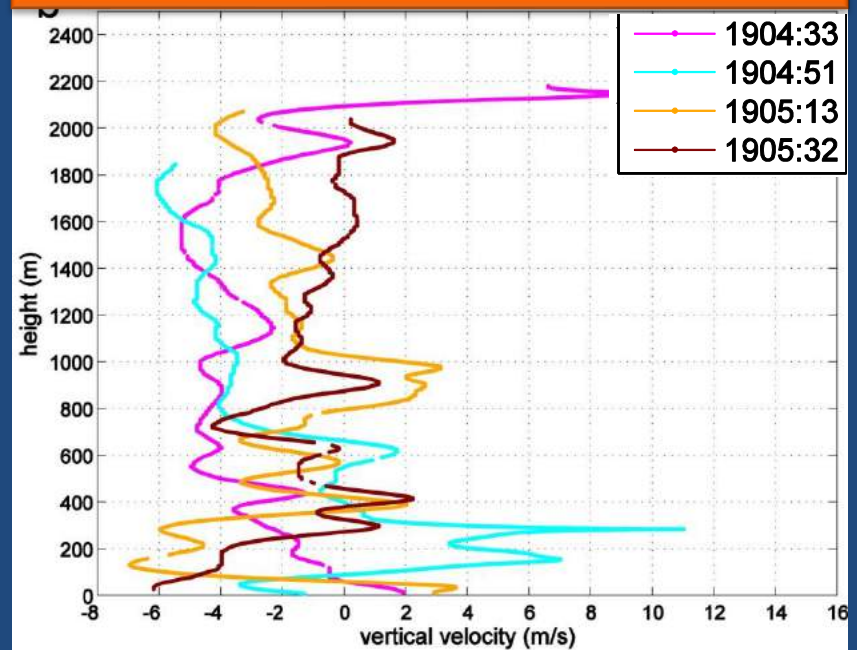
~30 km

Dropsonde Profiles in Hurricane Isabel

Vertical Velocity (ms^{-1})

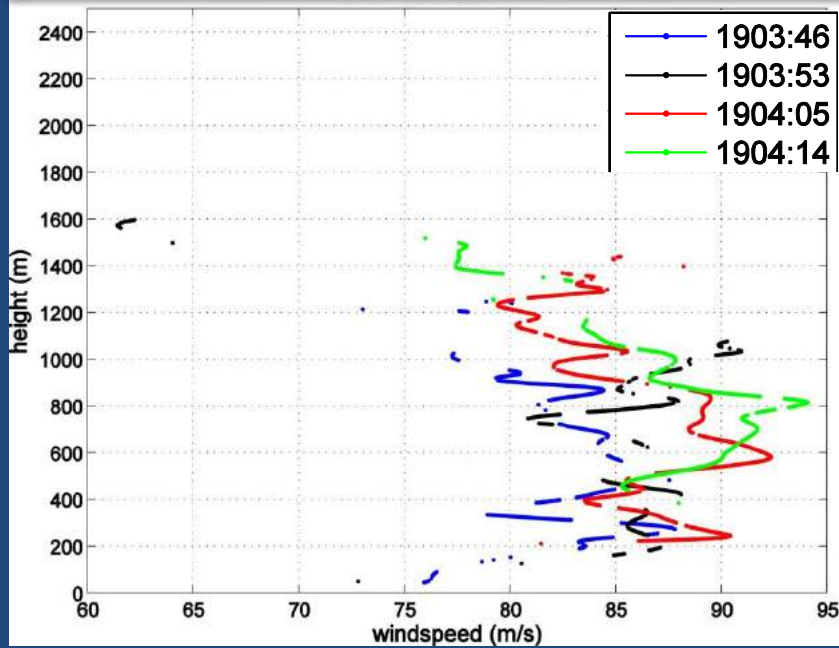


Vertical Velocity (ms^{-1})

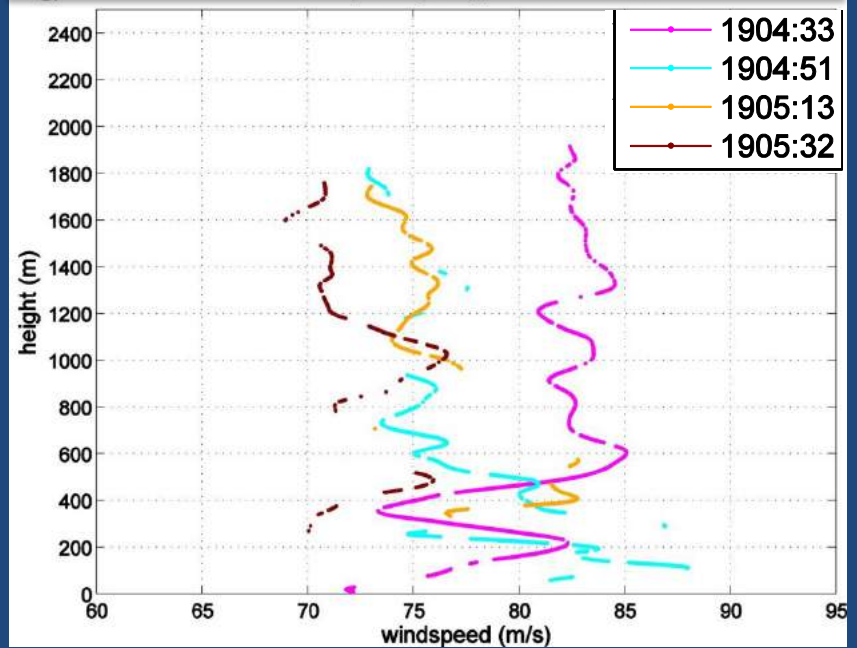


Dropsonde Profiles in Hurricane Isabel

Wind Speed (ms^{-1})



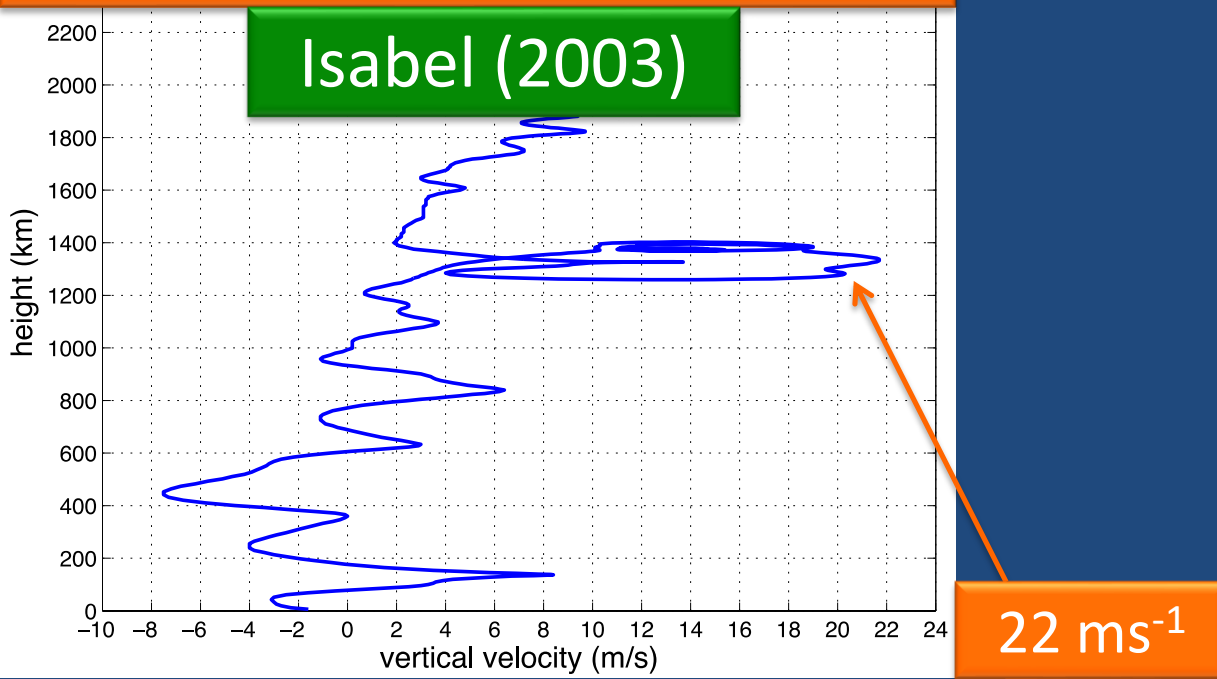
Wind Speed (ms^{-1})



Motivation

- Dropsondes occasionally sample 10-25 ms^{-1} updrafts within (and near) the TC boundary layer.

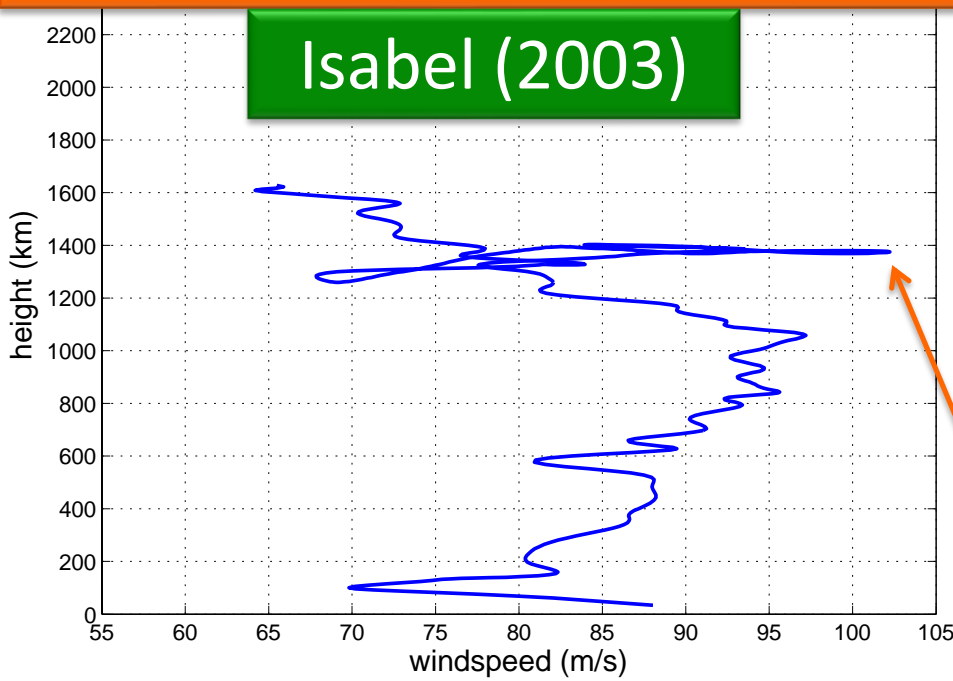
Vertical Velocity vs. Height



Motivation

- The extreme updrafts appear to be associated with extreme horizontal wind speeds ($> 90 \text{ ms}^{-1}$).

Wind Speed vs. Height



Hypothesis:

Extreme updrafts and near-surface wind speeds are associated with small-scale ($\leq 1 \text{ km}$) coherent vortices.

102 ms^{-1}

A New Dataset of Extreme Updrafts and Windspeeds

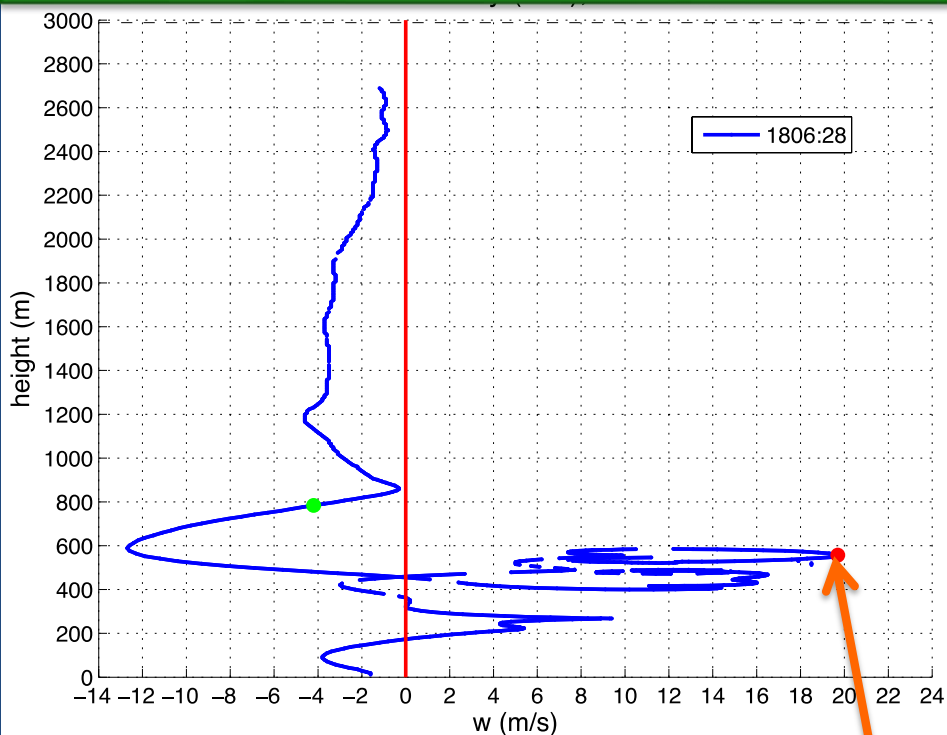
- ~12,000 sondes have been dropped into TCs
 - from NOAA and U.S. Air Force (1997-2013)

We found:

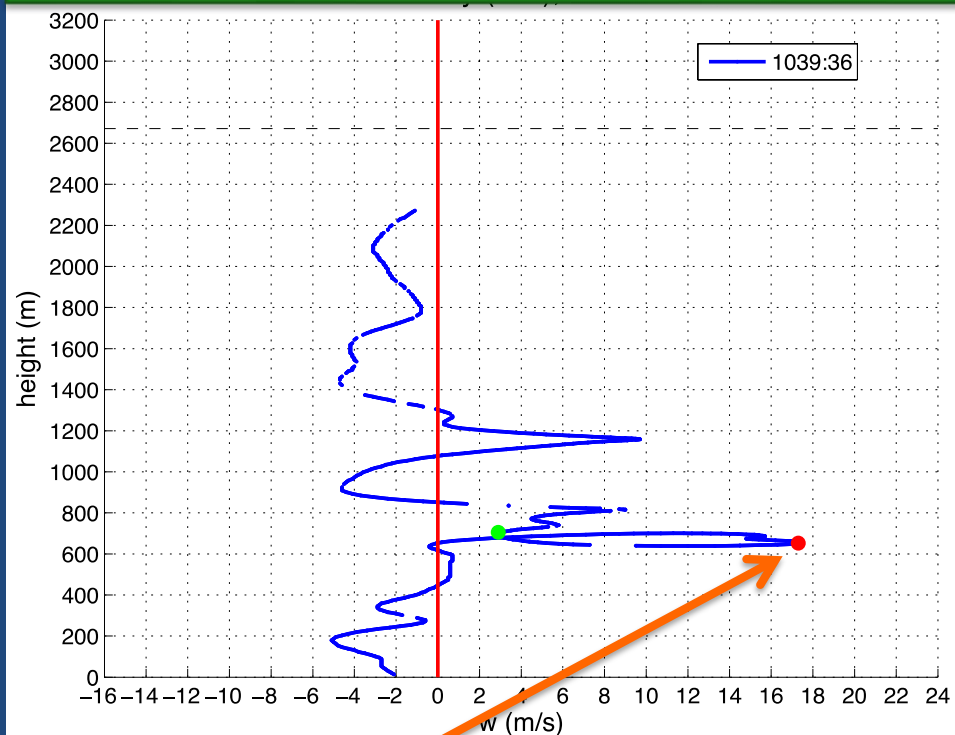
- 169 sondes (in 35 TCs) with $w > 10 \text{ ms}^{-1}$
- 64 sondes (in 12 TCs) with $wspd > 90 \text{ ms}^{-1}$

Examples: Extreme Updrafts

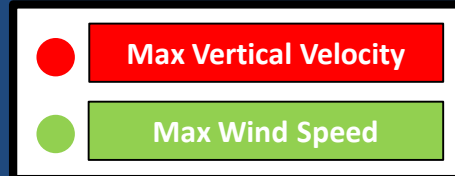
Hurricane Iris (2001)



Hurricane Frances (2004)

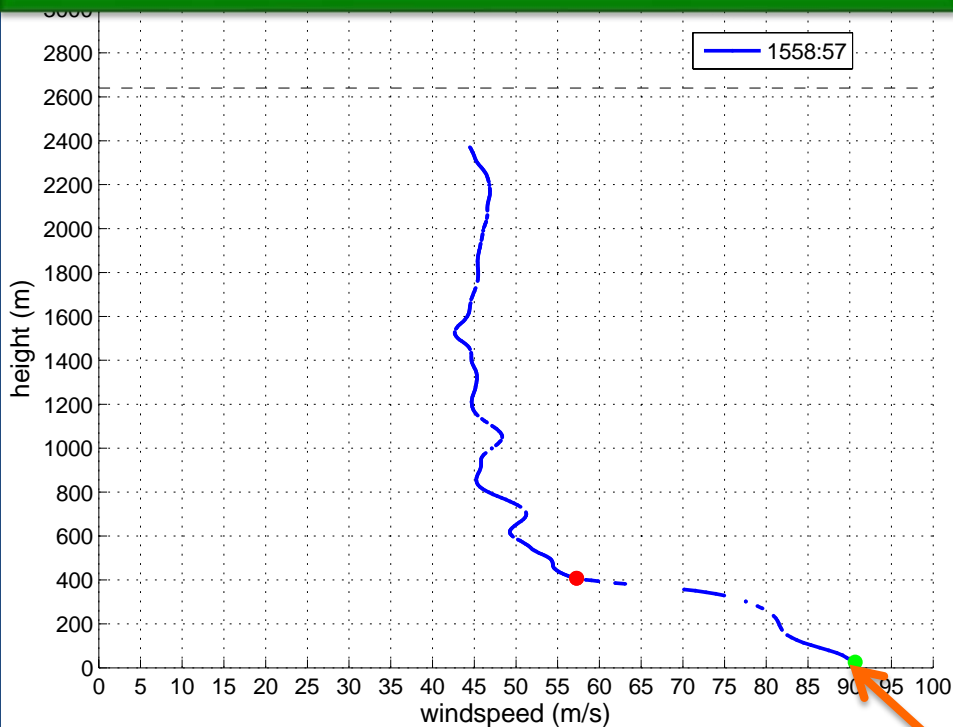


$>17 \text{ ms}^{-1}$ updrafts at $z=600 \text{ m}$

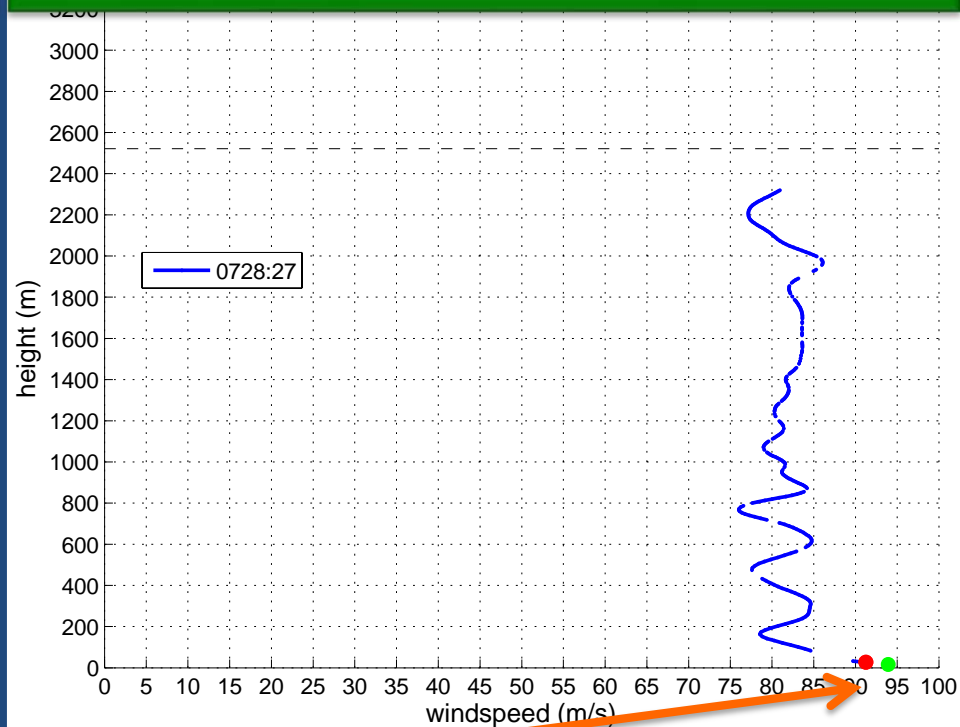


Examples: Extreme Windspeeds

Hurricane Rita (2005)



Hurricane Dean (2007)

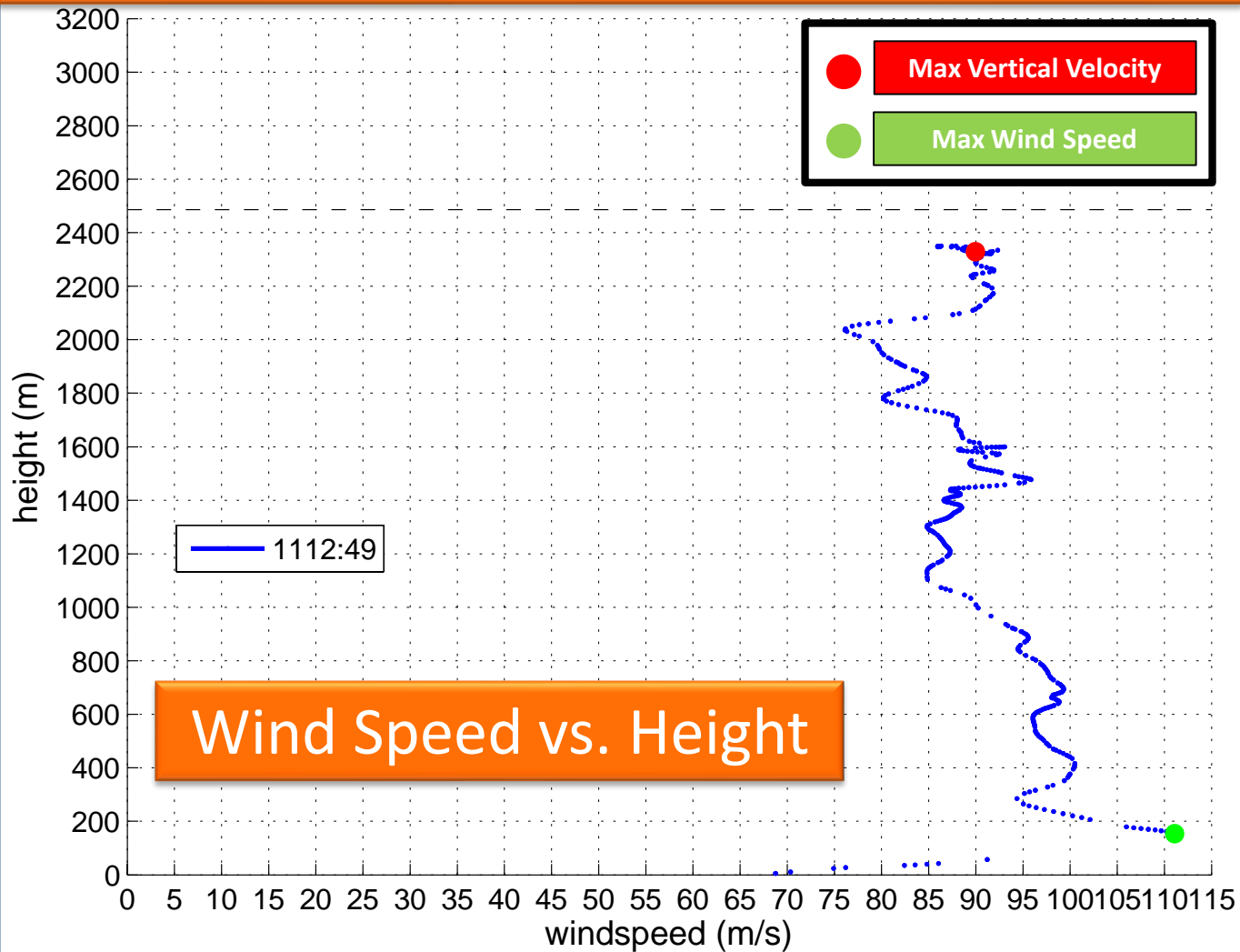


>90 ms^{-1} windspeed at $z < 100$ m

● Max Vertical Velocity

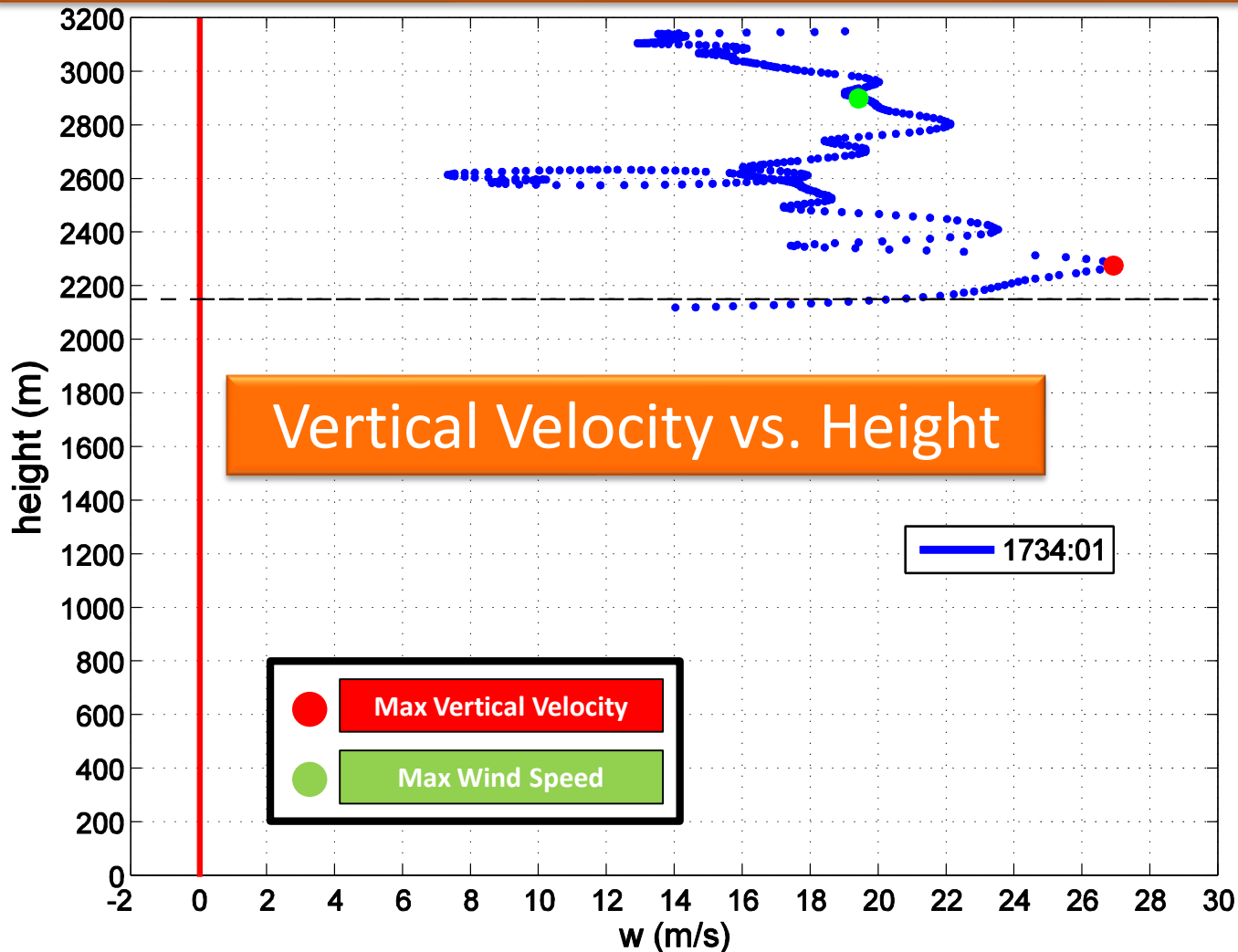
● Max Wind Speed

Oct 17th 2010, Super Typhoon Megi



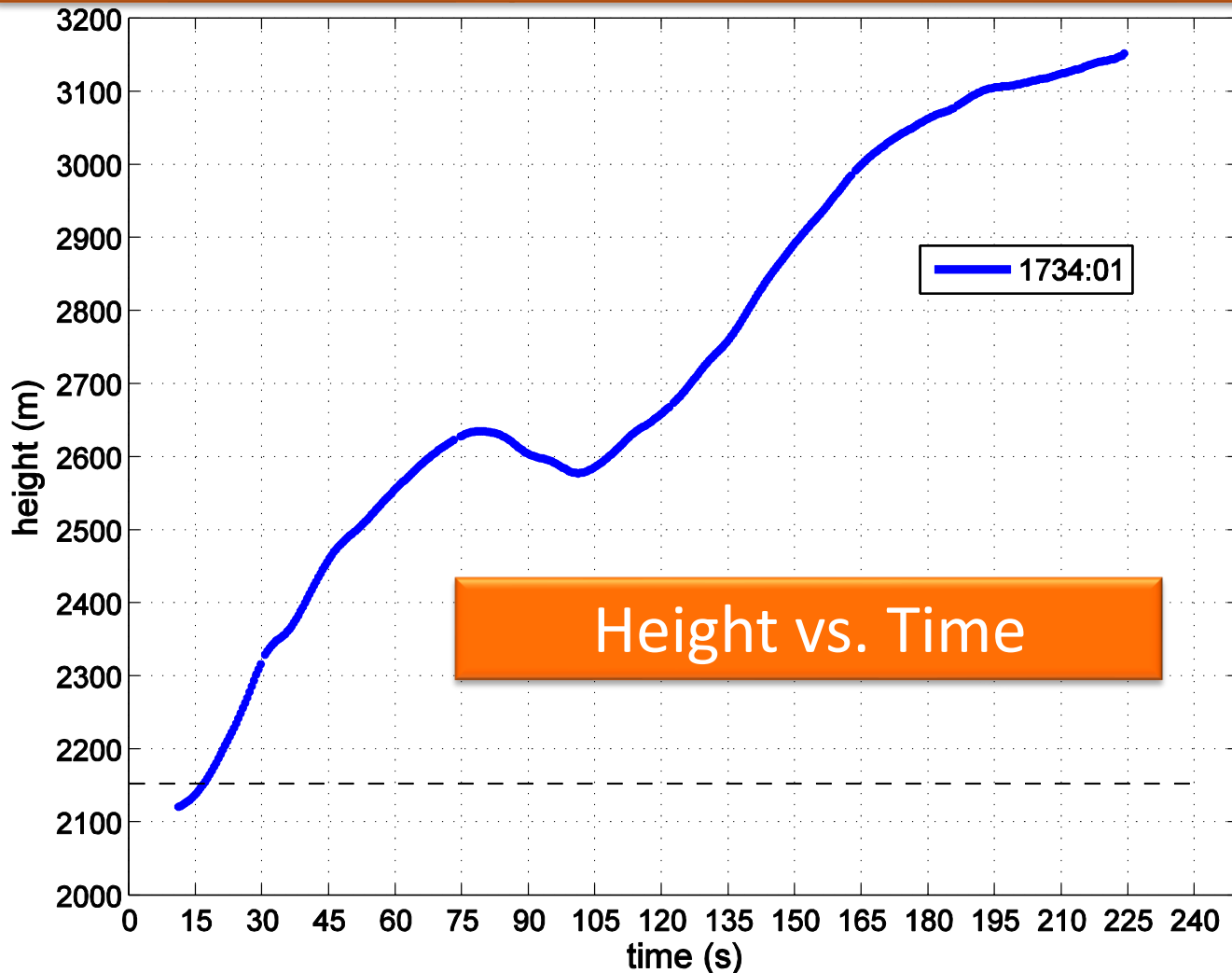
Maximum winds measured by any dropsonde: 111 ms^{-1} at $\sim 150 \text{ m ASL}$

Oct 23rd 2015, Hurricane Patricia



Maximum updraft measured by any dropsonde: 26.9 ms^{-1} at $\sim 2300 \text{ m ASL}$

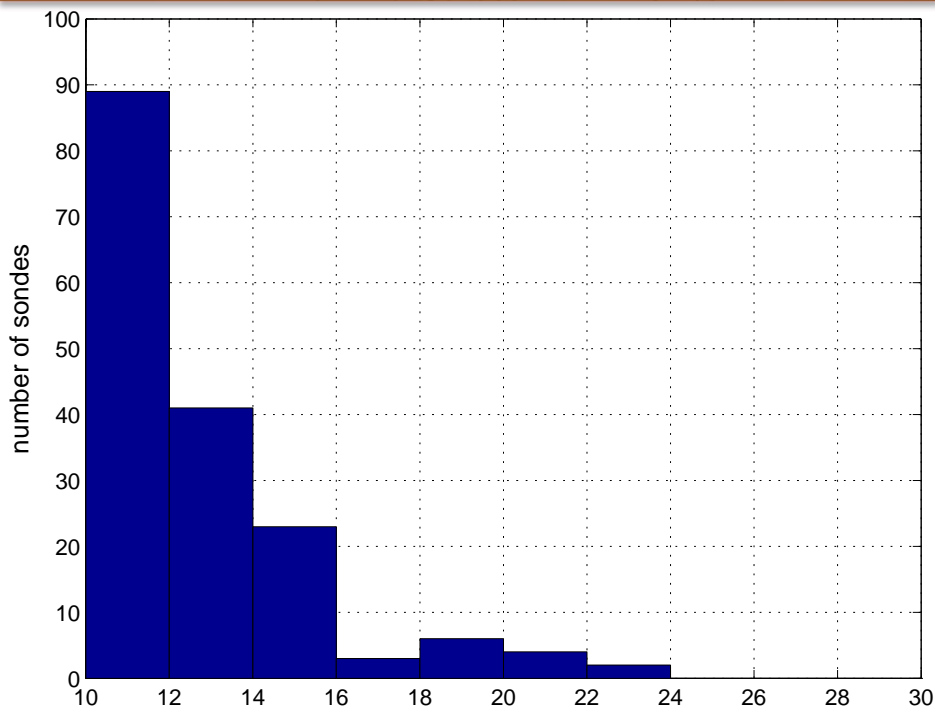
Oct 23rd 2015, Hurricane Patricia



Sonde was dropped at 2100 m, rose by 1000 m, then failed at 3100 m

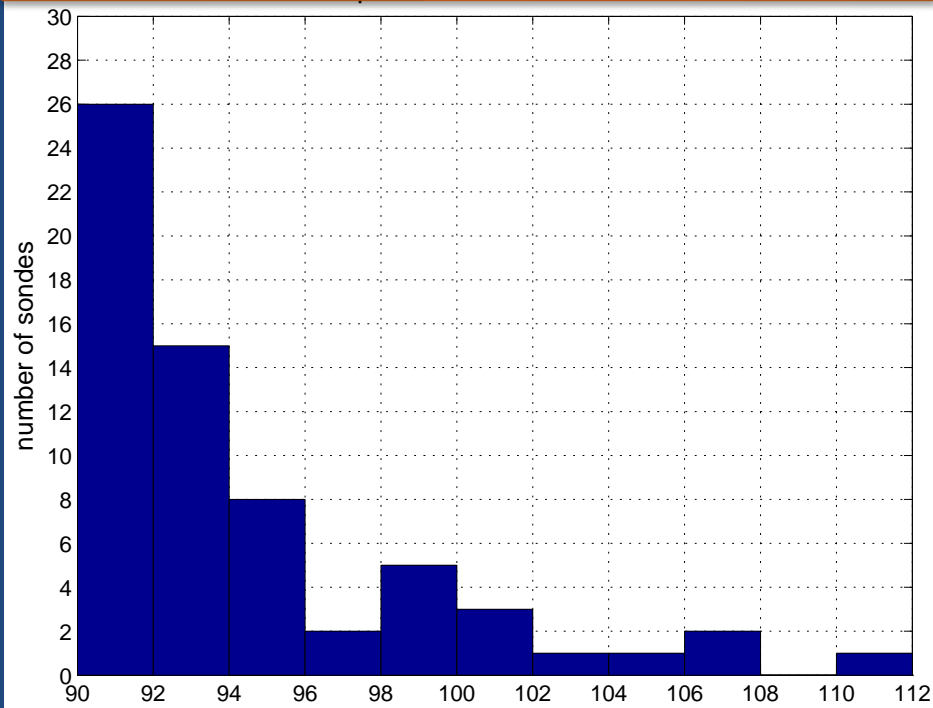
Distribution of Updraft/Windspeed Magnitudes

Sondes with $w > 10 \text{ ms}^{-1}$



Maximum Vertical Velocity (ms^{-1})

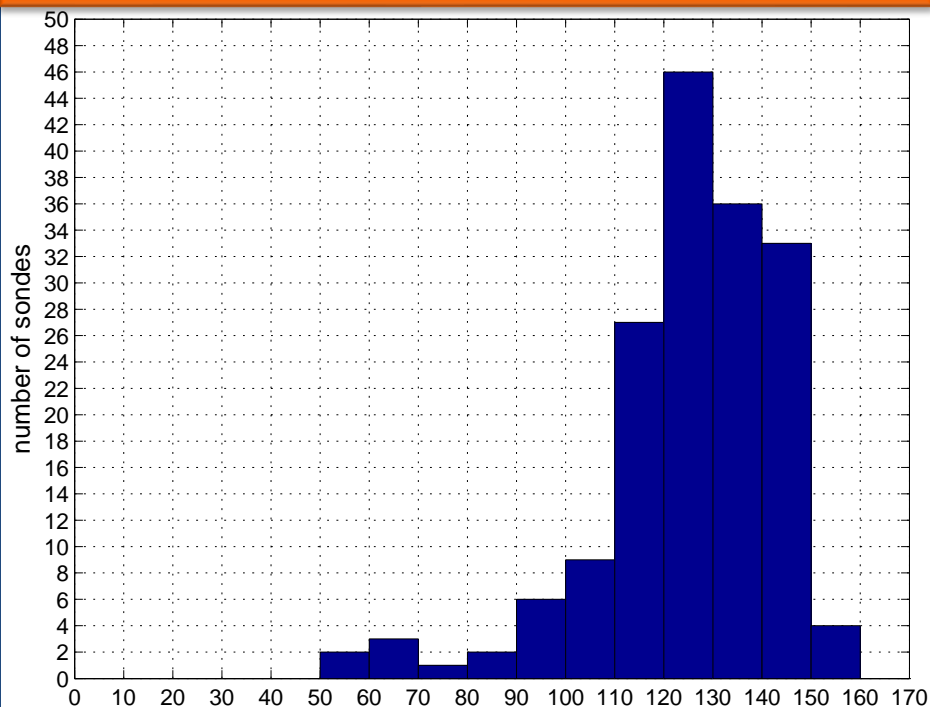
Sondes with $\text{wspd} > 90 \text{ ms}^{-1}$



Maximum Wind Speed (ms^{-1})

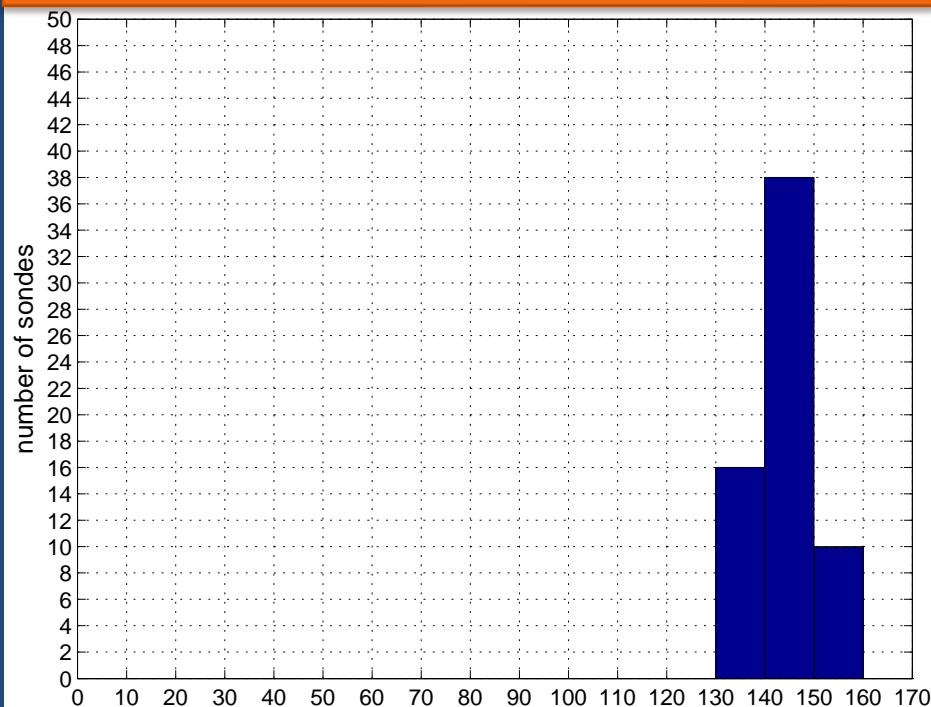
Relationship with Storm Intensity

Sondes with $w > 10 \text{ ms}^{-1}$



Best Track Intensity (kt)

Sondes with $wspd > 90 \text{ ms}^{-1}$



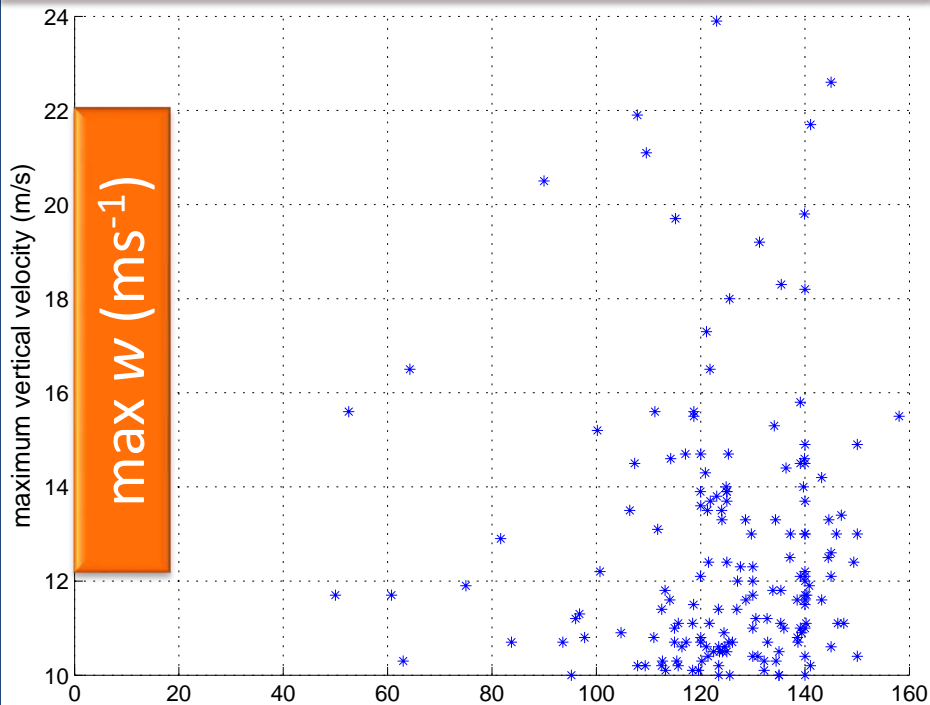
Best Track Intensity (kt)

92% of extreme updrafts from Cat 3-5

All extreme wind gusts from Cat 4-5

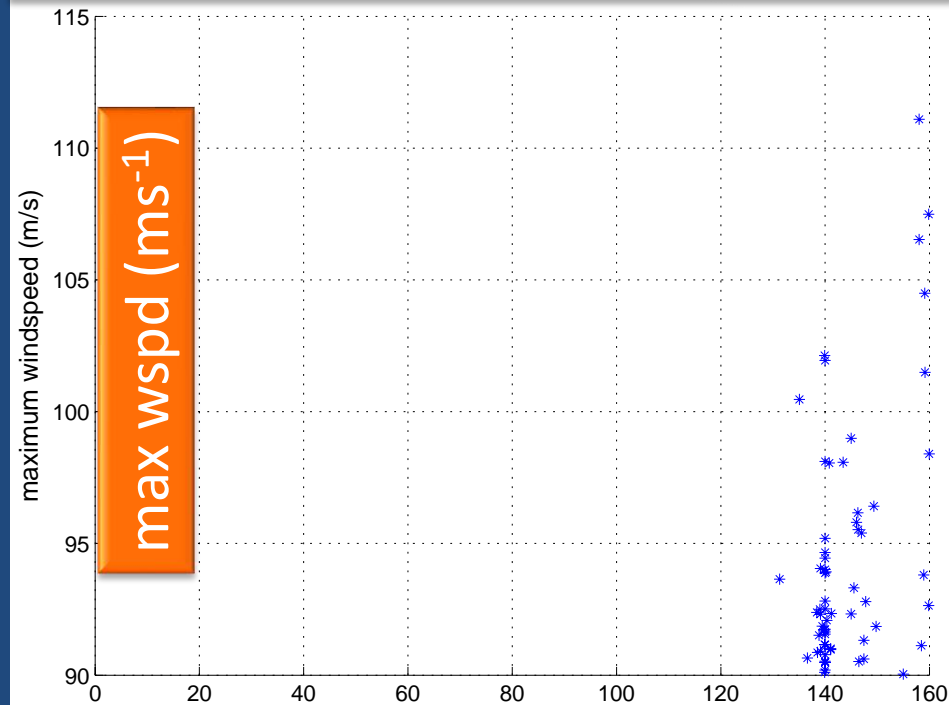
Relationship with Storm Intensity

Sondes with $w > 10 \text{ ms}^{-1}$



Best Track Intensity (kt)

Sondes with $wspd > 90 \text{ ms}^{-1}$

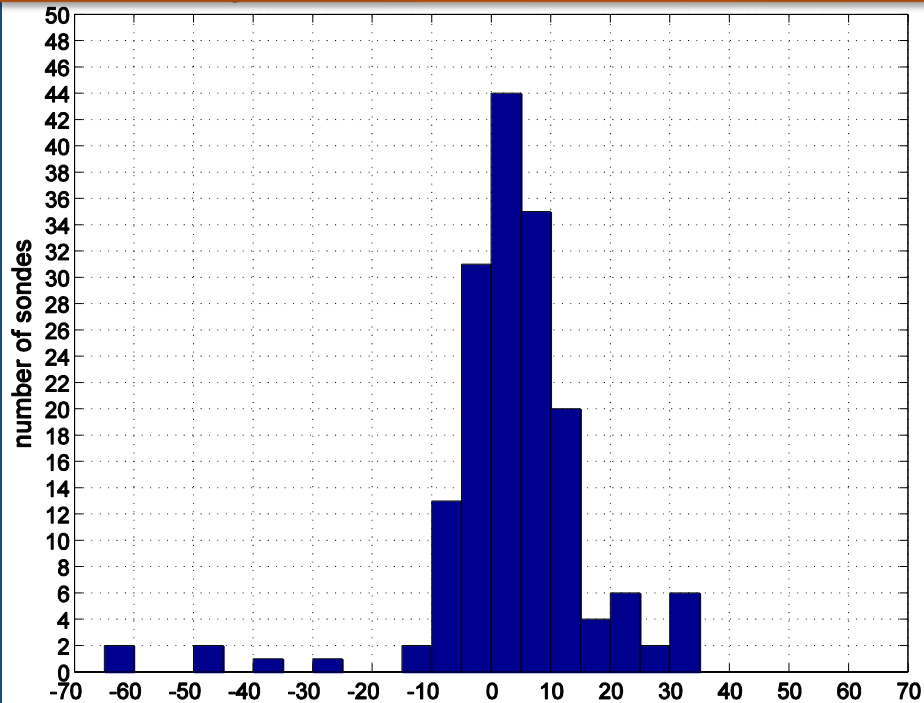


Best Track Intensity (kt)

- No correlation of updraft/windspeed magnitude with TC intensity.
- This is likely related to limited sampling.

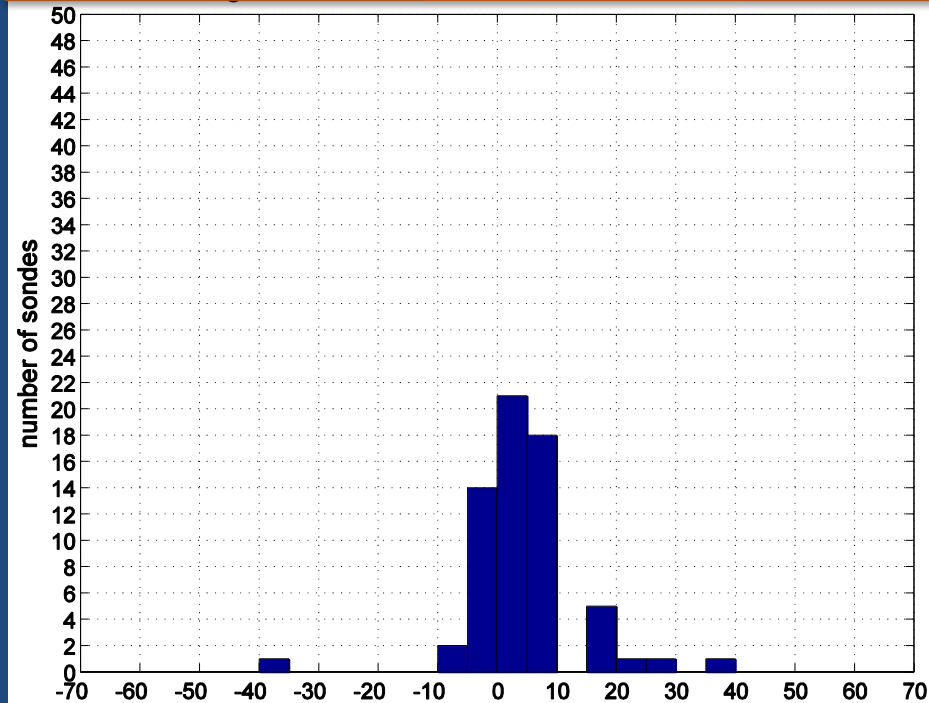
Relationship with Intensity Change

Sondes with $w > 10 \text{ ms}^{-1}$



6-h Intensity Change (kt)

Sondes with $wspd > 90 \text{ ms}^{-1}$

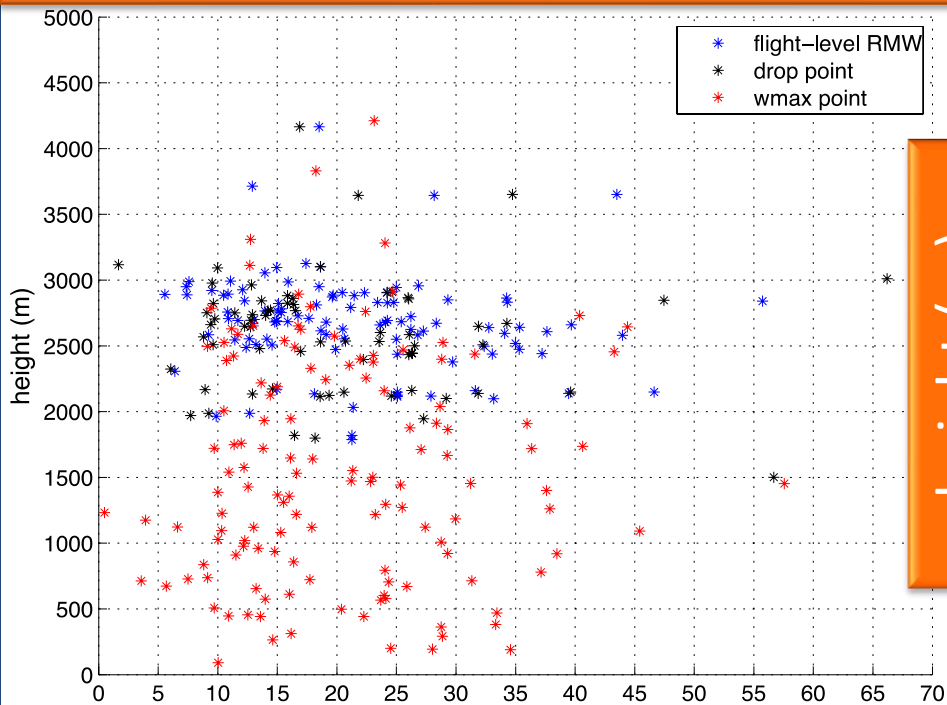


6-h Intensity Change (kt)

No clear relationship between TC intensity change and the frequency of extreme updrafts and wind gusts.

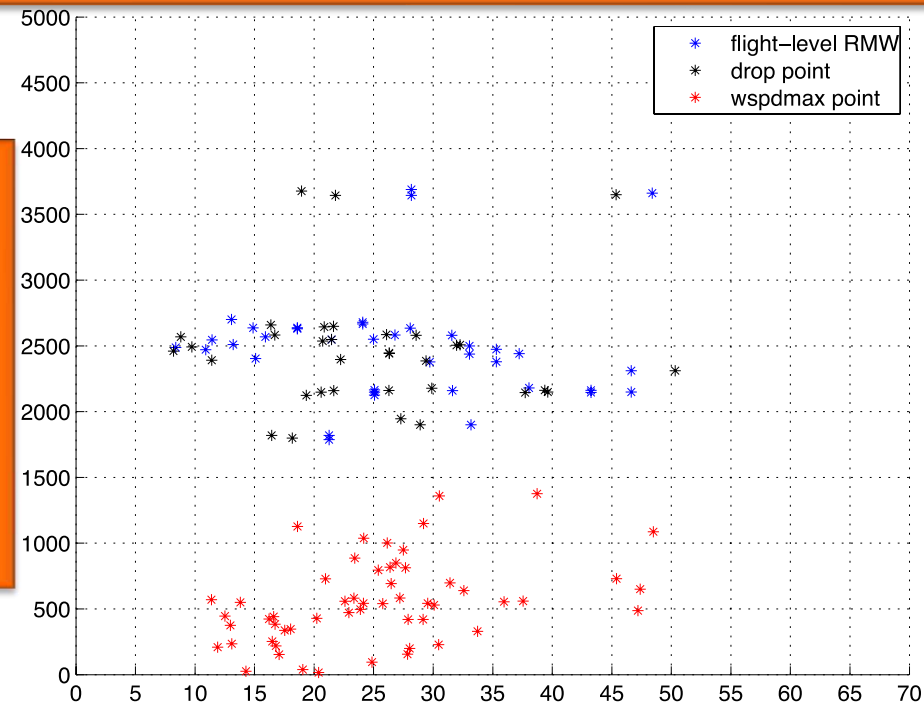
Radius-Height Location

Sondes with $w > 10 \text{ ms}^{-1}$



radius (km)

Sondes with $wspd > 90 \text{ ms}^{-1}$



radius (km)

height (m)

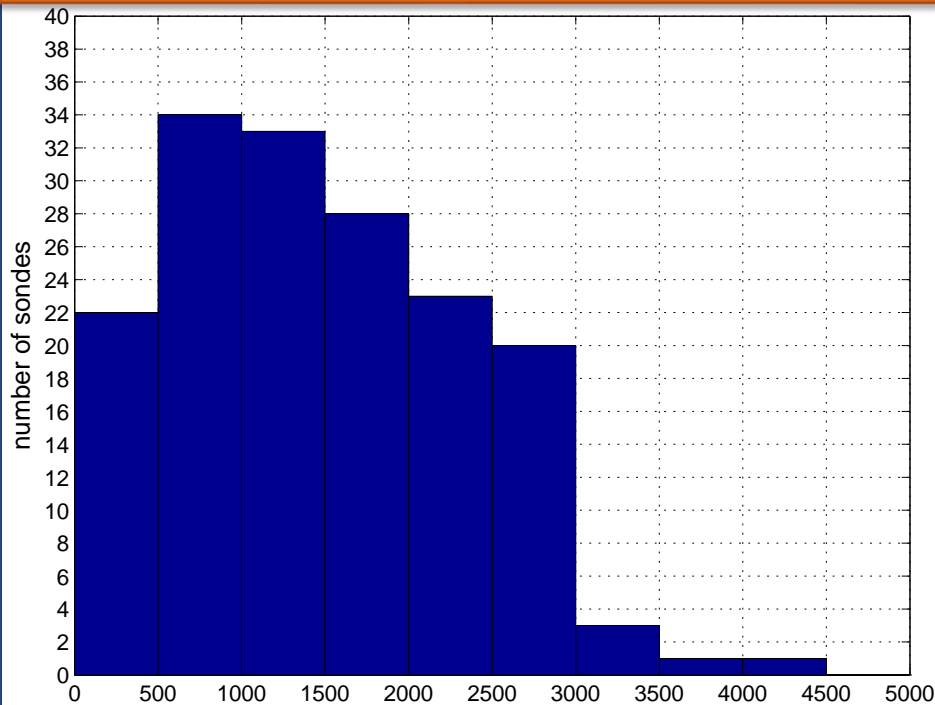
Red = Maximum

Black = Drop Point

Blue = Flight-Level RMW

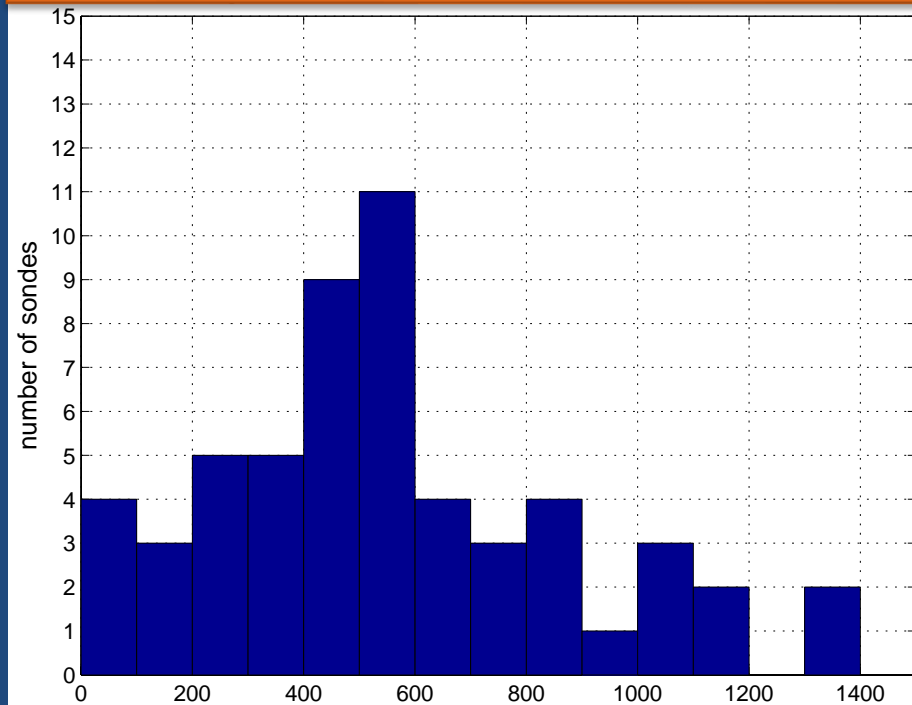
Heights of Maximum Updraft and Wind Speed

Sondes with $w > 10 \text{ ms}^{-1}$



Height of Max Vertical Velocity (m)

Sondes with $wspd > 90 \text{ ms}^{-1}$

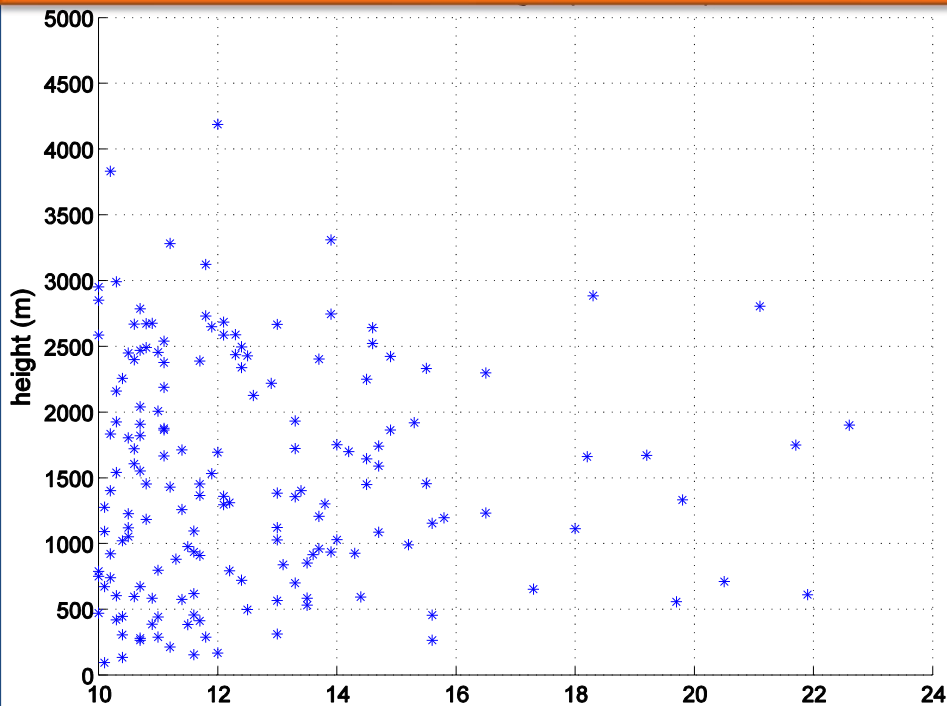


Height of Max Windspeed (m)

Extreme updrafts occur over a much deeper layer (note different axes)

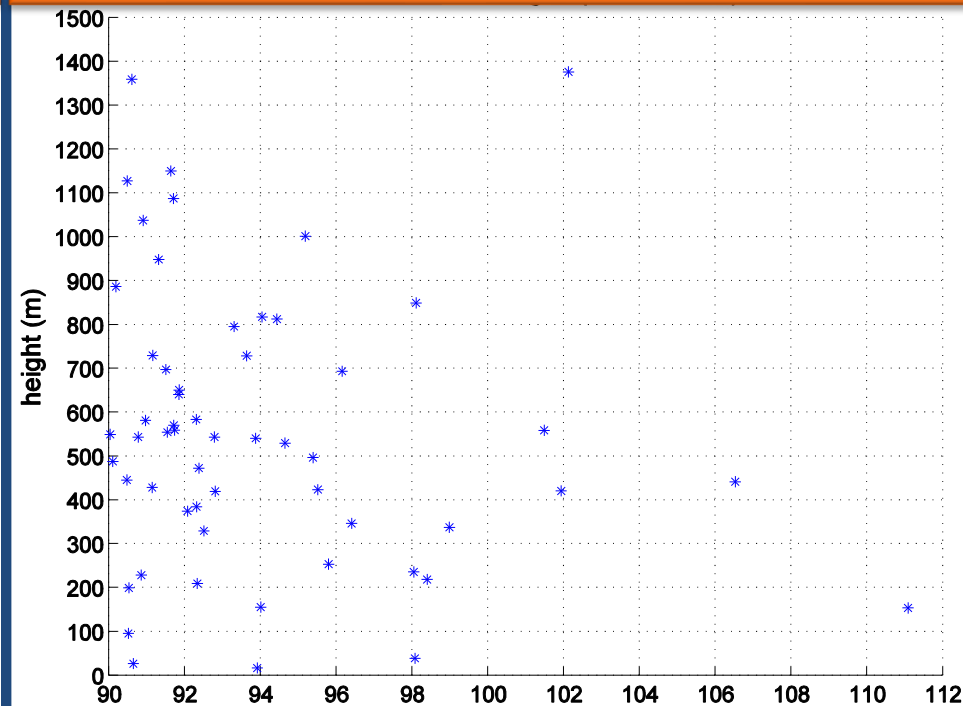
Maximum Updraft and Wind Speed vs. Height

Sondes with $w > 10 \text{ ms}^{-1}$



Maximum Vertical Velocity (ms^{-1})

Sondes with $wspd > 90 \text{ ms}^{-1}$



Maximum Wind Speed (ms^{-1})

No clear relationship between magnitudes and heights of extremes.

The Limits of Observations

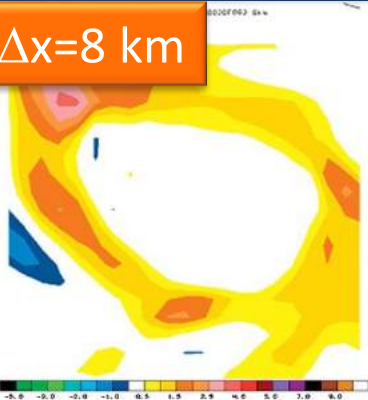
- Aircraft are unable to fly safely within the boundary layer in the eyewall.
- Dropsondes sample sparsely in space and time.
- We can gain further insight using Large-Eddy Simulations (LES).



Mesoscale Cloud-Permitting Modeling

Simulated Vertical Velocity, Hurricane Ivan (2004)

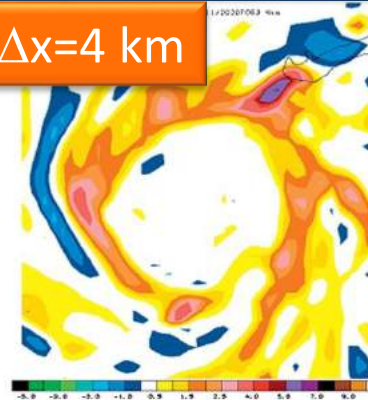
$\Delta x=8$ km



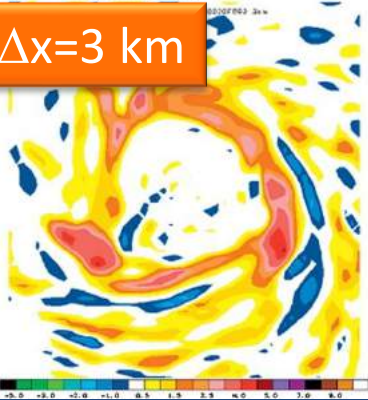
$\Delta x=6$ km



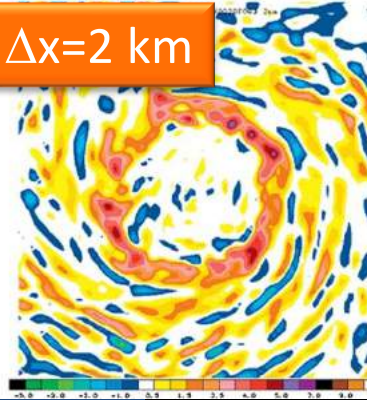
$\Delta x=4$ km



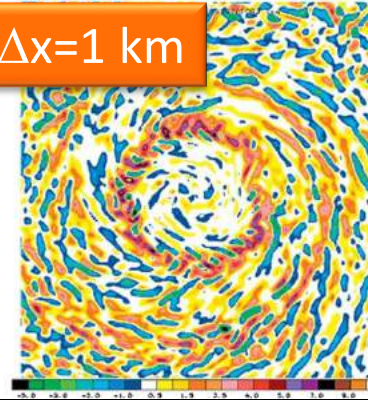
$\Delta x=3$ km



$\Delta x=2$ km



$\Delta x=1$ km



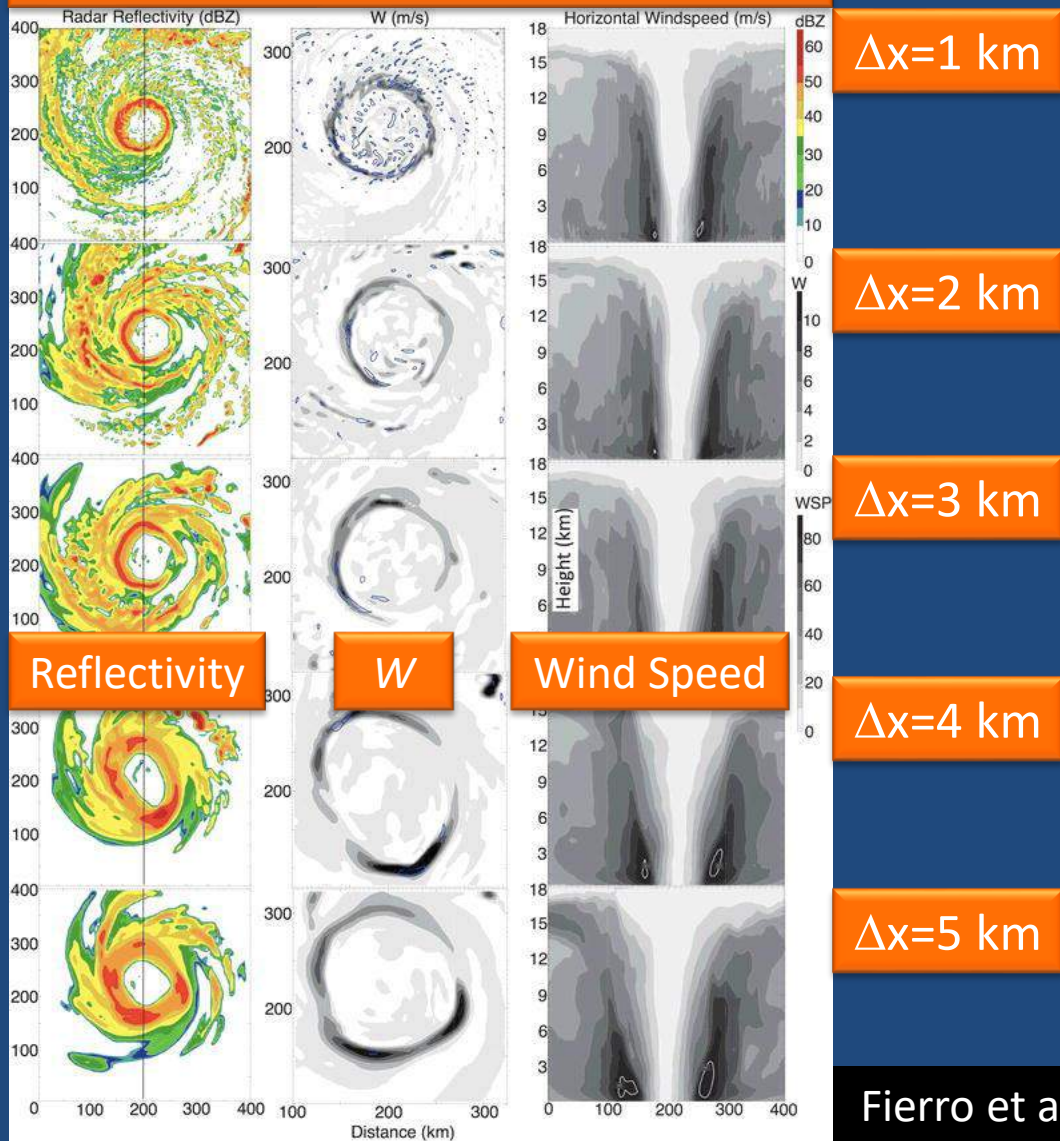
As grid spacing decreases:

- More detailed and (hopefully) realistic structures are resolved.
- Updrafts tend to get stronger.
- The inner core tends to get smaller.

Gentry and Lackmann (2010)

Mesoscale Cloud-Permitting Modeling

Simulated Hurricane Rita (2005)



Fierro et al. (2009)

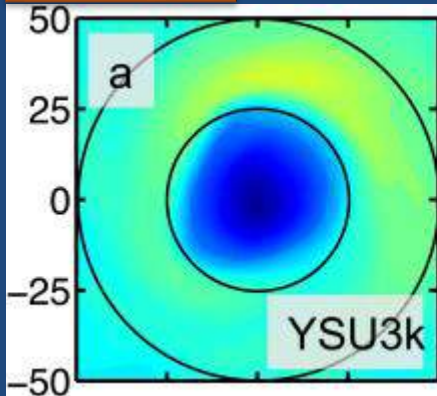
As grid spacing decreases:

- More detailed and (hopefully) realistic structures are resolved.
- Updrafts tend to get stronger.
- The inner core tends to get smaller.
- The eyewall decreases in width and the area of strong winds decreases.

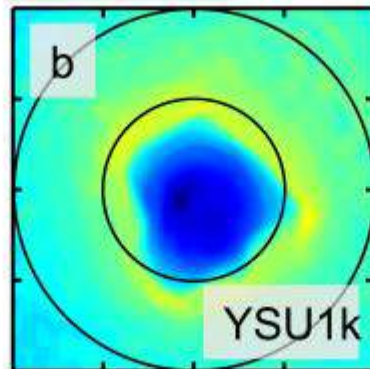
The Turbulent “Gray Zone”

- PBL schemes assume that ALL turbulence must be parameterized.
- But as we go to sub-kilometer grid spacing, we start to partially resolve large eddies, which can be problematic.

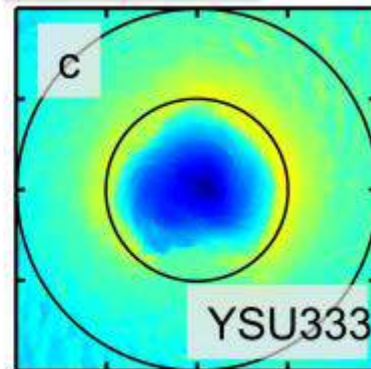
$\Delta x = 3$ km



$\Delta x = 1$ km



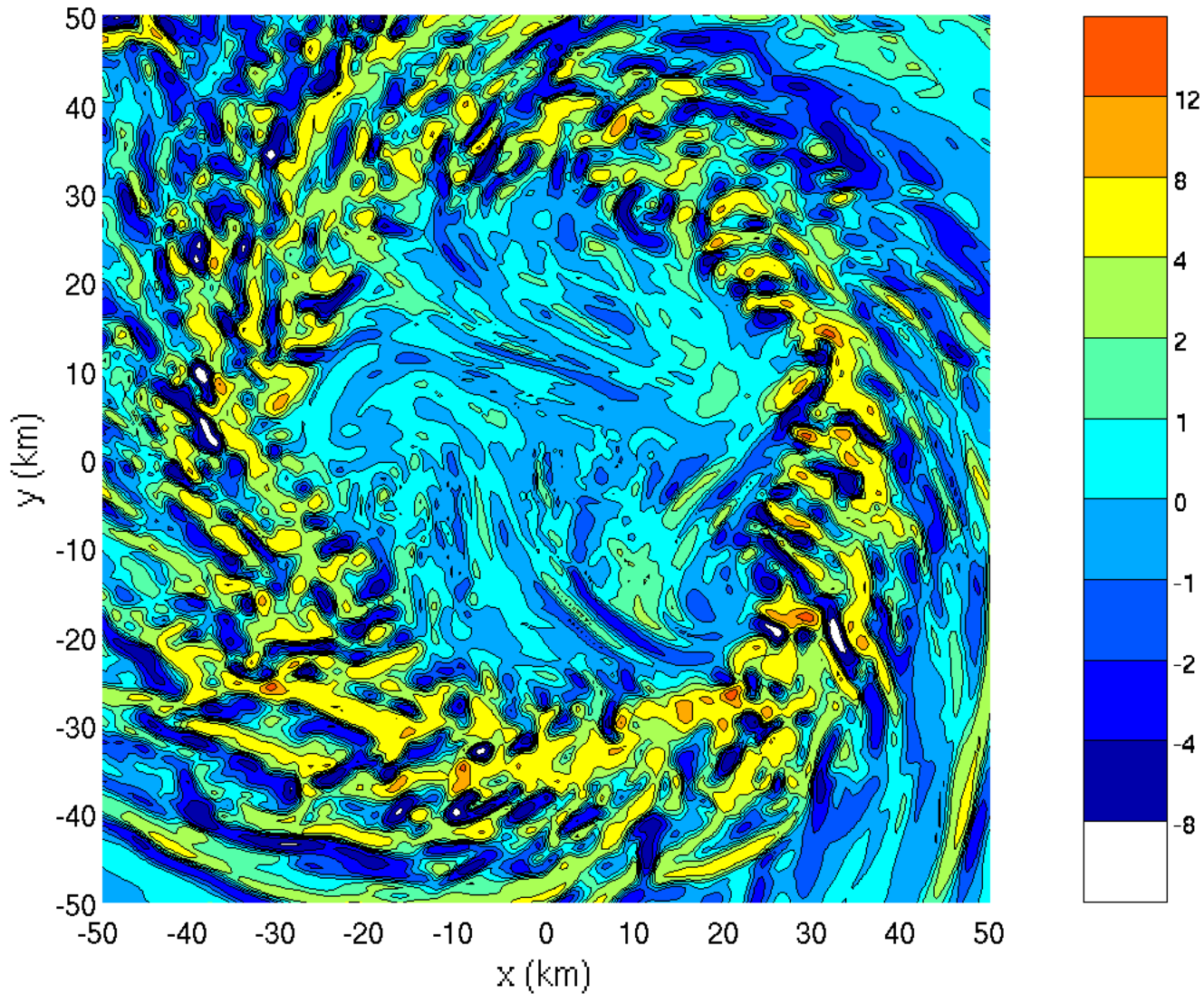
$\Delta x = 333$ m



Simulated Hurricane Katrina (2005), 10-m Wind Speed

Vertical Velocity, Isabel, $\Delta x=444$ m

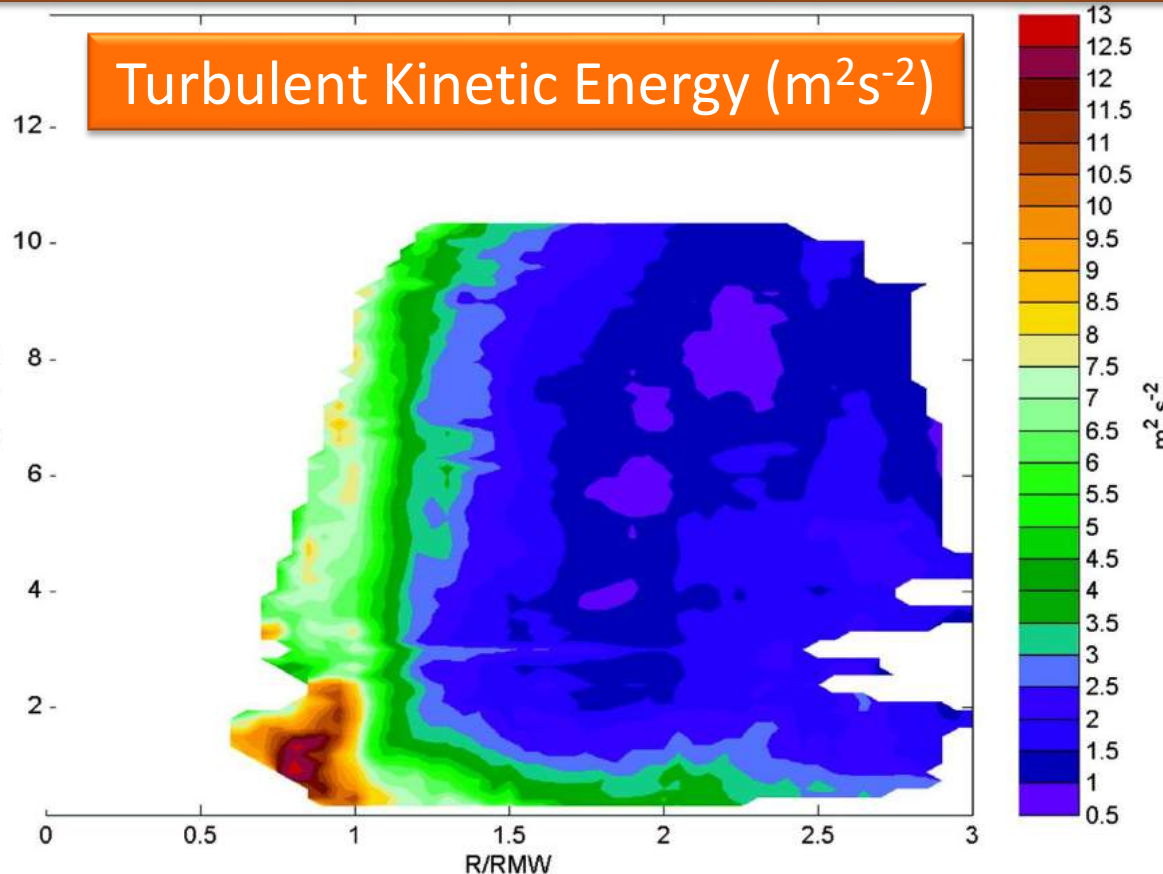
Isabel 18:00:00Z, Vertical Velocity (m/s), $z \sim 1$ km, max = 16 m/s



The Hurricane Eyewall is Turbulent

Composite of Major Hurricanes

Turbulent Kinetic Energy (m^2s^{-2})

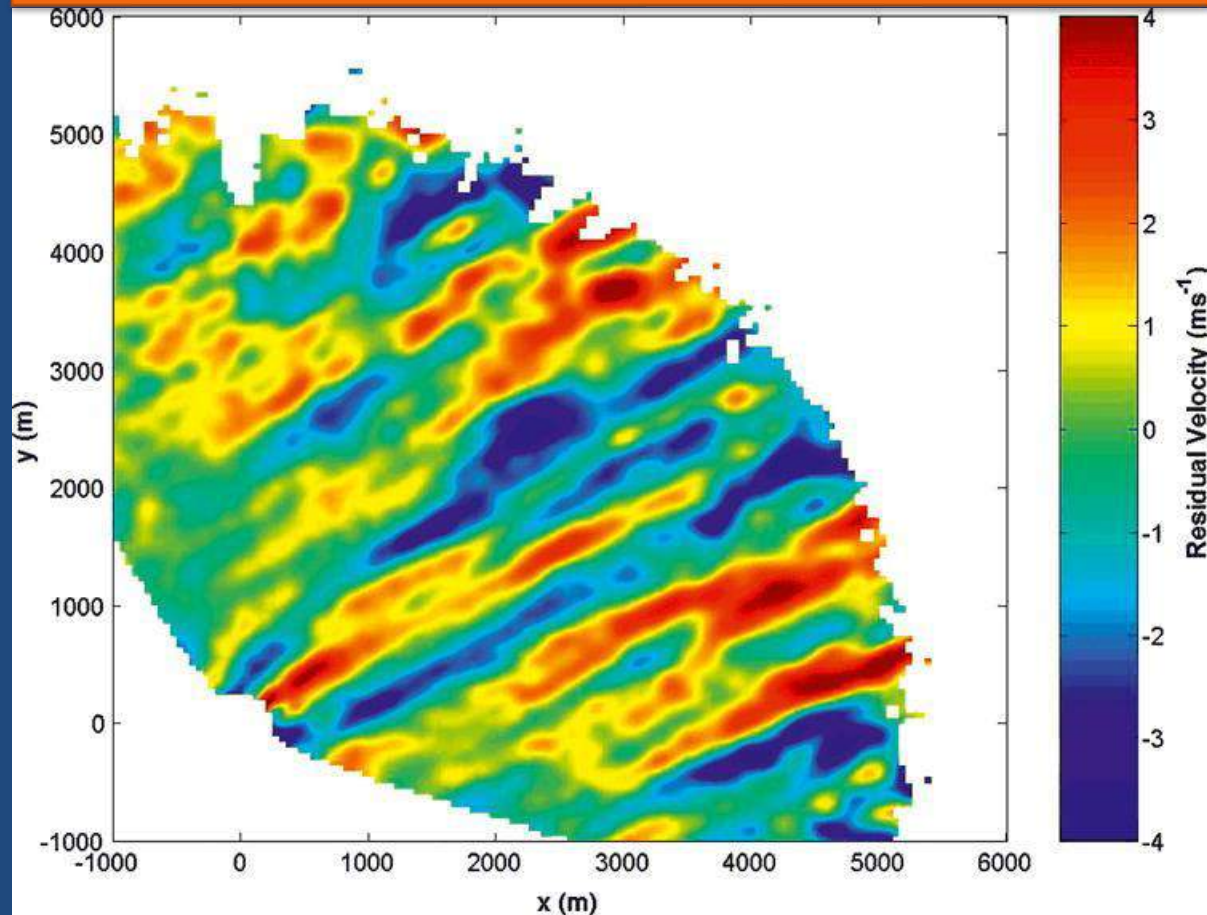


- The outer core is turbulent within the boundary layer.
- The eyewall is turbulent at all heights, especially in the lowest 2 km.

$$\text{TKE} = \frac{1}{2} (u'^2 + v'^2 + w'^2)$$

Boundary Layer Rolls

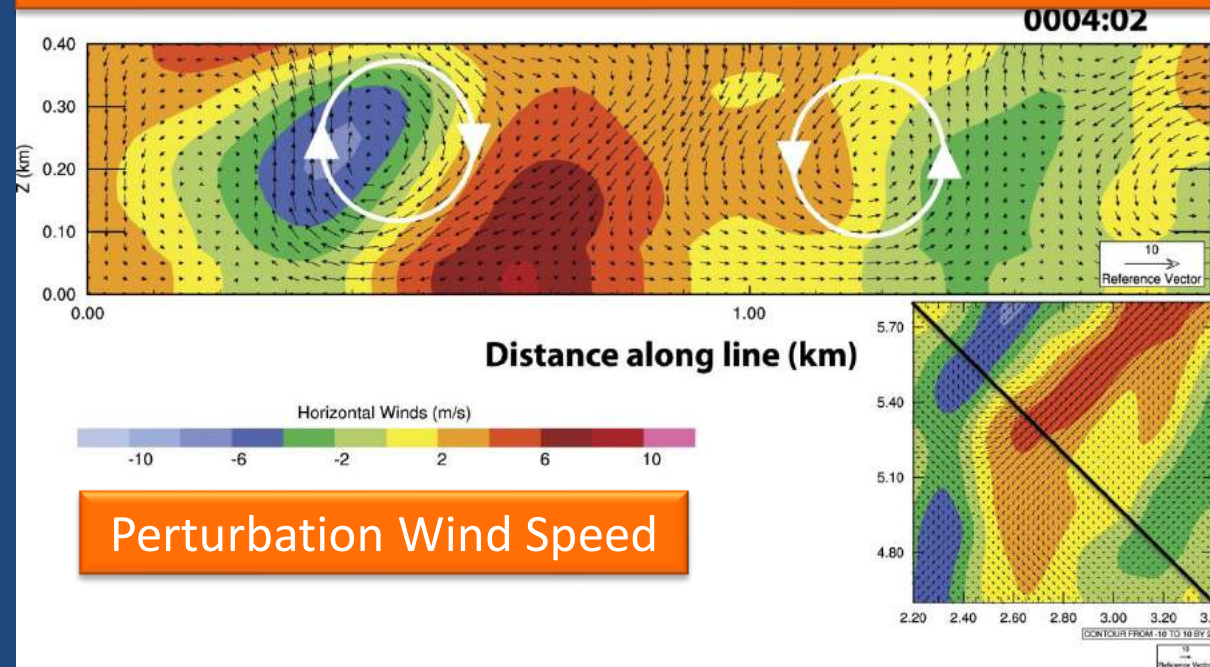
Perturbation Radial Velocity, $z=300$ m
Hurricane Frances (2004)



- Roll vortices are very common in the hurricane boundary layer.
- Scales vary, but are generally sub-kilometer.

Boundary Layer Rolls

Vertical Cross Section through Roll Hurricane Frances (2004)

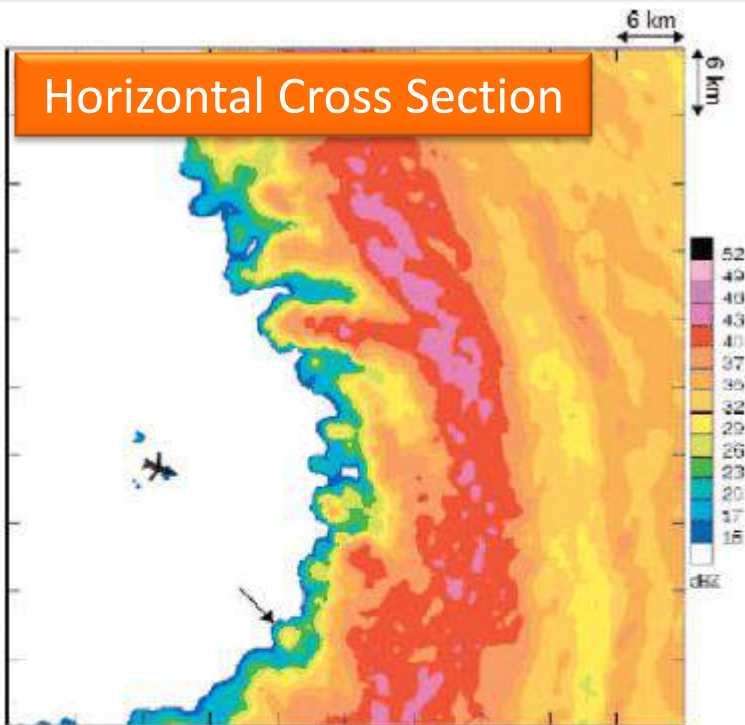


- Rolls tend to transport high momentum air upwards and low momentum air downwards.
- These features are generally unresolved by mesoscale models.

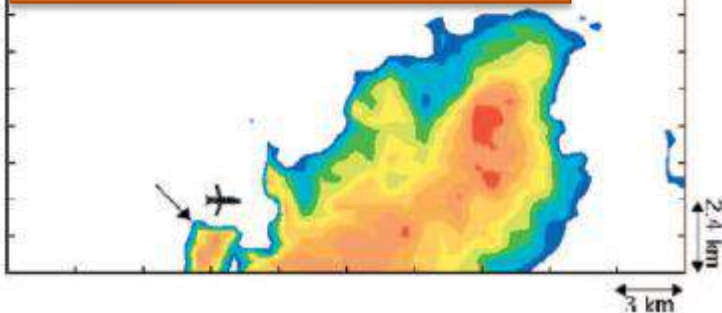
Coherent Eyewall Vortices

Hurricane Isabel (2003)

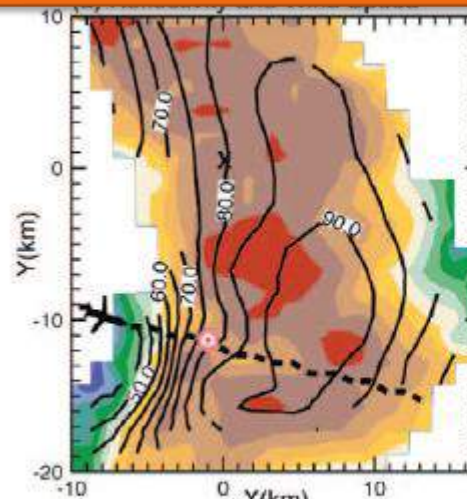
Horizontal Cross Section



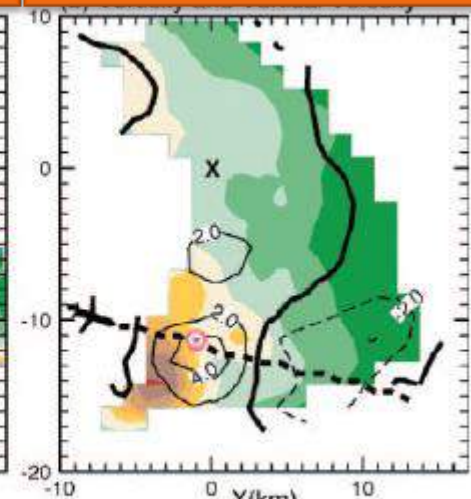
Vertical Cross Section



Reflectivity, Wind Speed



Vorticity, Vert. Velocity



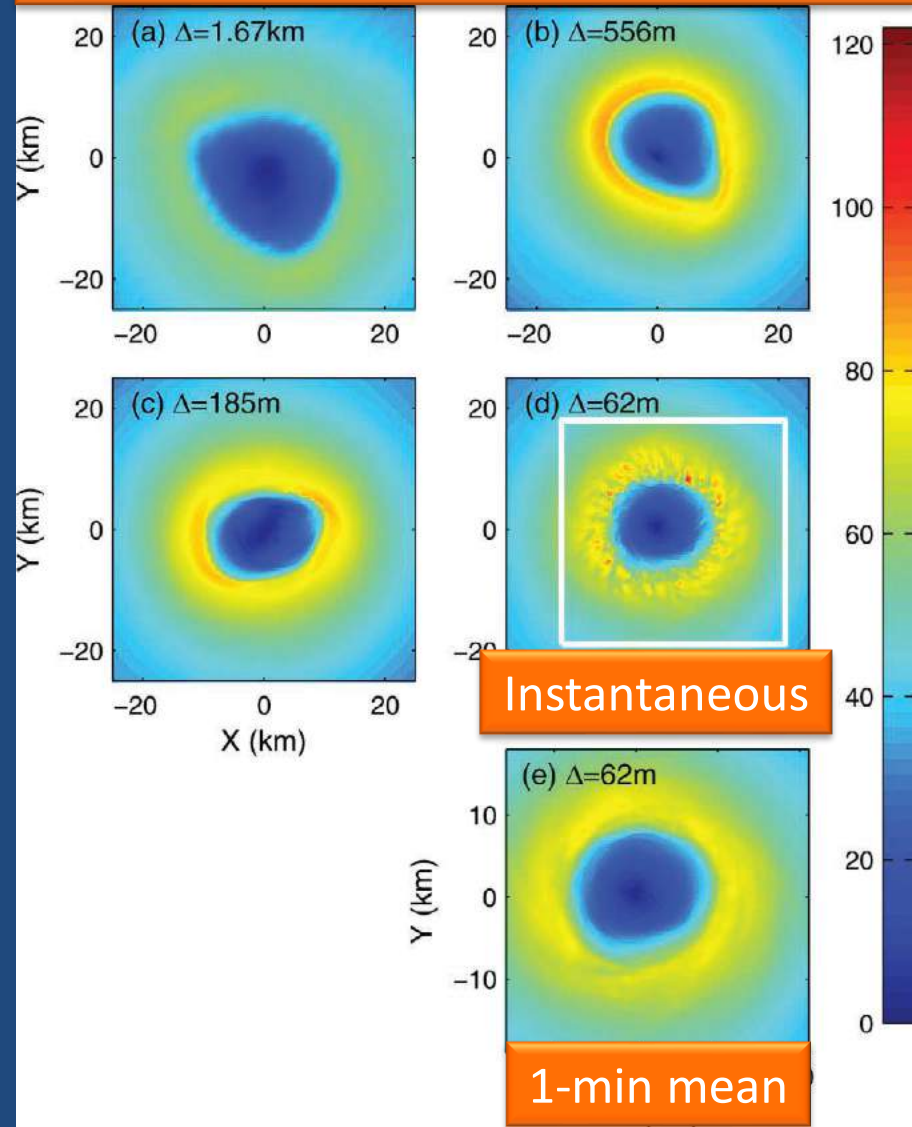
- Observations suggest that small-scale three-dimensional vortices can form on the inner edge of the eyewall.

Limitations of Cloud-Permitting Simulations

- Tropical cyclone simulations typically use grid spacing $\Delta x = 1-4$ km.
- Important structures (e.g., boundary layer rolls, eyewall vortices) are unresolved at these grid spacings.
- To properly resolve features, grid spacing needs to be at least 4-6 times smaller than the scale of the feature.
- To resolve turbulent structures in TCs, need $\Delta x < \sim 100$ m.
- At this resolution, the “large eddies” are mostly resolved, and we no longer need to parameterize the boundary layer.

Large Eddy Simulations (LES)

Wind Speed at $z=10$ m

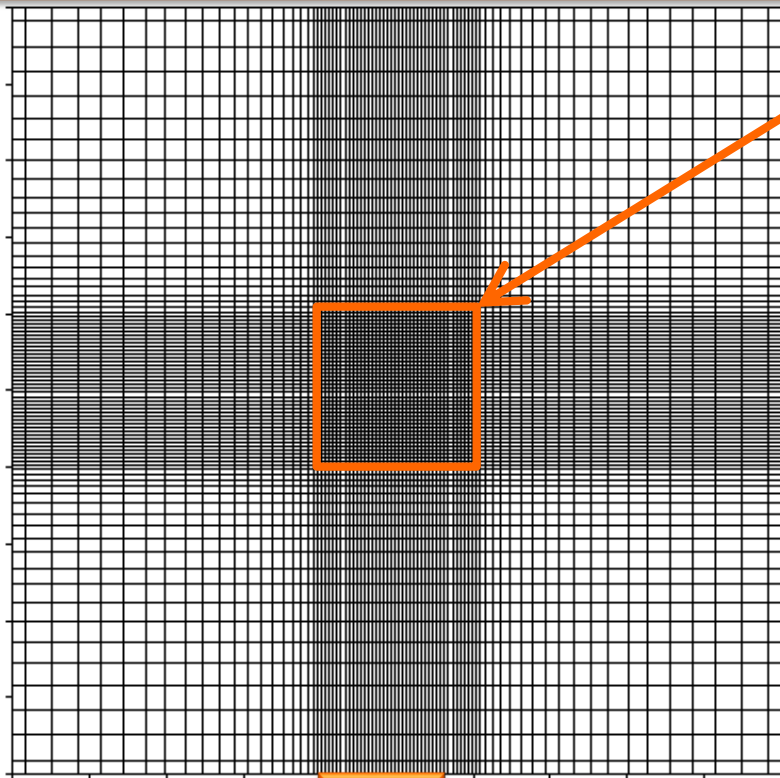


- WRF-LES idealized simulation with nested grids.
- PBL scheme on for $\Delta x=1.67$ km, off for finer domains.
- Turbulence develops only when grid spacing < 100 m.
- 1-min average winds are much less than instantaneous winds.

CM1 Model

- A non-hydrostatic, cloud-resolving model, similar in numerics to WRF
- Utilizes a single domain, with grid stretching
- Can be configured for Large-Eddy Simulations

Layout of Horizontal Grid

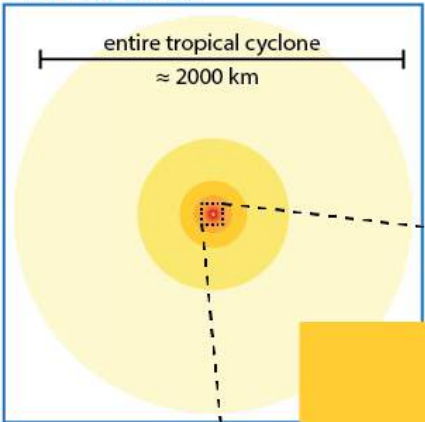


Fine-mesh part of domain:

- Fine Mesh Region:
 - 80 km × 80 km × 3 km
 - No PBL Scheme (only LES subgrid model; Deardorff 1980)
- Rest of domain:
 - $\Delta x, \Delta y, \Delta z$ increase gradually
 - Parameterized turbulence (i.e., PBL scheme)

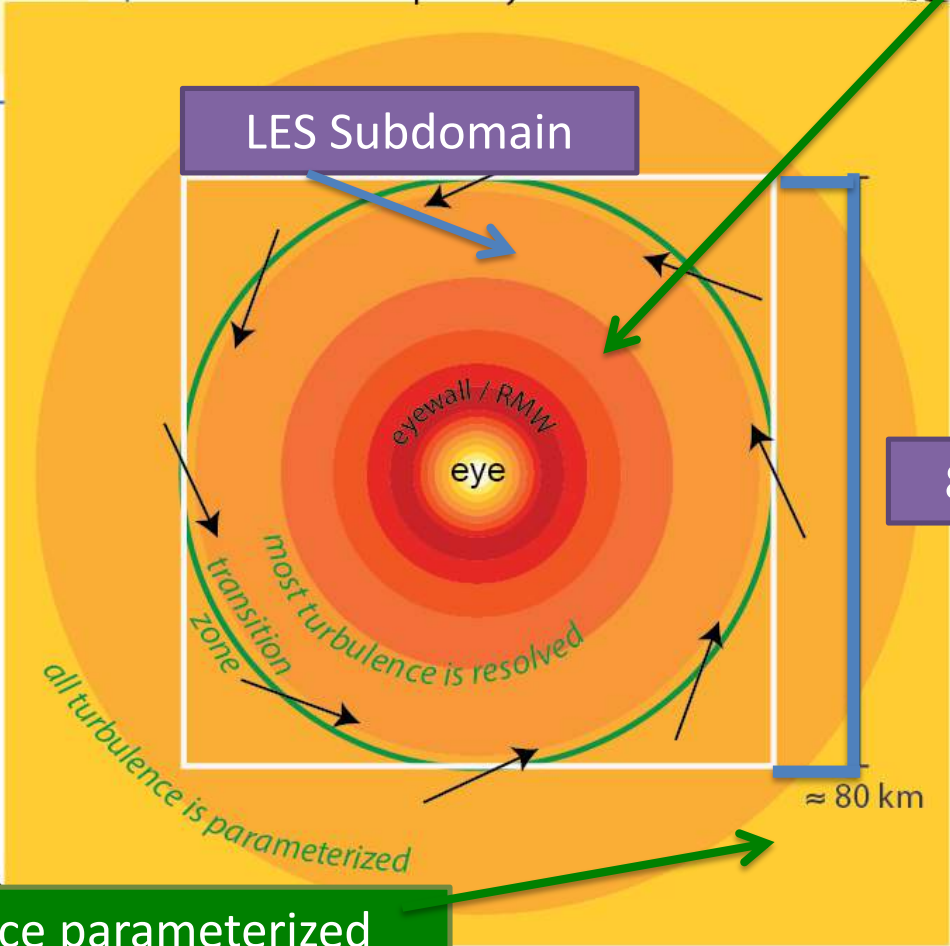
entire model domain

LES with CM1 Model

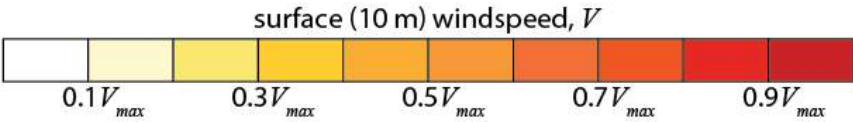


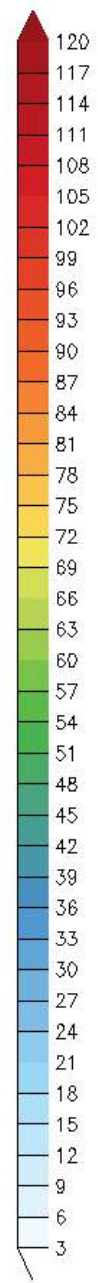
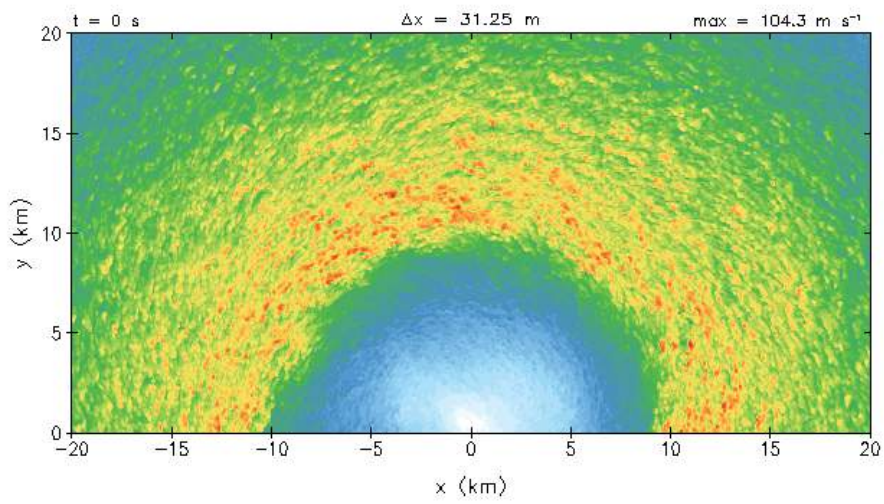
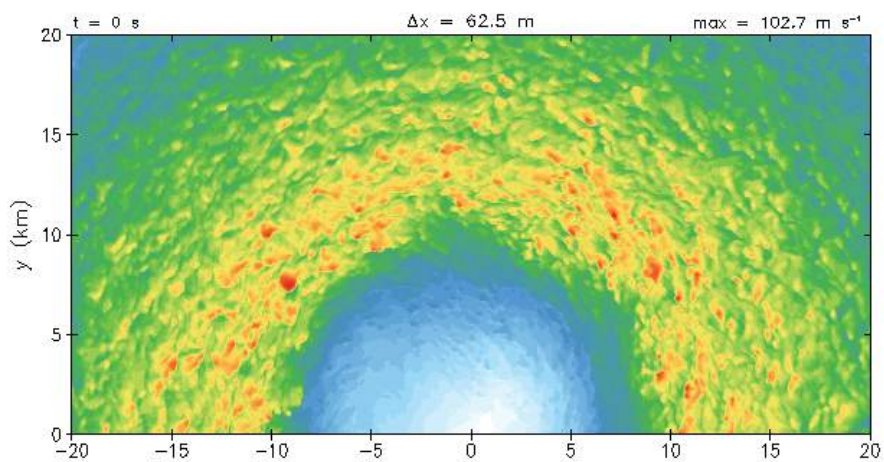
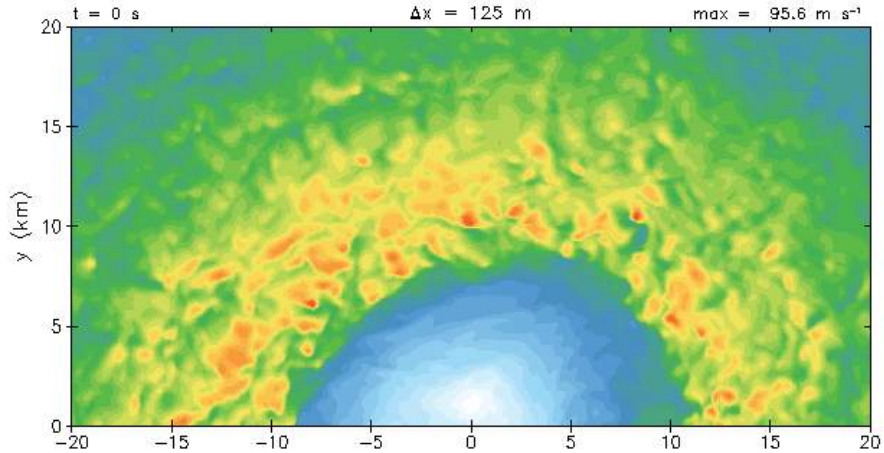
Most turbulence resolved

"inner core" of tropical cyclone



All turbulence parameterized





Wind Speed at z=10 m

$\Delta x=125$ m

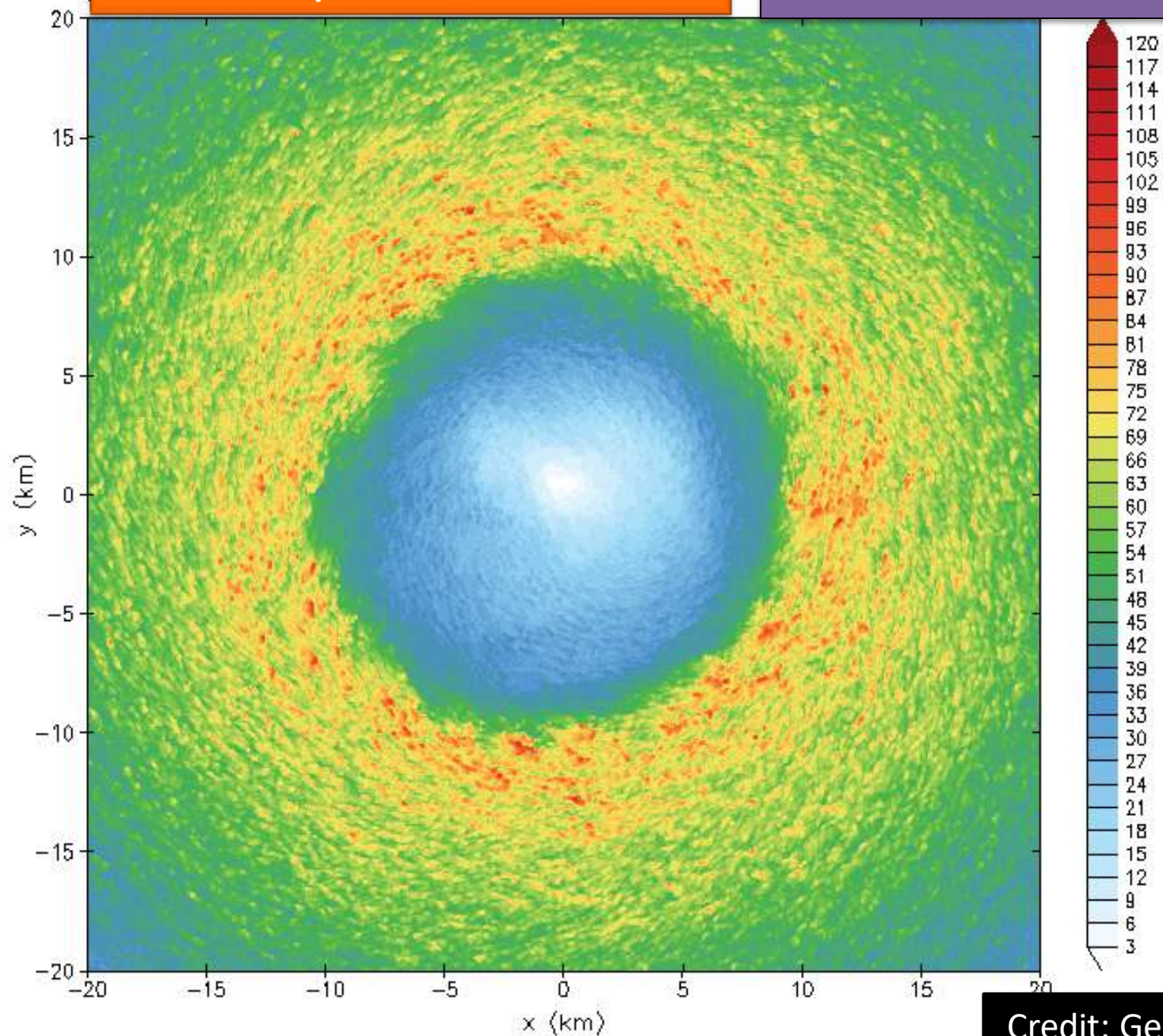
$\Delta x=62.5$ m

$\Delta x=31.25$ m

Credit: George Bryan

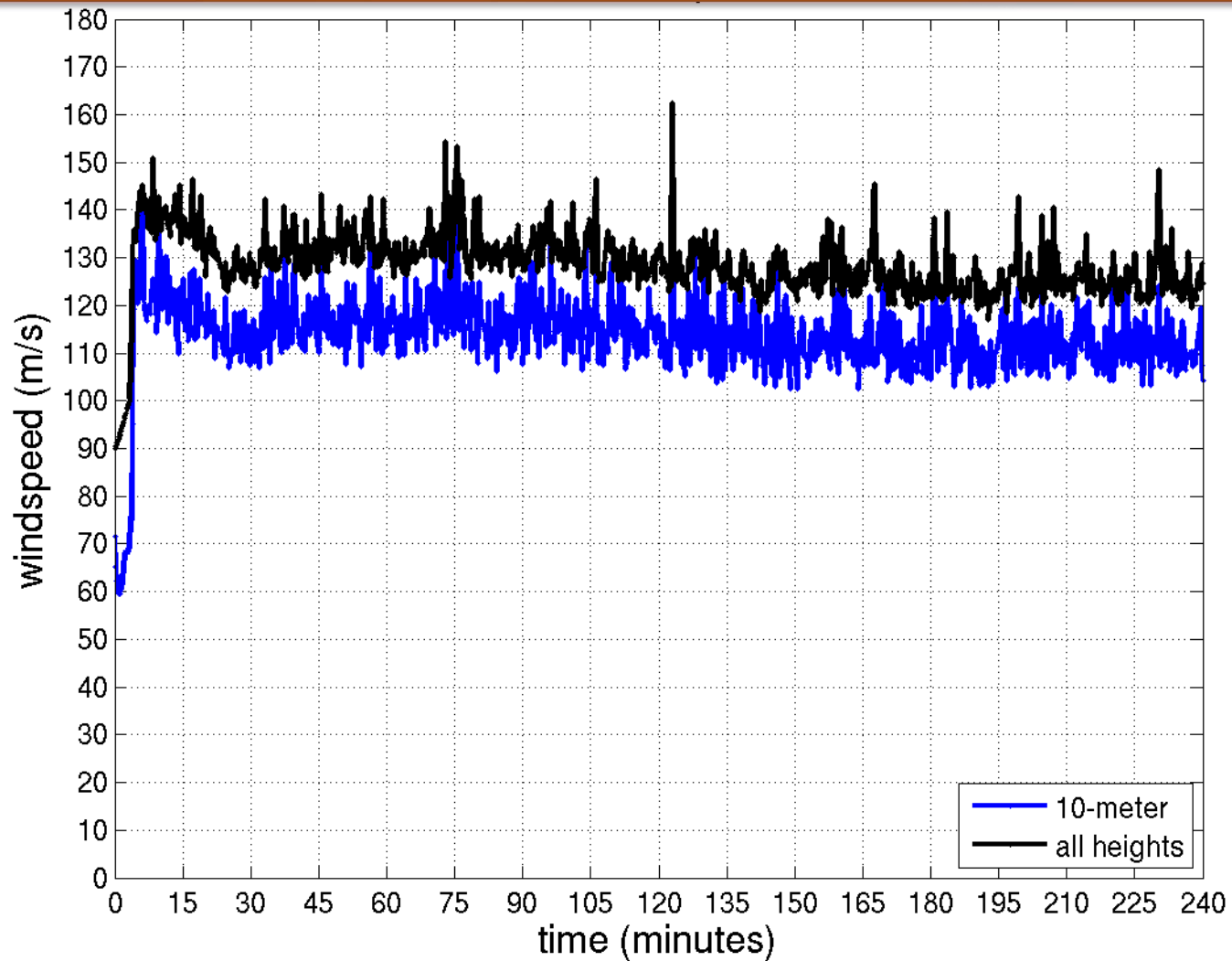
Wind Speed at z=10 m

$\Delta x=31.25$ m

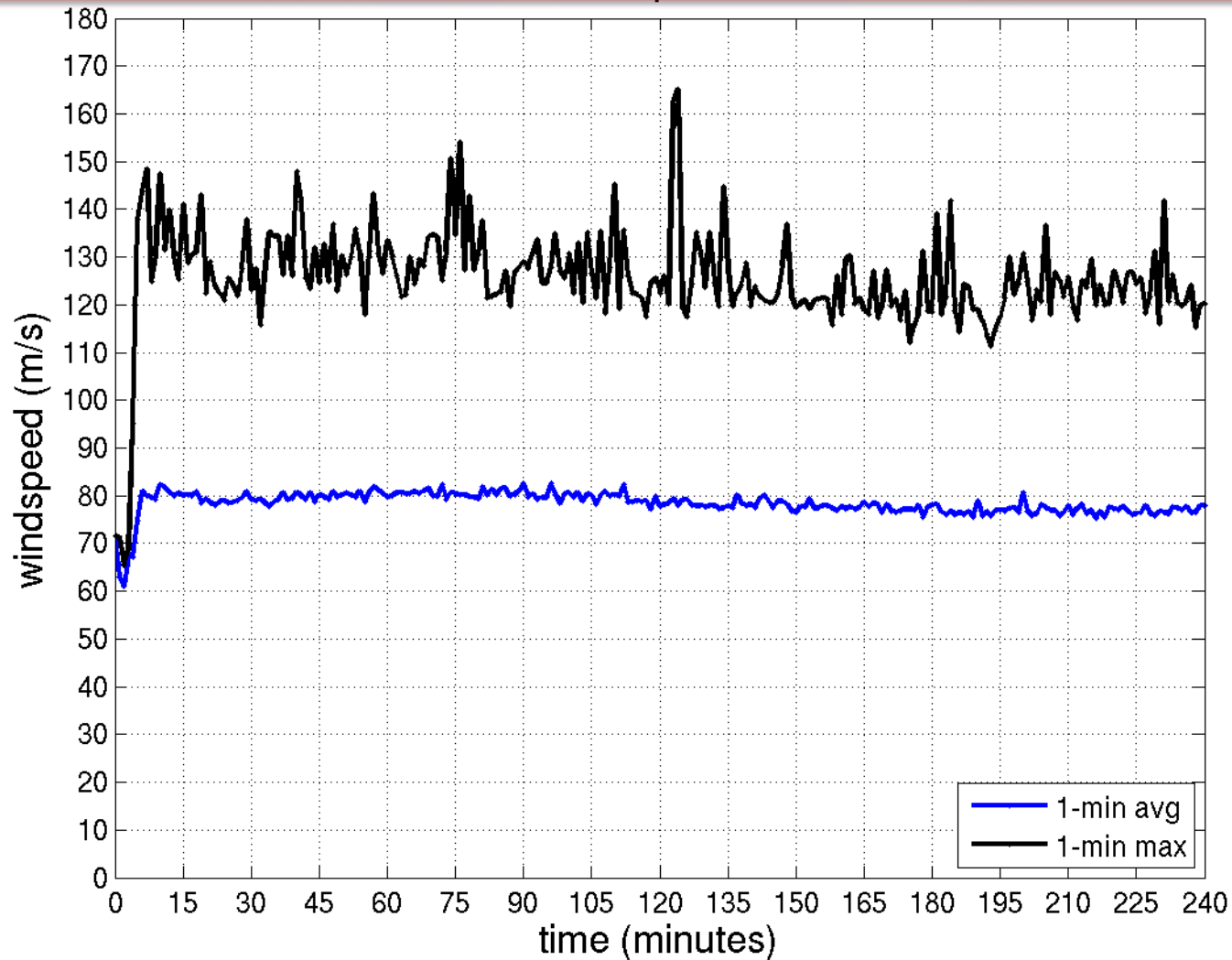


Credit: George Bryan

Peak Instantaneous Wind Speed Vs. Time



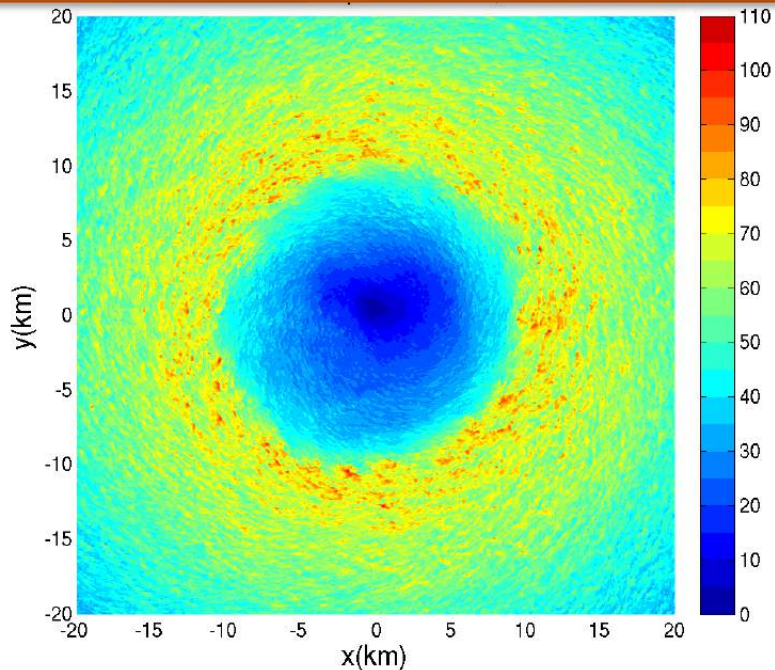
Peak 10-meter Wind Speed Vs. Time



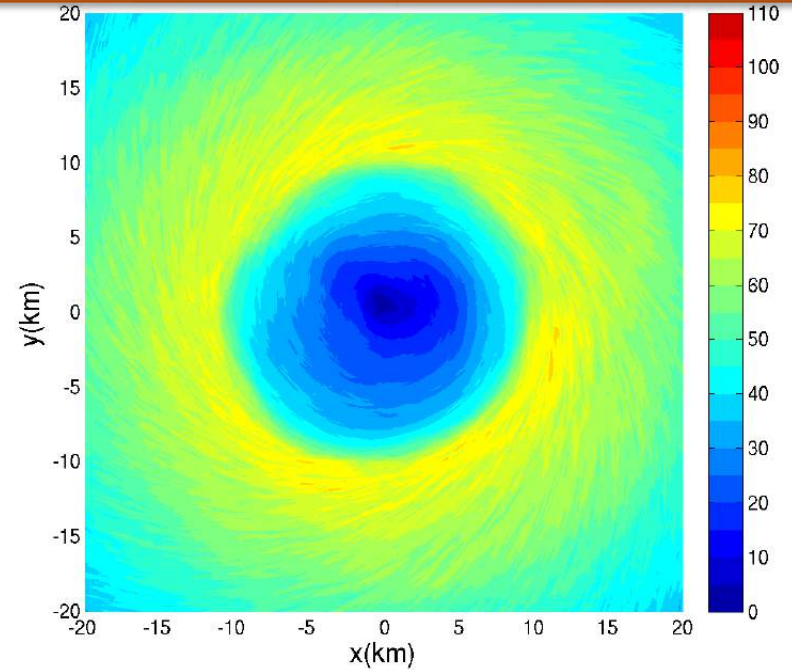
- Instantaneous wind gusts are extremely strong
- Peak 1-min average wind speed is representative of category-5

Wind Speed (ms^{-1}) at $z=10$ m

Instantaneous Wind Speed

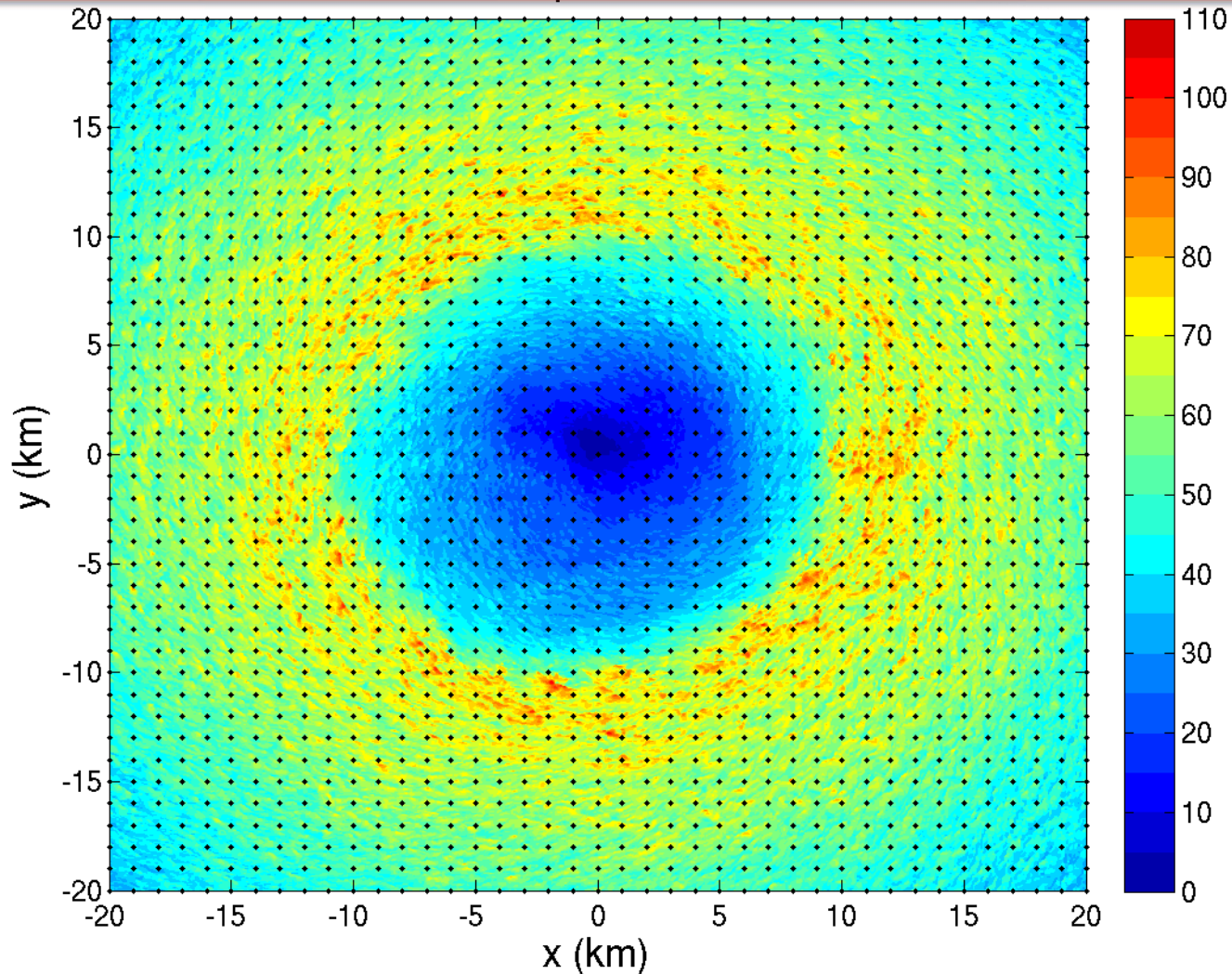


1-min Avg. Wind Speed



- Numerous instantaneous wind gusts in excess of 100 ms^{-1}
- The 1-min mean wind speed is much weaker, $70\text{-}80 \text{ ms}^{-1}$ in the eyewall

10-m Wind Speed at t=4h; Simulated Dropsondes



- Sondes released every 8 grid points (250 m); every 1 km shown
- 103,041 “virtual” dropsondes

Simulated Dropsonde

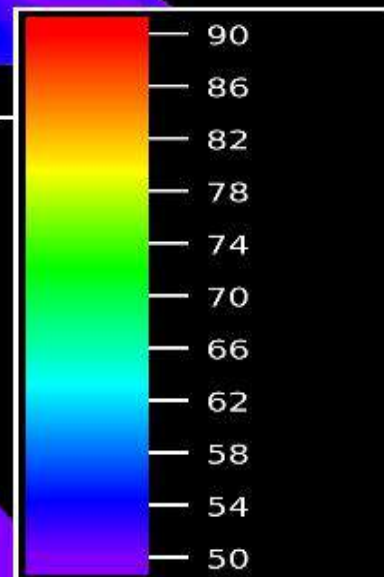
Wind Speed=75 m/s

Wind Speed > 90 m/s

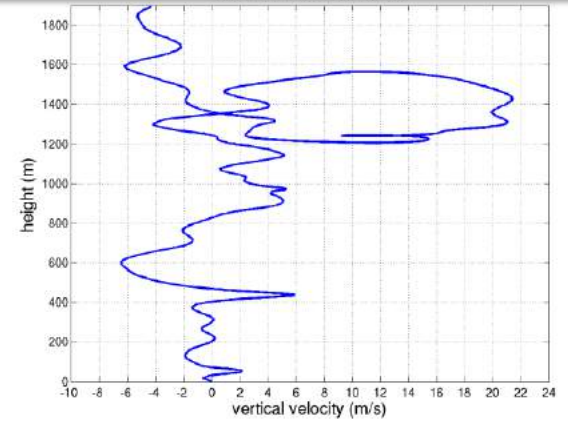
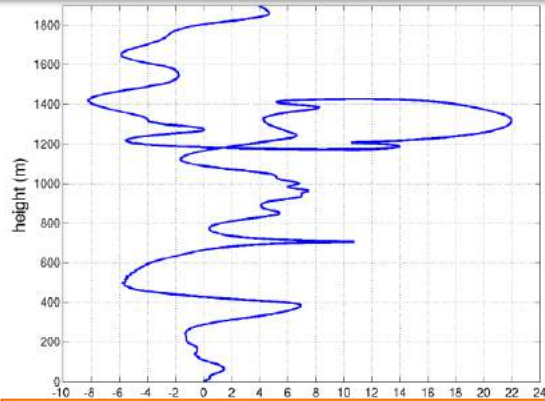
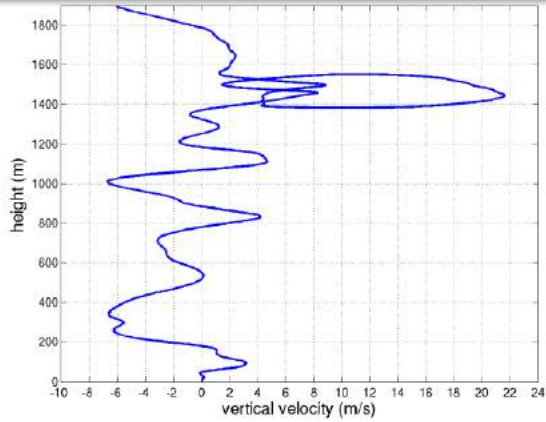
z=560 m

z=10 m

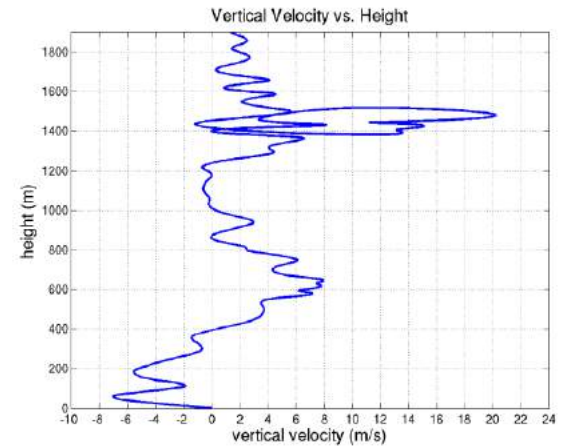
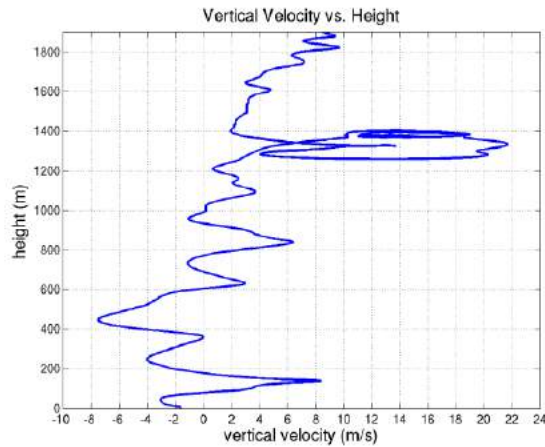
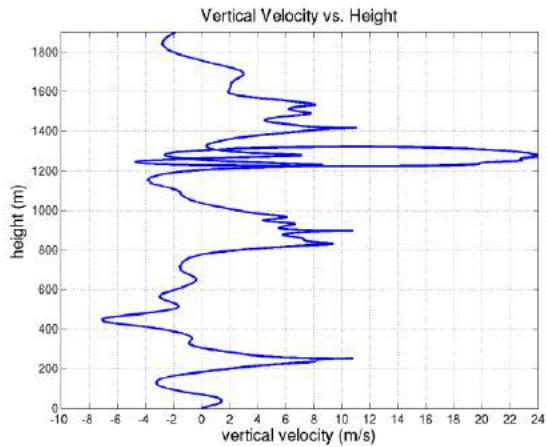
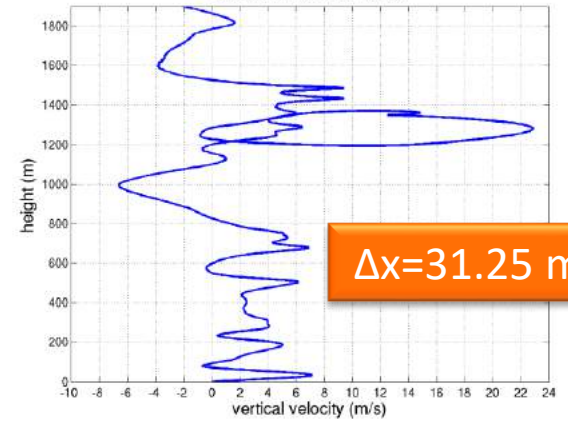
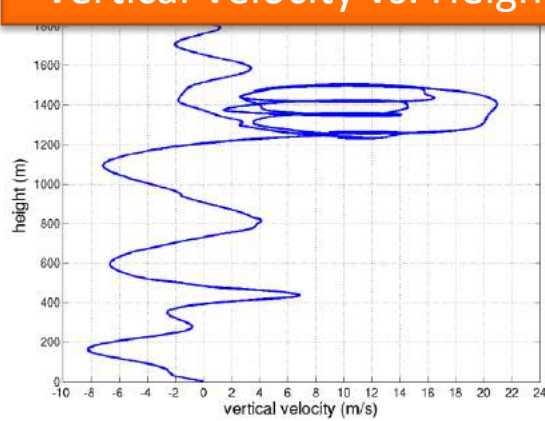
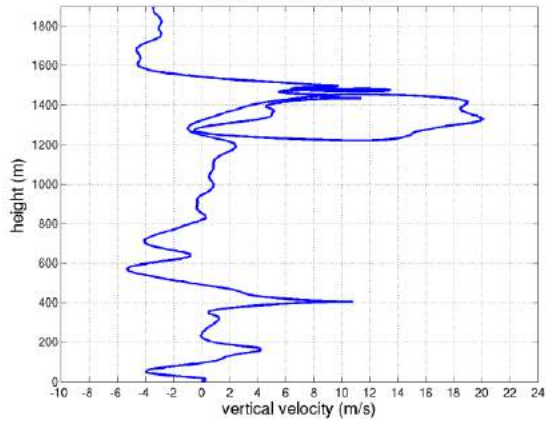
$\Delta x = 125$ m



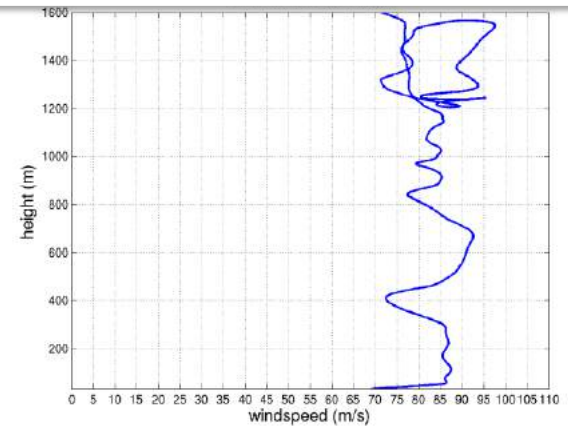
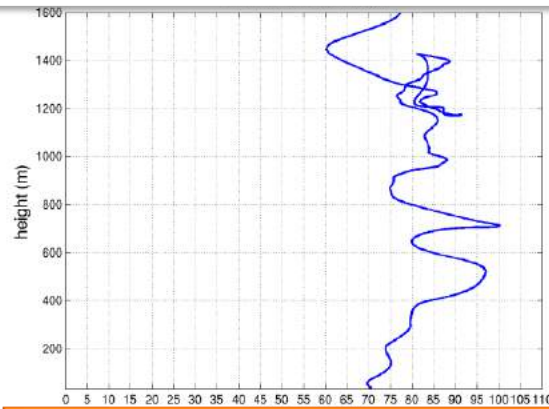
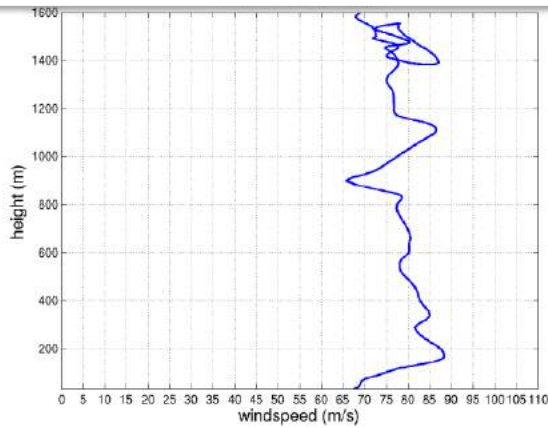
Which is the real dropsonde?



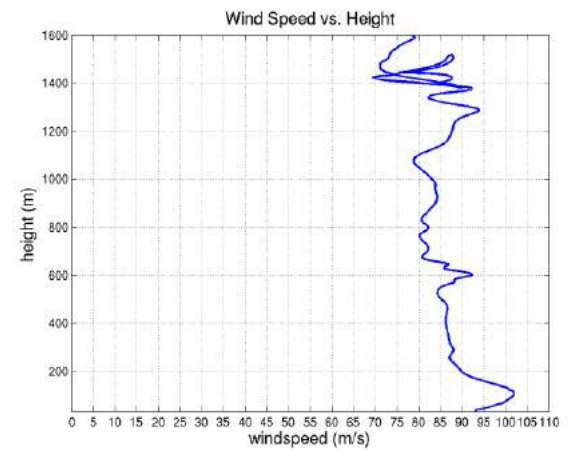
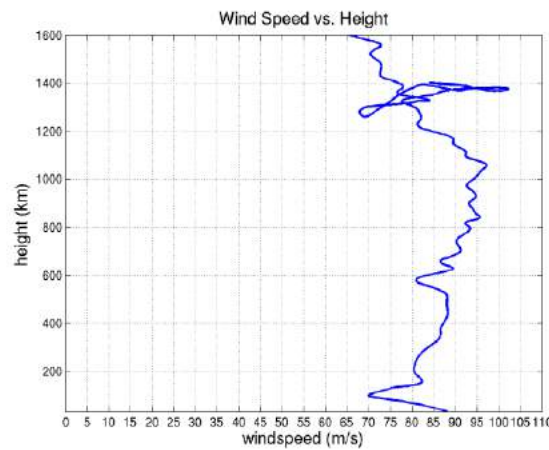
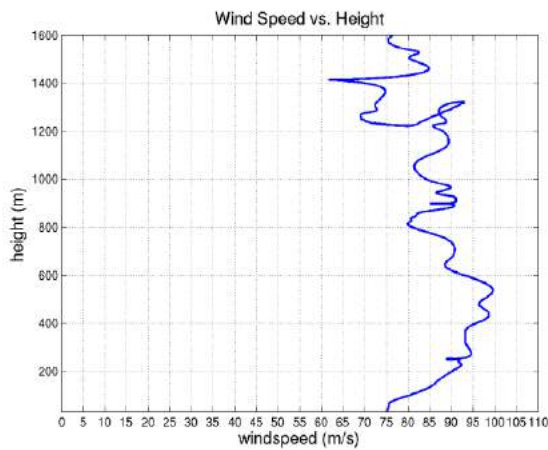
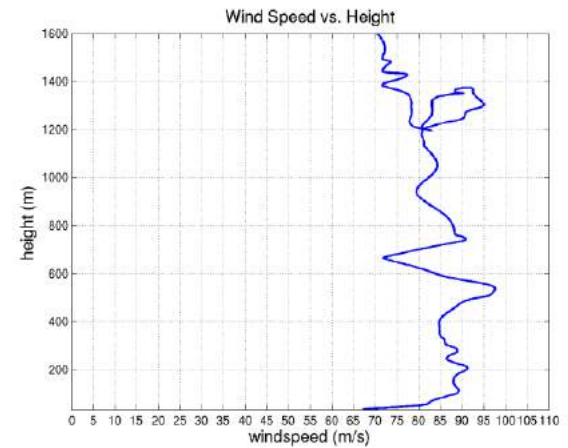
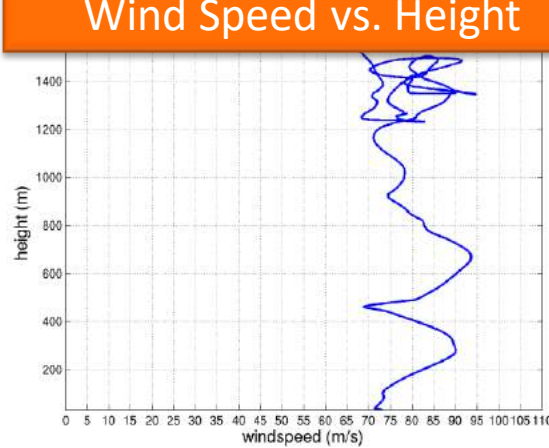
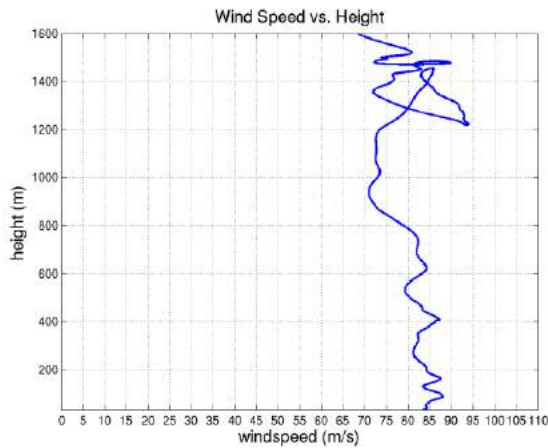
Vertical Velocity vs. Height



Which is the real dropsonde?



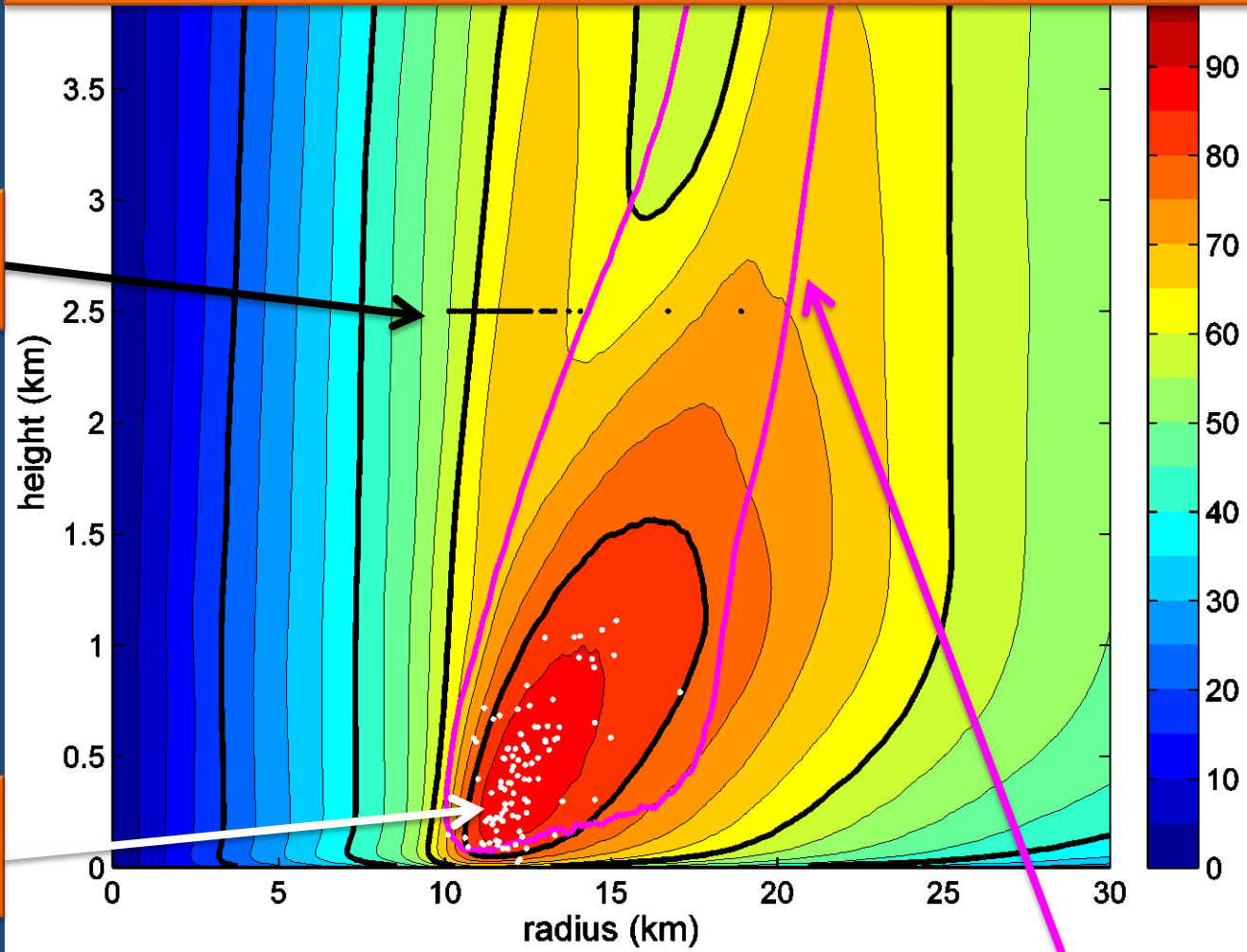
Wind Speed vs. Height



Azimuthal Mean Tangential Wind

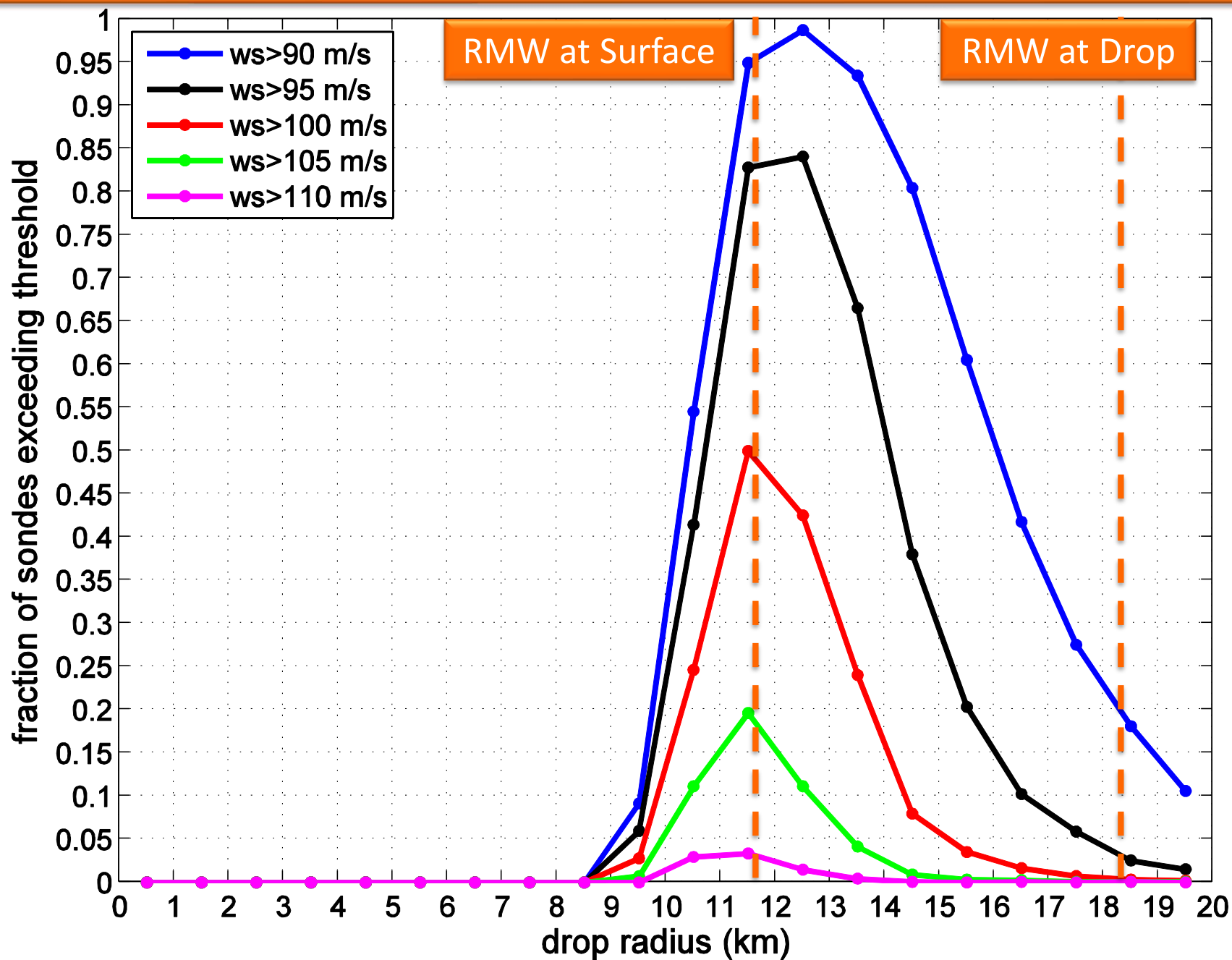
Drop points for sondes
with $w_s > 110 \text{ ms}^{-1}$

Locations of Max winds
(for $w_s > 110 \text{ ms}^{-1}$)



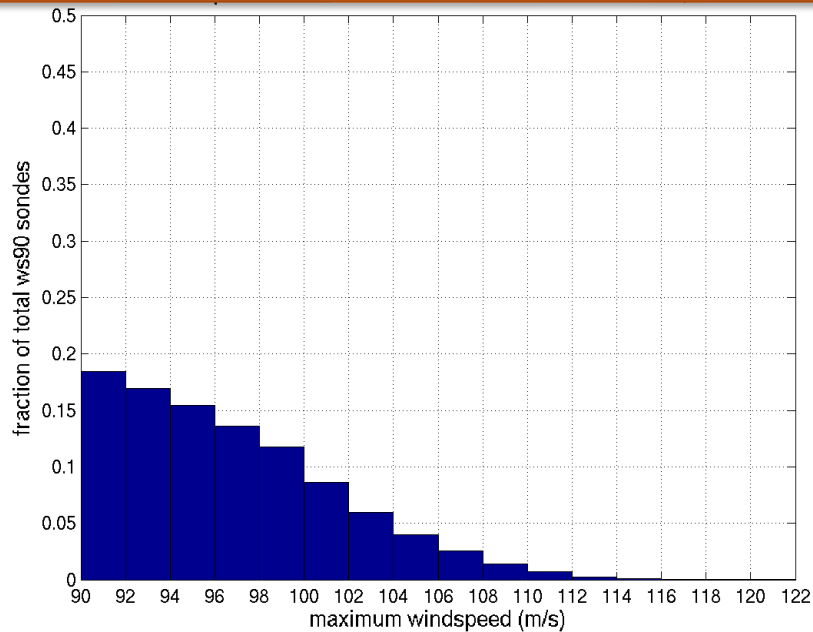
Azimuthal mean $w > 1 \text{ m/s}$

Fraction of Sondes Sampling Extreme Winds

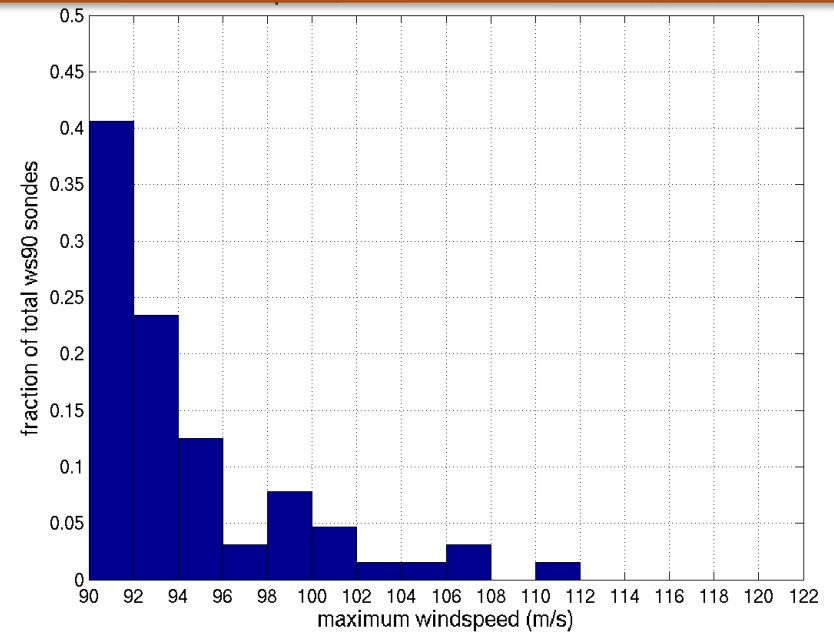


Comparison of Simulated to Observed Dropsondes

Simulated Sonde WS > 90 ms⁻¹

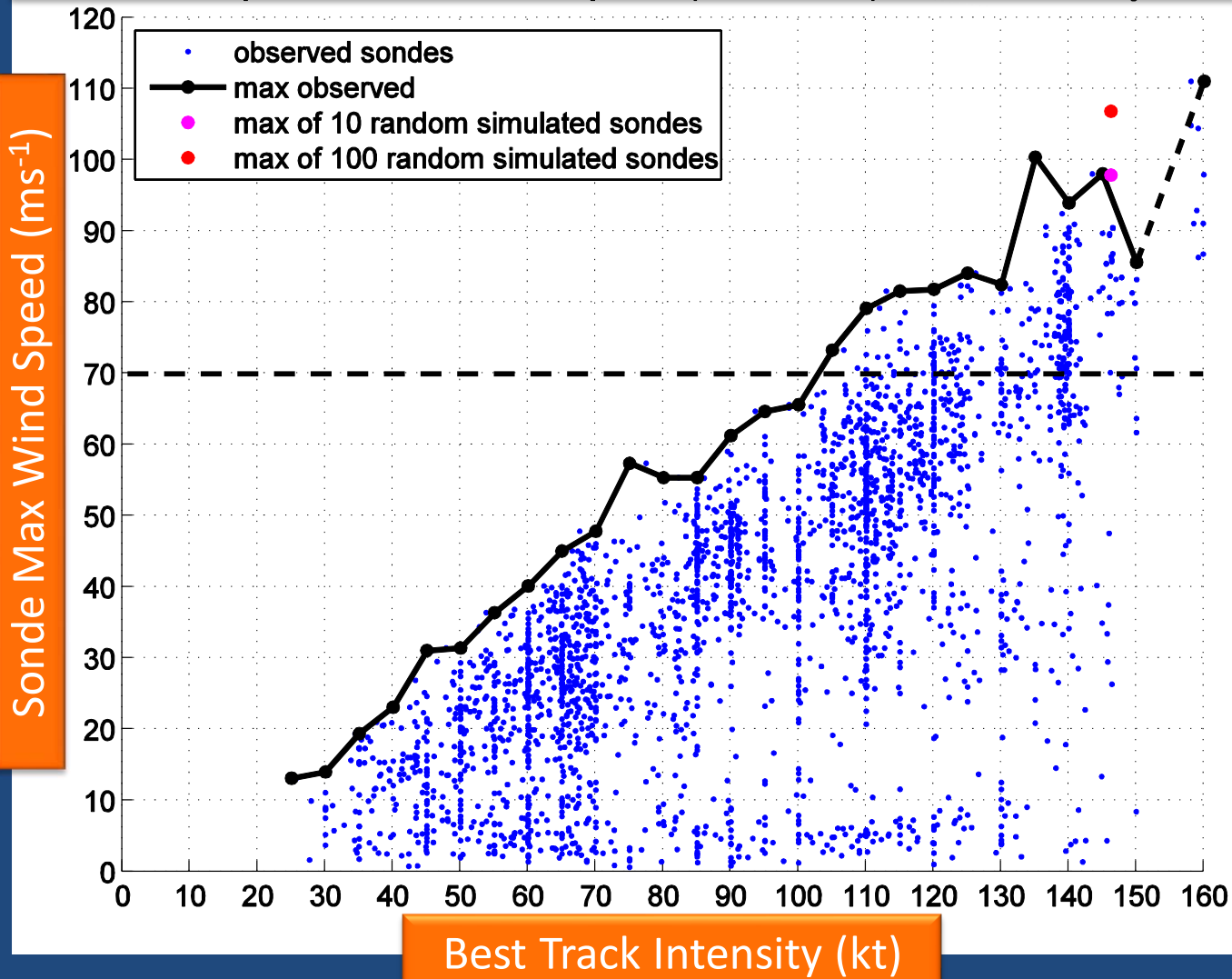


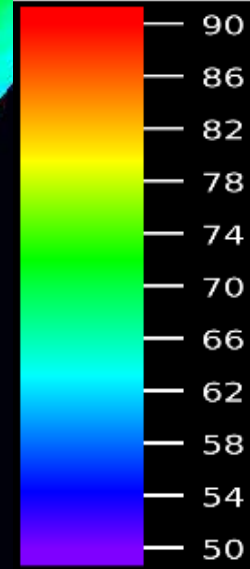
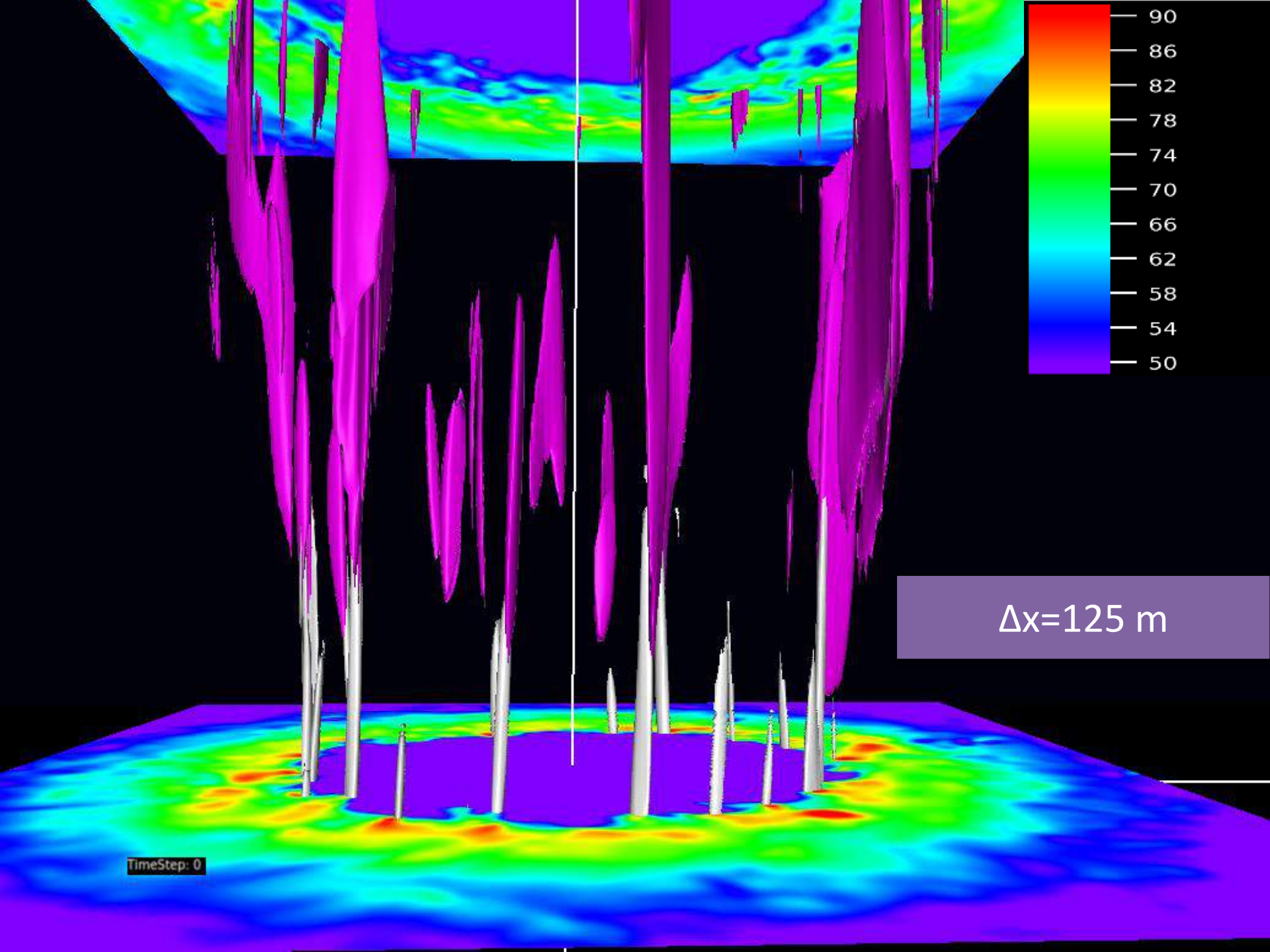
Observed Sonde WS > 90 ms⁻¹



- Magnitude of strongest sampled wind gusts is comparable
- Simulated sondes sample the most extreme values more frequently than observed
- This is likely because the simulated TC is slightly stronger than the average observed TC

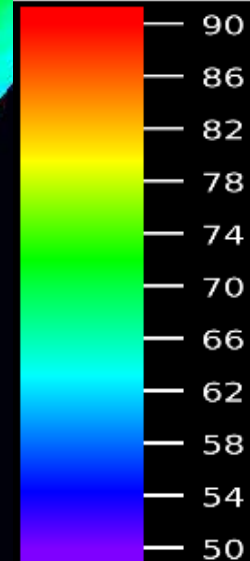
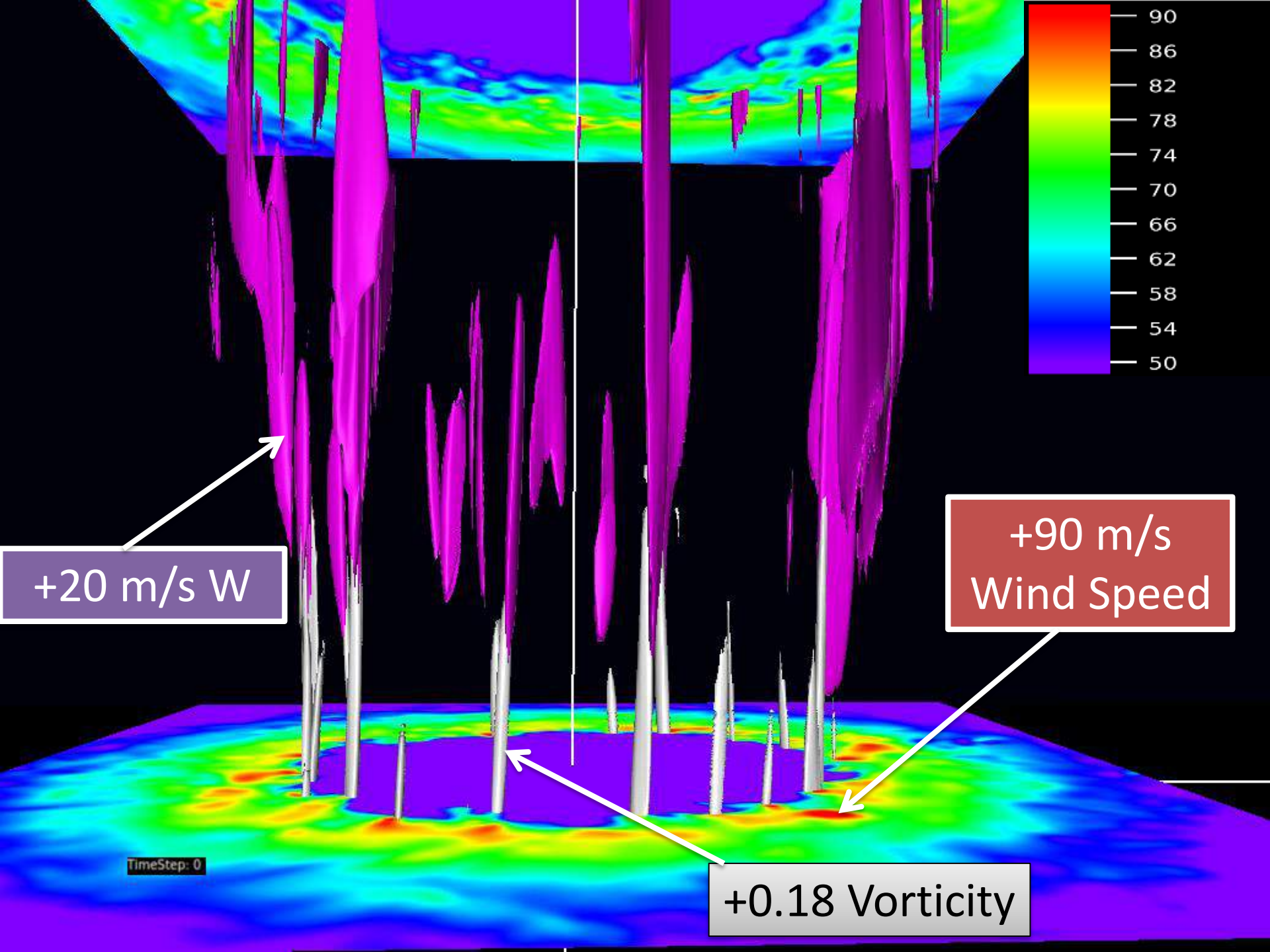
Observed Sonde Max Wind Speed (z=0-200m)





$\Delta x = 125 \text{ m}$

TimeStep: 0

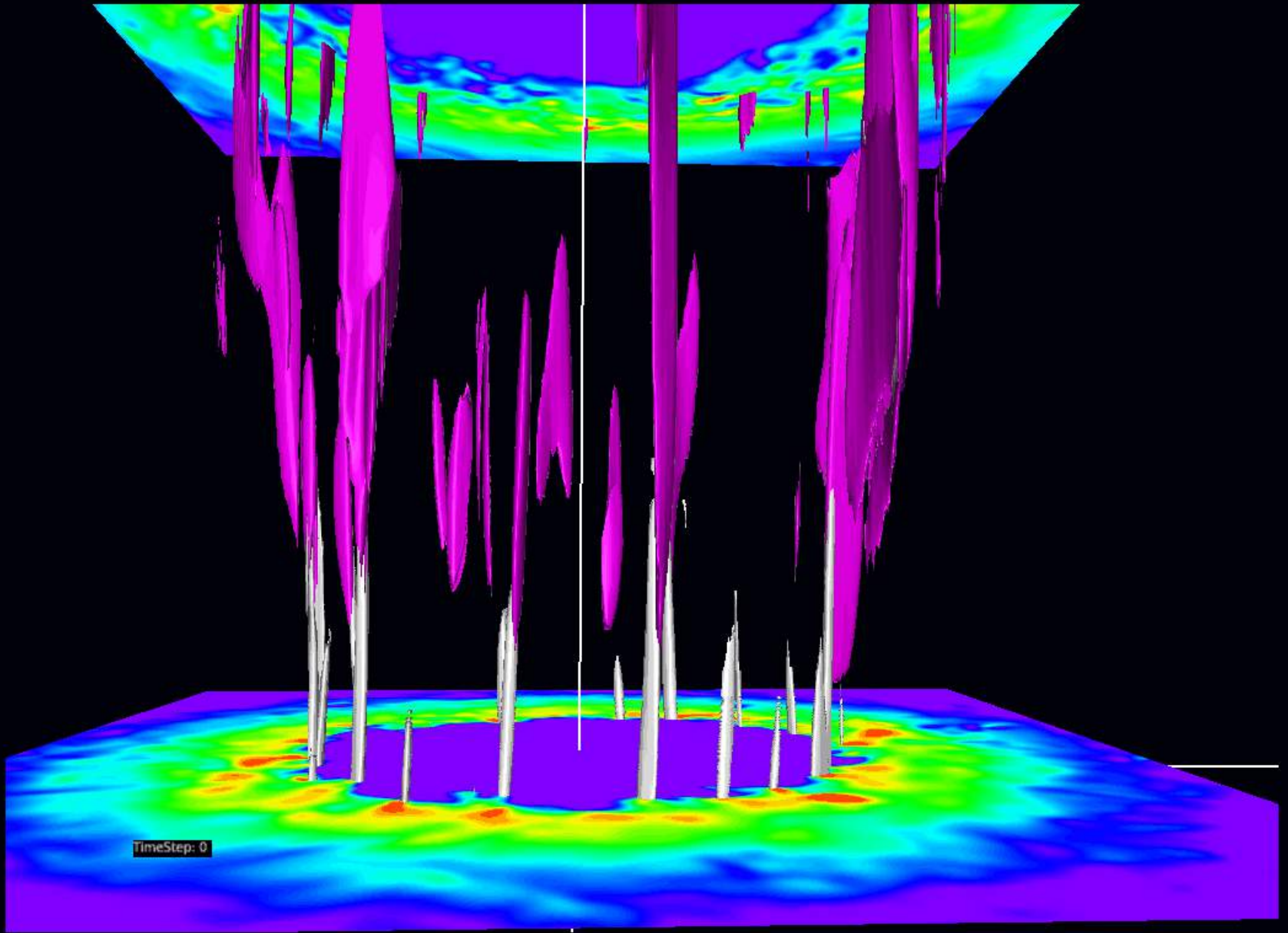


+20 m/s W

+90 m/s
Wind Speed

+0.18 Vorticity

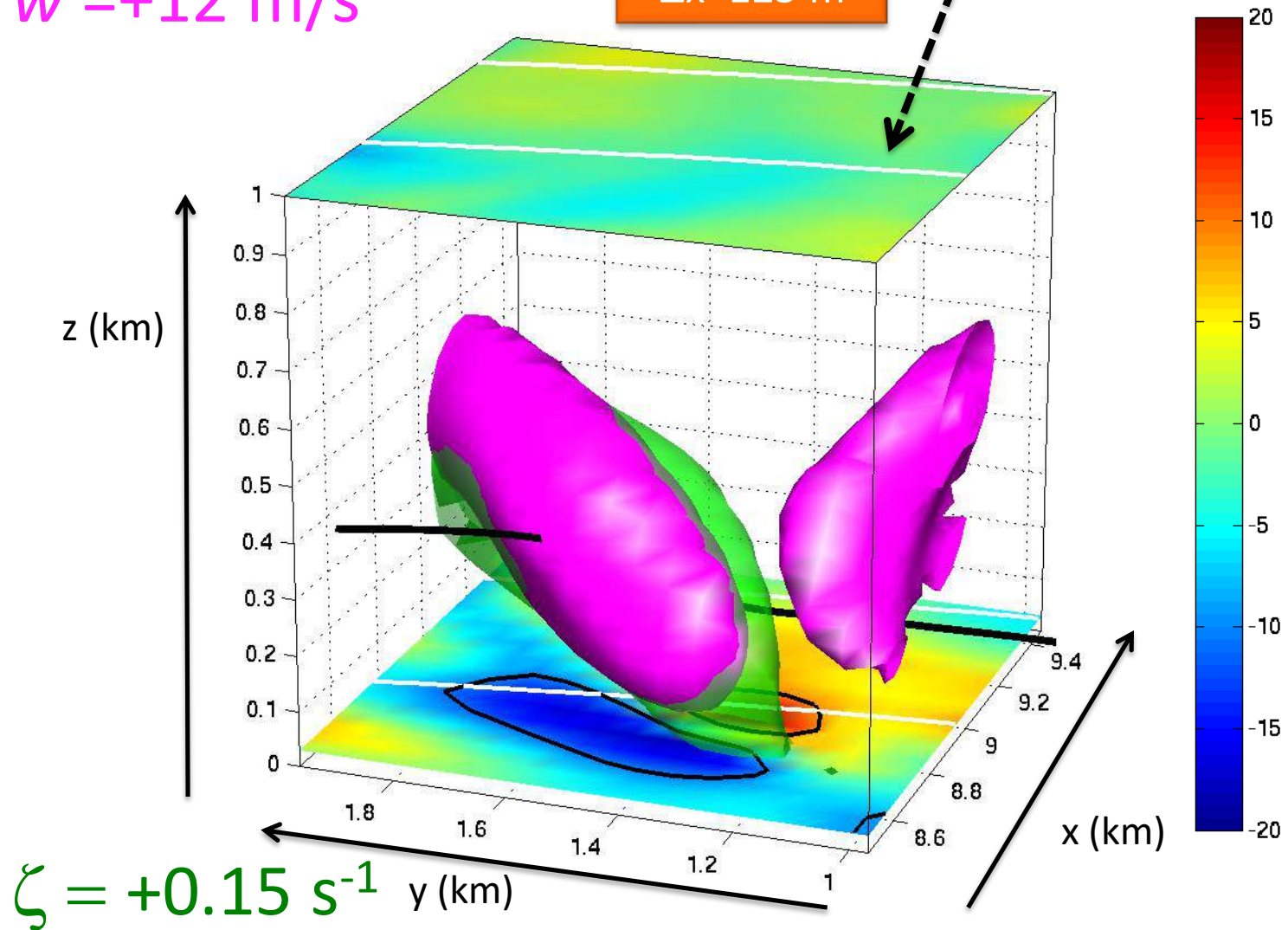
TimeStep: 0



Slices of Perturbation V_t (ms^{-1} ; +/- 10 black)

$w = +12 \text{ m/s}$

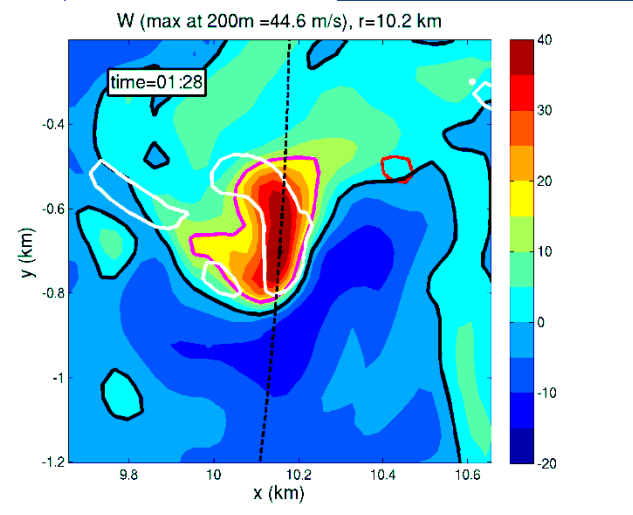
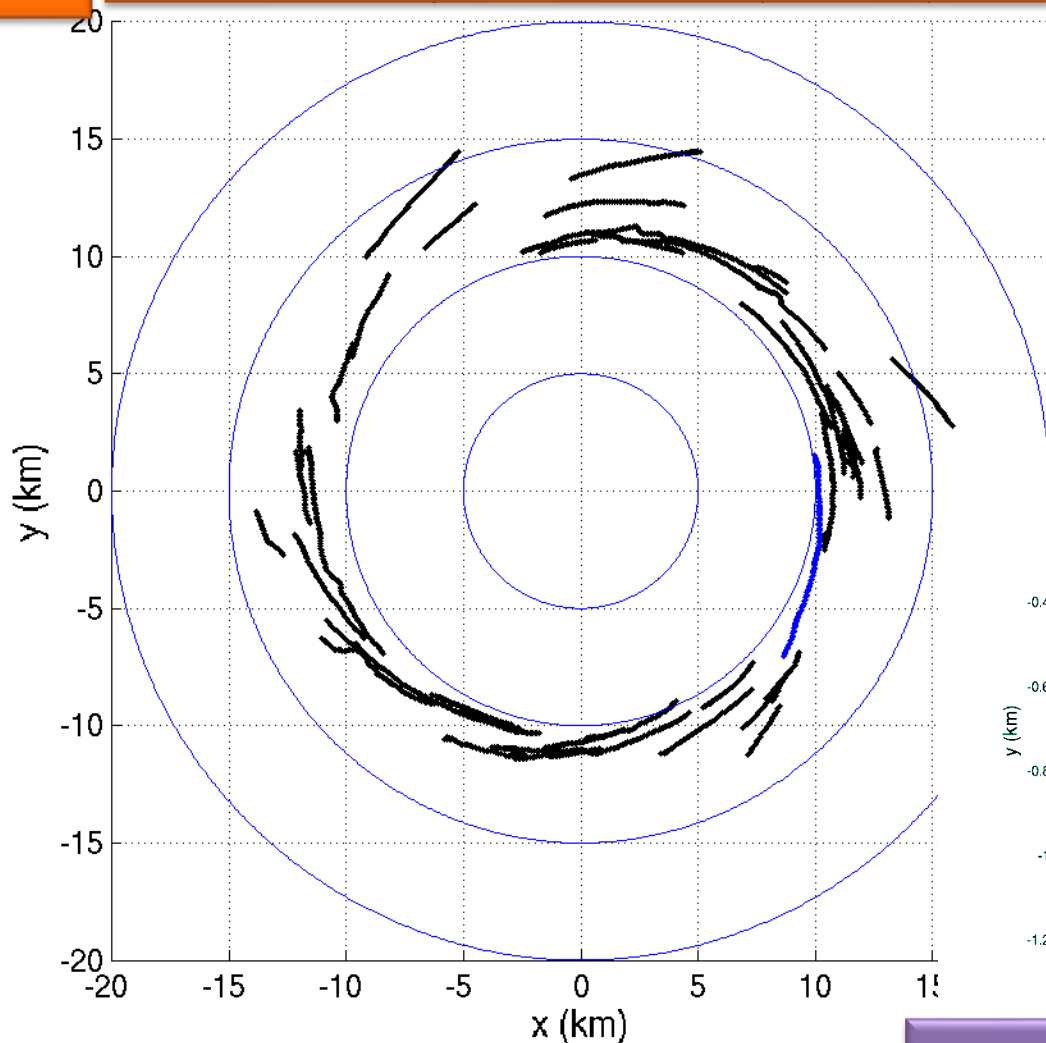
$\Delta x = 125 \text{ m}$



Objective Tracking of Updrafts and Wind Gusts

$\Delta x = 31.25 \text{ m}$

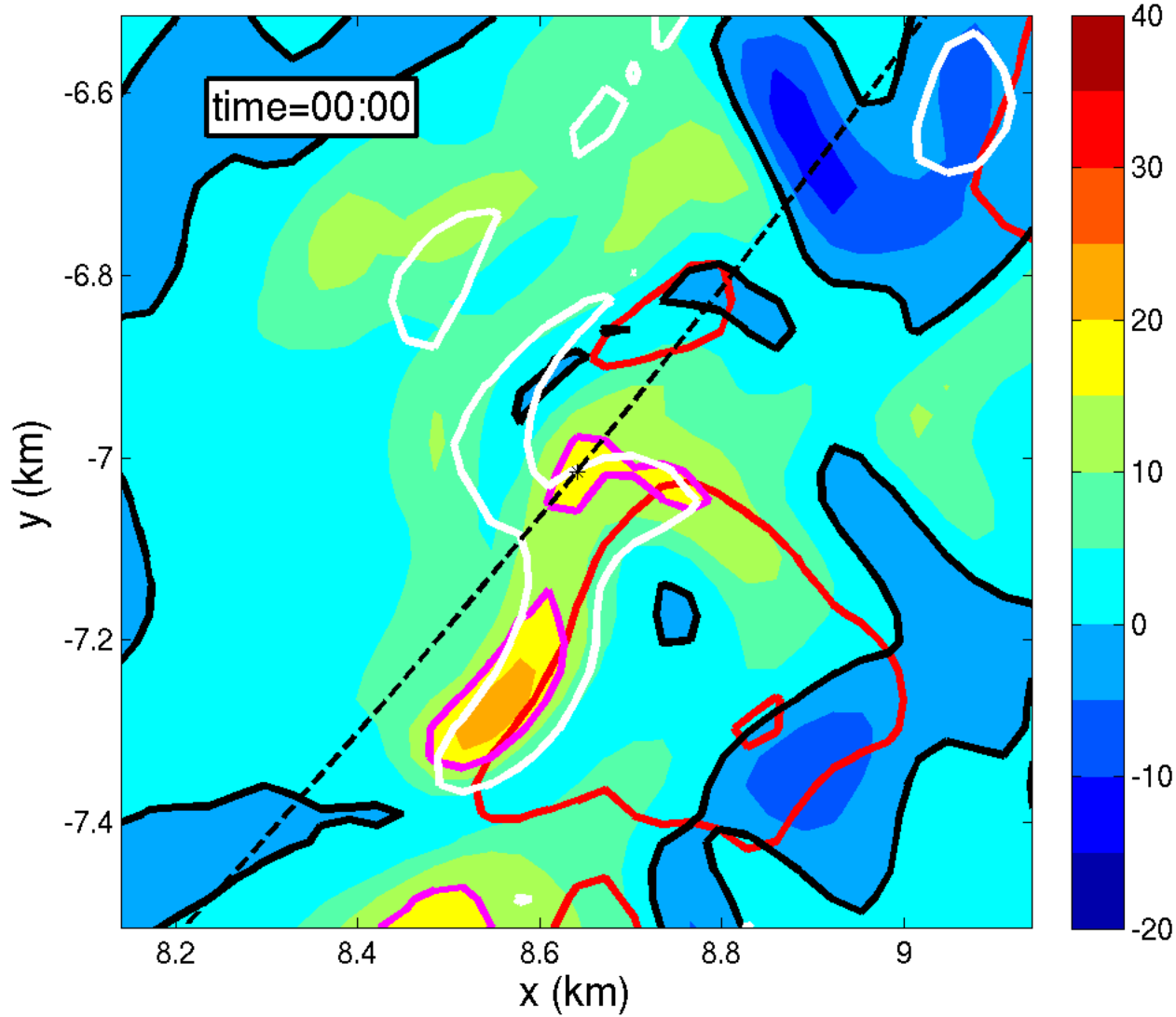
Updrafts $> 15 \text{ ms}^{-1}$ at $z = 200 \text{ m}$



- Azimuthal Translation: 74 ms^{-1}
- Radial Translation: -12 ms^{-1}

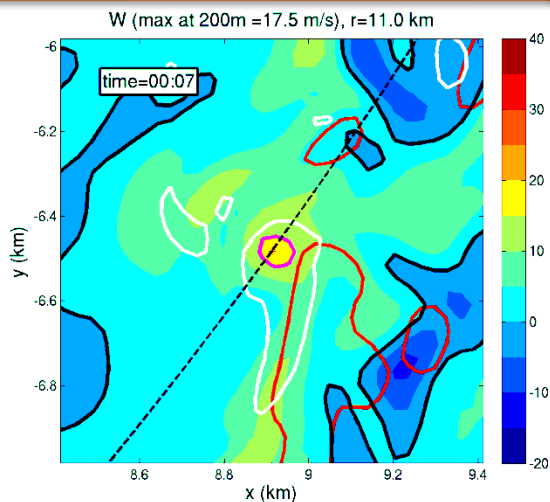
Horizontal Cross Section Following Updraft

W (max at 200m = 16.8 m/s), r=11.1 km

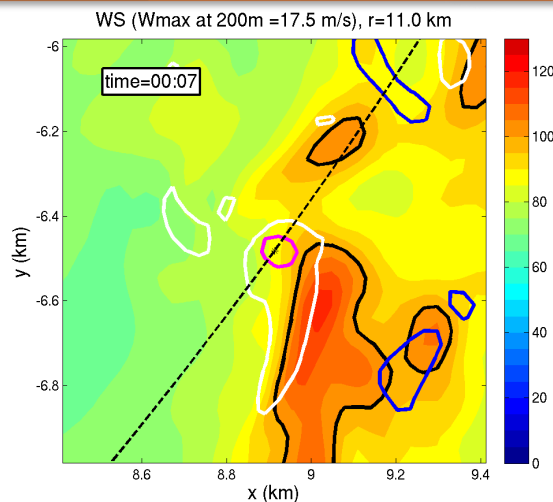


Horizontal Cross Sections at Times of Peak W, WSPD

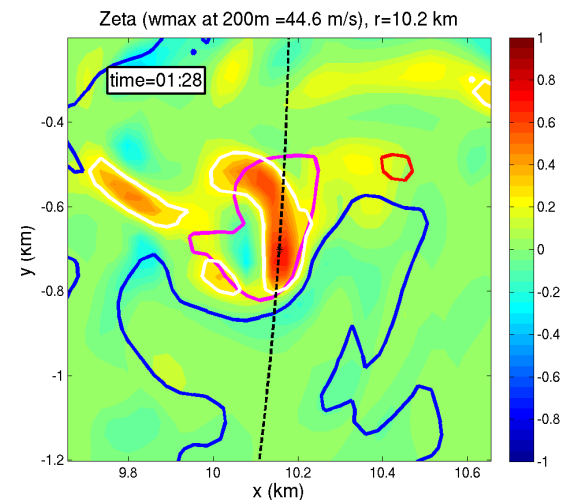
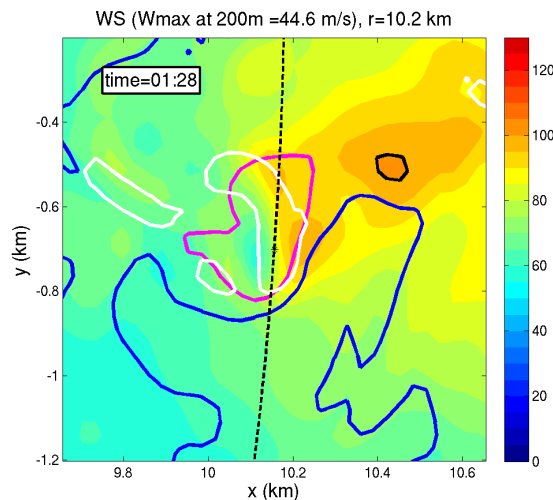
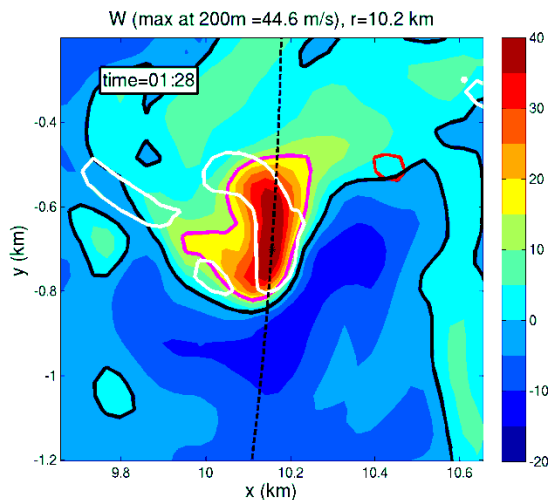
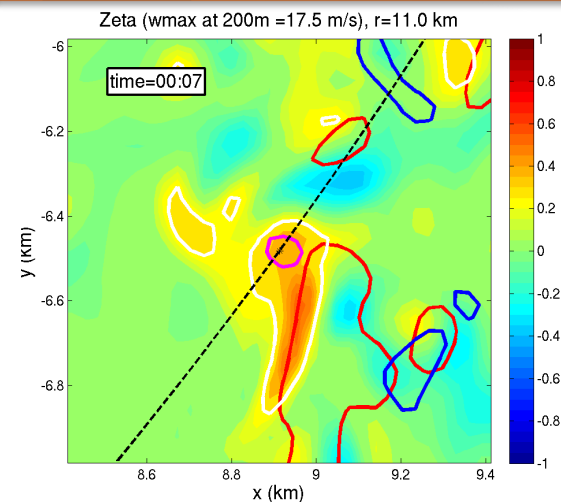
Vertical Velocity



Wind Speed

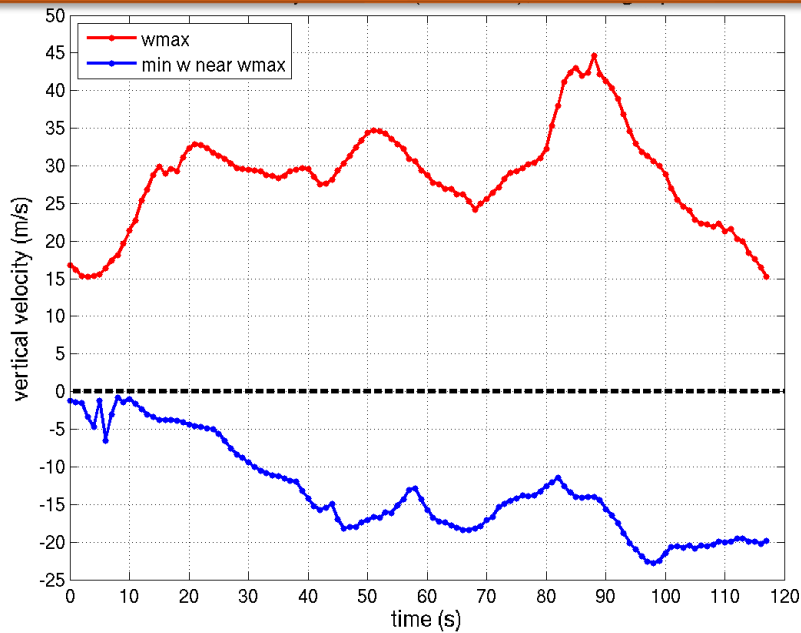


Vertical Vorticity

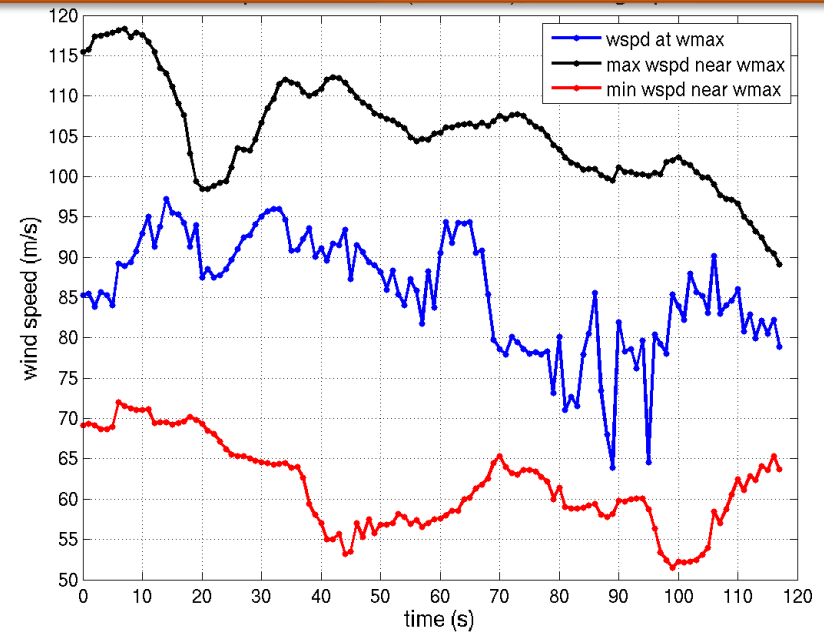


Evolution of Updraft and Wind Gust

Vertical Velocity vs. Time



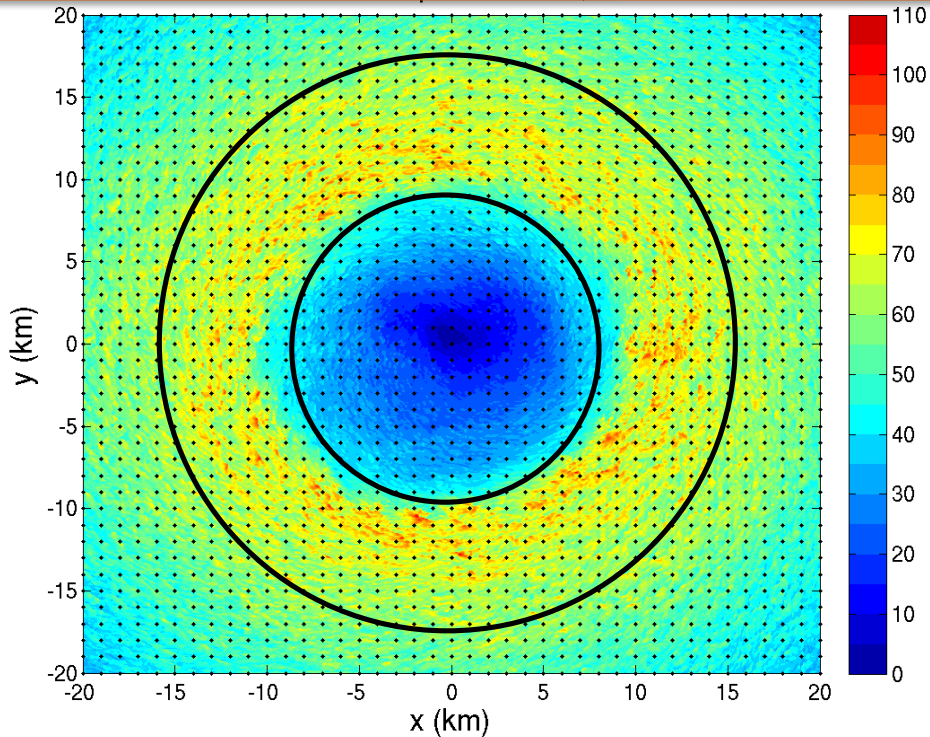
Wind Speed vs. Time



- 118 ms^{-1} gust, remains $>100 \text{ ms}^{-1}$ for almost two minutes
- Such extreme gusts are unlikely to be sampled observationally.

What is the Maximum *Sampled* Wind Speed?

10-m Wind Speed; Dropsonde Locations



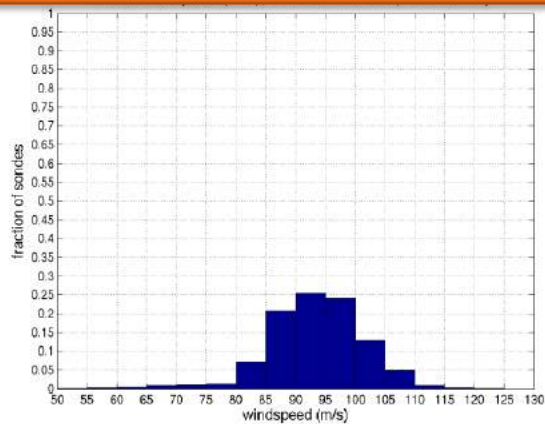
Sonde density is 16 times what is shown

1. Randomly sample combinations of simulated sondes in eyewall
2. Find the maximum wind speed among sondes
3. Repeat 10,000 times to obtain distribution

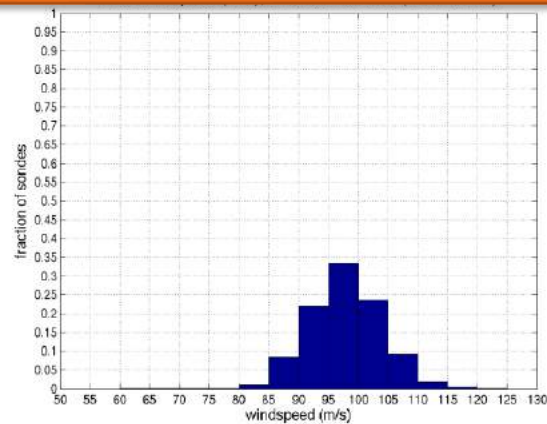
What is the Maximum *Sampled* Wind Speed?

PDFs of Peak Gust Sampled by Simulated Sondes

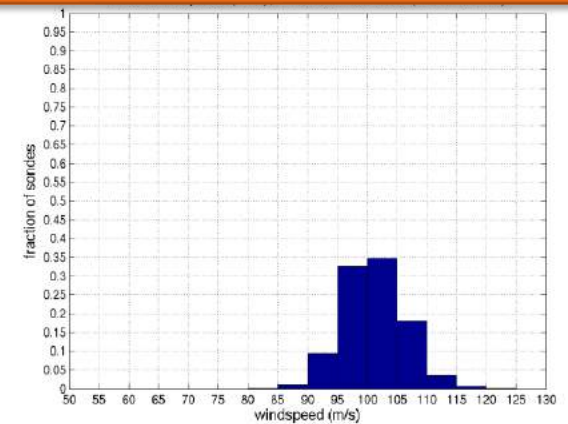
1 Random Sonde



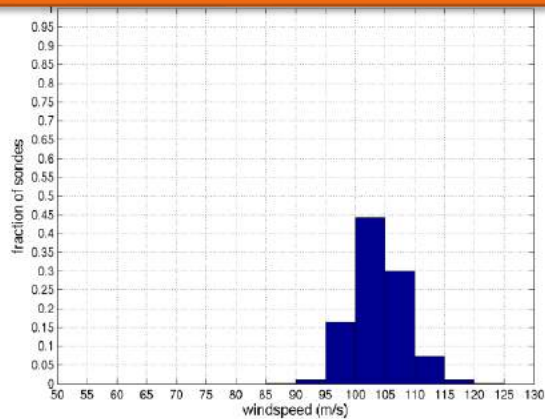
2 Random Sondes



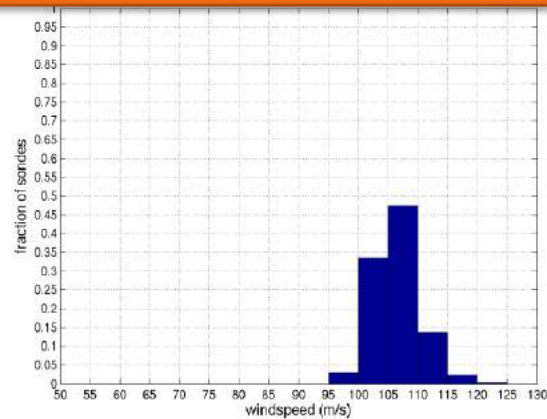
4 Random Sondes



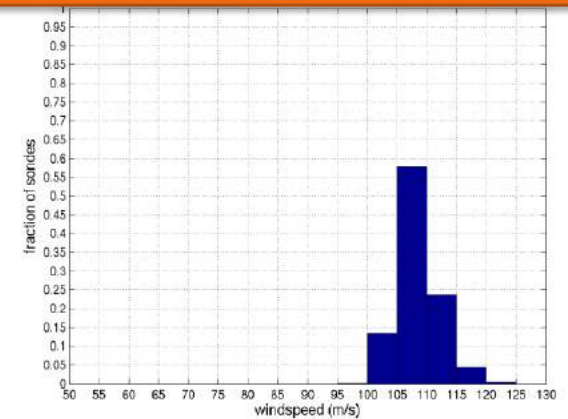
8 Random Sondes



16 Random Sondes



32 Random Sondes



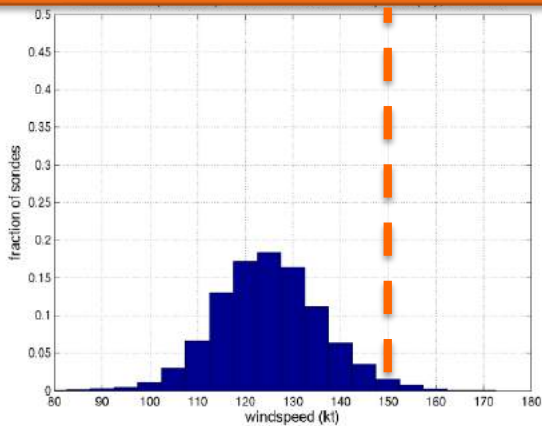
Summary

- Observations indicate the existence of small-scale vortices along the eye/eyewall interface, which are associated with extreme updrafts ($10\text{-}30\text{ ms}^{-1}$) and wind speeds ($90\text{-}110\text{ ms}^{-1}$).
- LES produces these observed structures, and with the simulations, we can learn about dynamics that are difficult to observe.
- Gusts of $120\text{-}140\text{ ms}^{-1}$ are always present in the simulation, substantially stronger than have ever been observed.
- Such gusts are likely realistic, as simulated dropsondes very rarely sample gusts exceeding 110 ms^{-1} , consistent with observations.

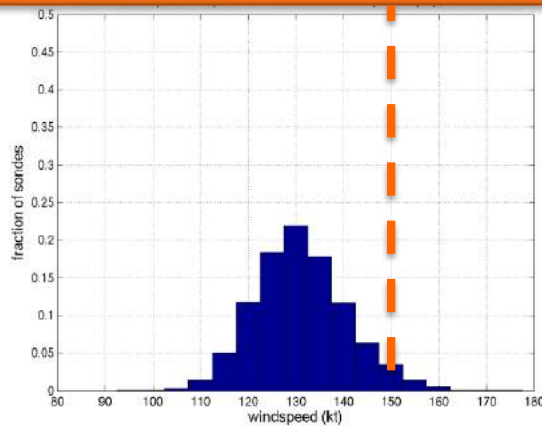
Bonus Slides!

What is the Estimated TC Intensity?

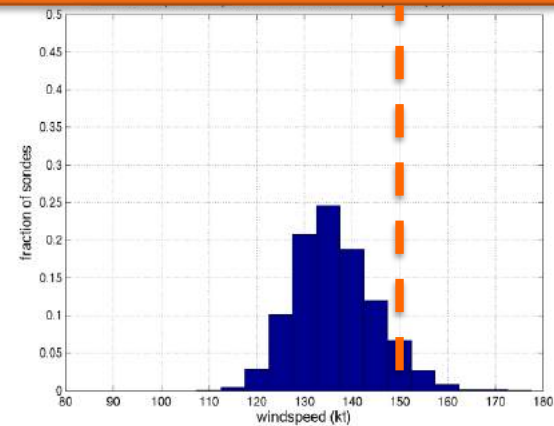
1 Random Sonde



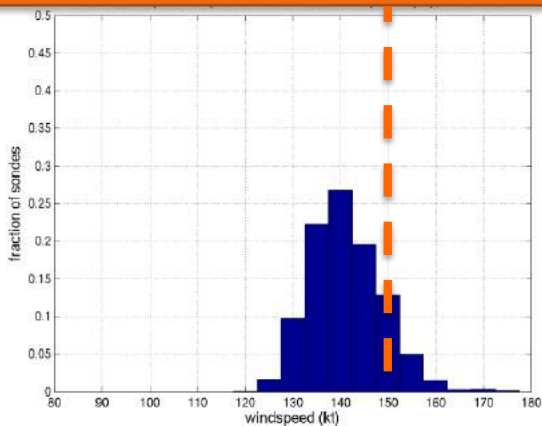
2 Random Sondes



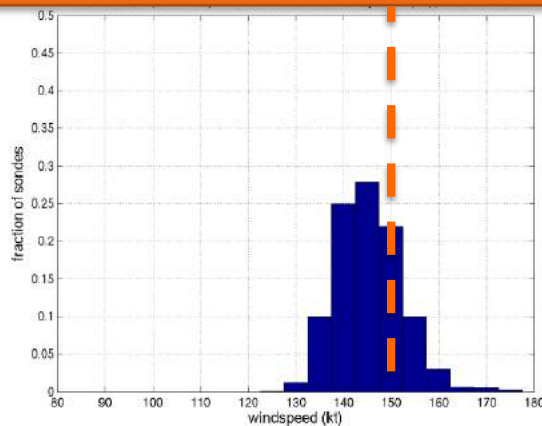
4 Random Sondes



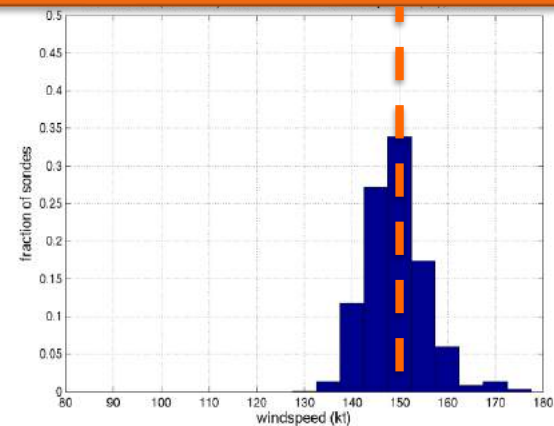
8 Random Sondes



16 Random Sondes

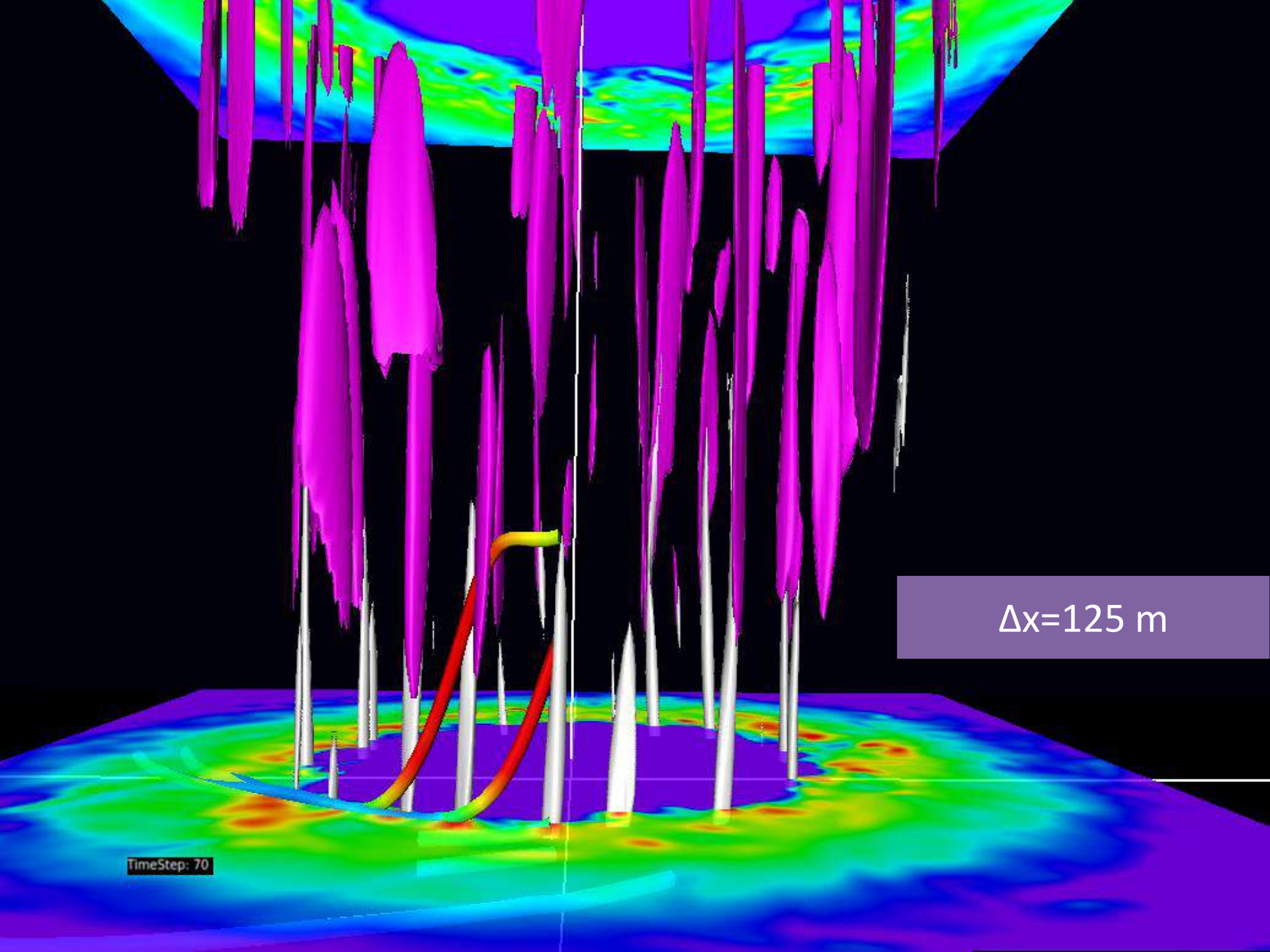


32 Random Sondes



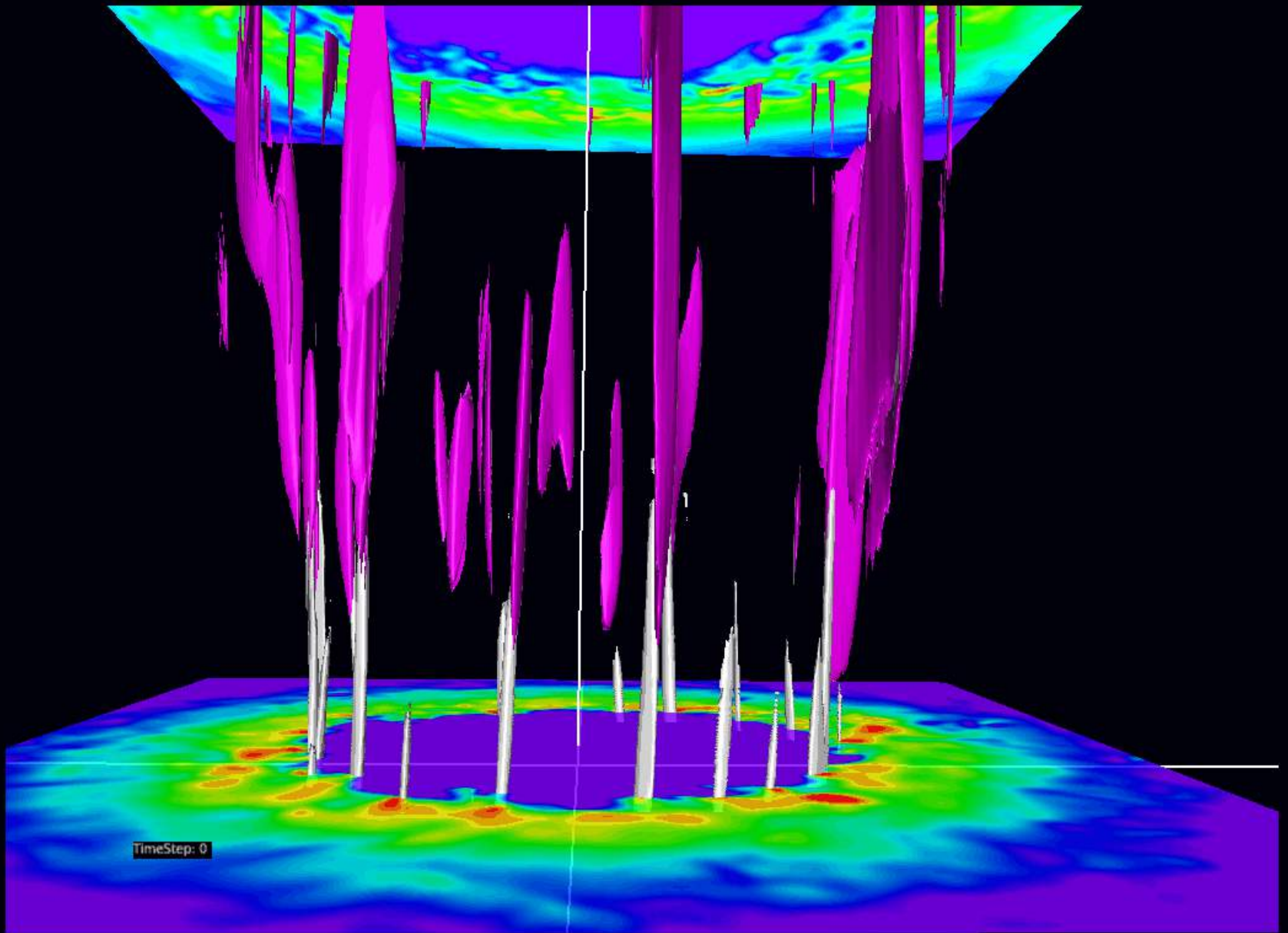
Using Parcel Trajectories to Explore Dynamics

- Calculate backward and forward trajectories for air parcels that enter an extreme low-level updraft.
- We can learn where such parcels originate and how the wind speed changes along the trajectory path.

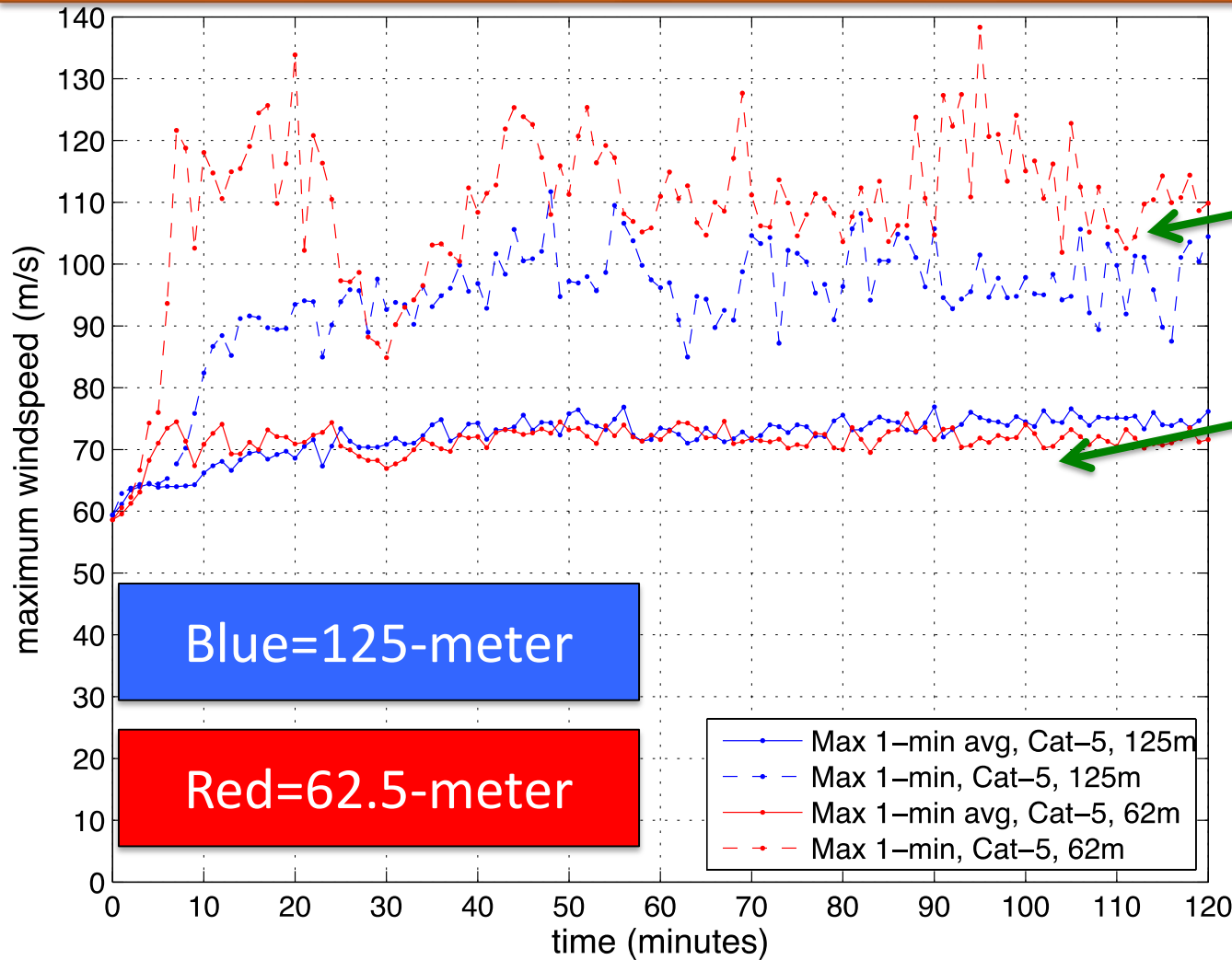


$\Delta x = 125 \text{ m}$

TimeStep: 70



Maximum Surface Wind Speed vs. Time

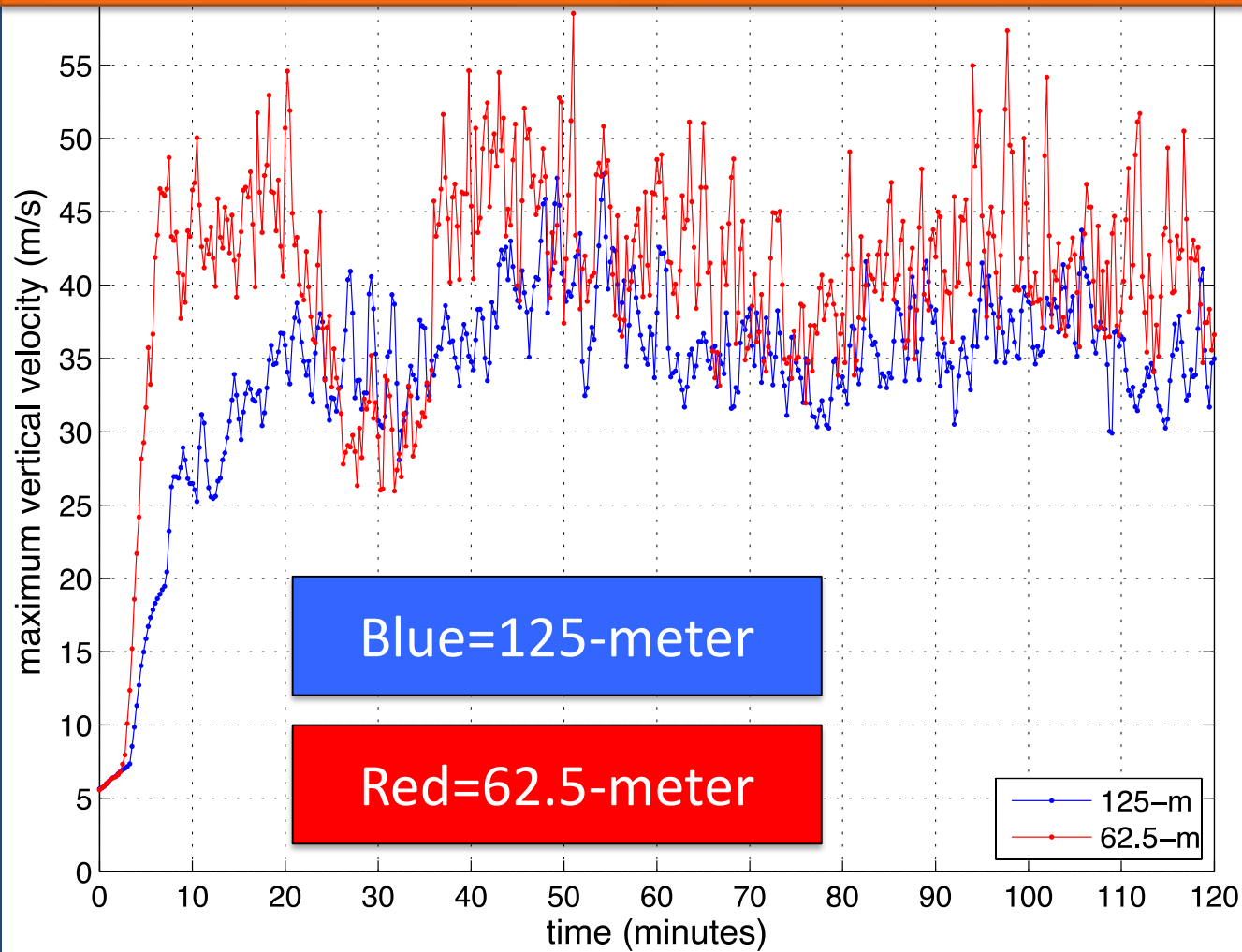


Maximum over previous minute

Average over previous minute

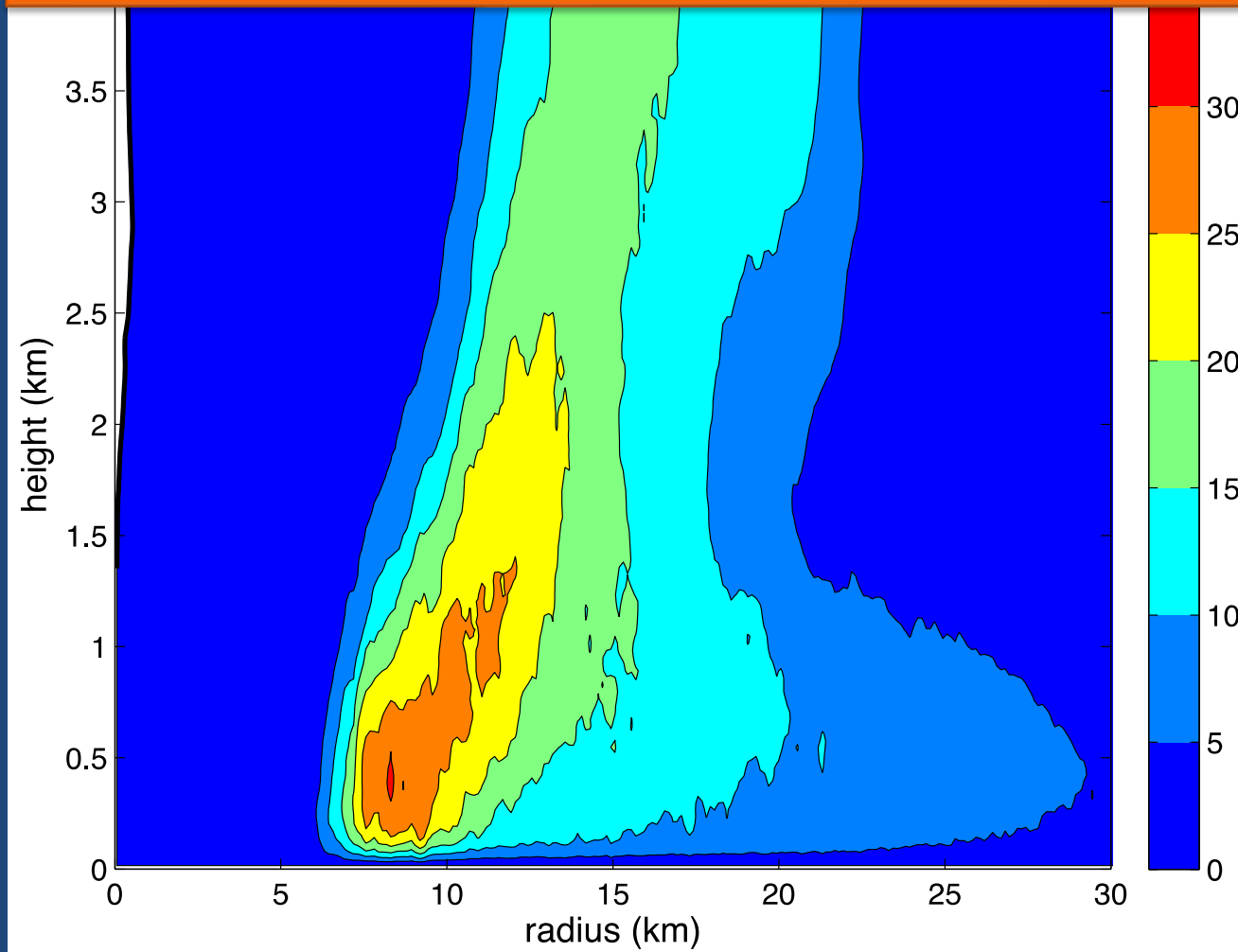
- 1-min mean winds are not that sensitive to grid spacing.
- Instantaneous gusts become stronger with finer resolution.

Maximum Vertical Velocity vs. Time



- Starting from an initially axisymmetric state, small-scale extreme updrafts develop quite quickly.
- Updraft strength is sensitive to grid spacing.

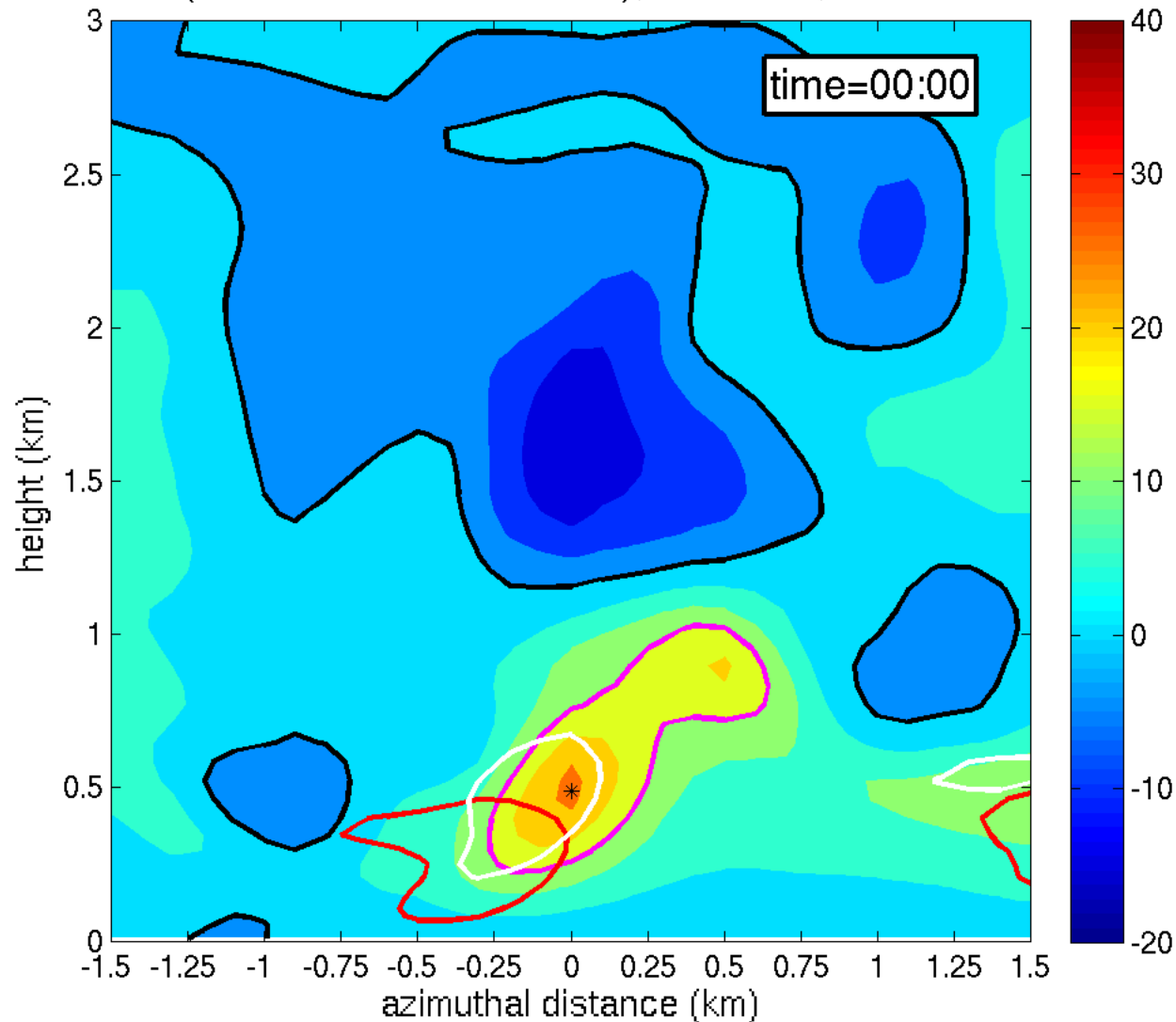
Maximum Vertical Velocity



Strongest updrafts are within or just above the boundary layer

Azimuth-Height Cross Section Following Updraft

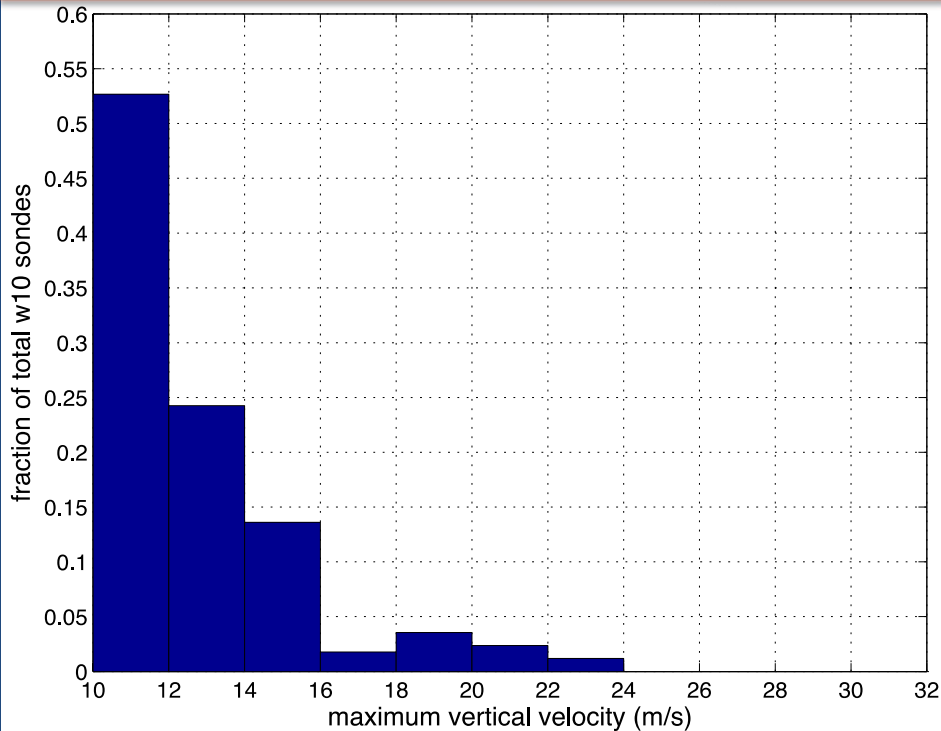
W (max at 500m = 26.7 m/s), r=8.9 km, az=301.8



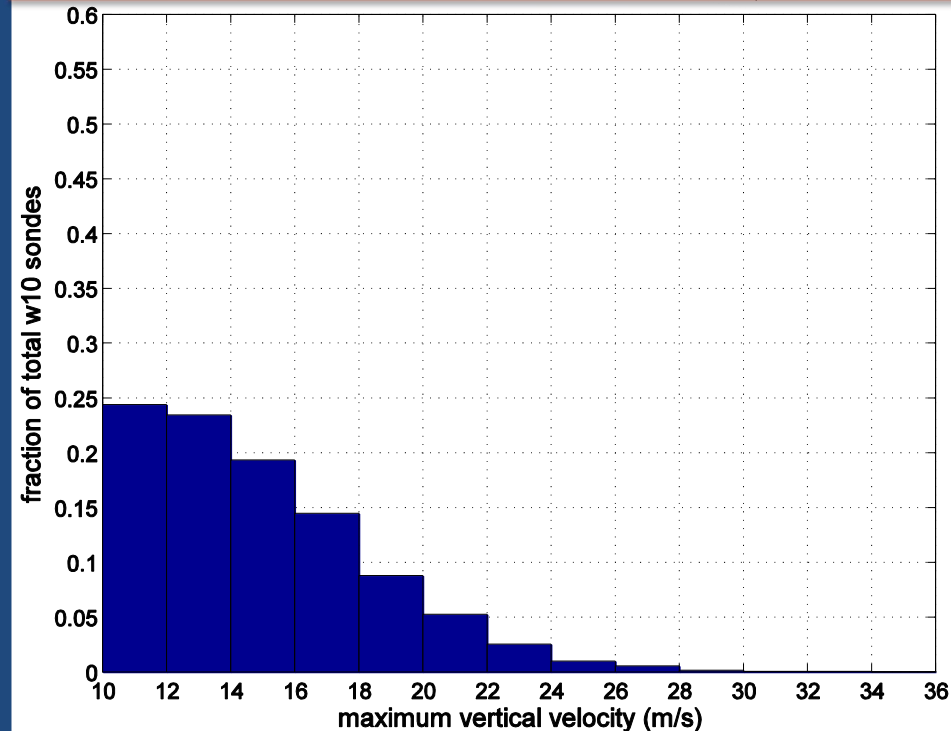
How can we test the realism of the LES?

PDF of Max Vertical Velocity for sondes with $w > 10$ m/s

Observed

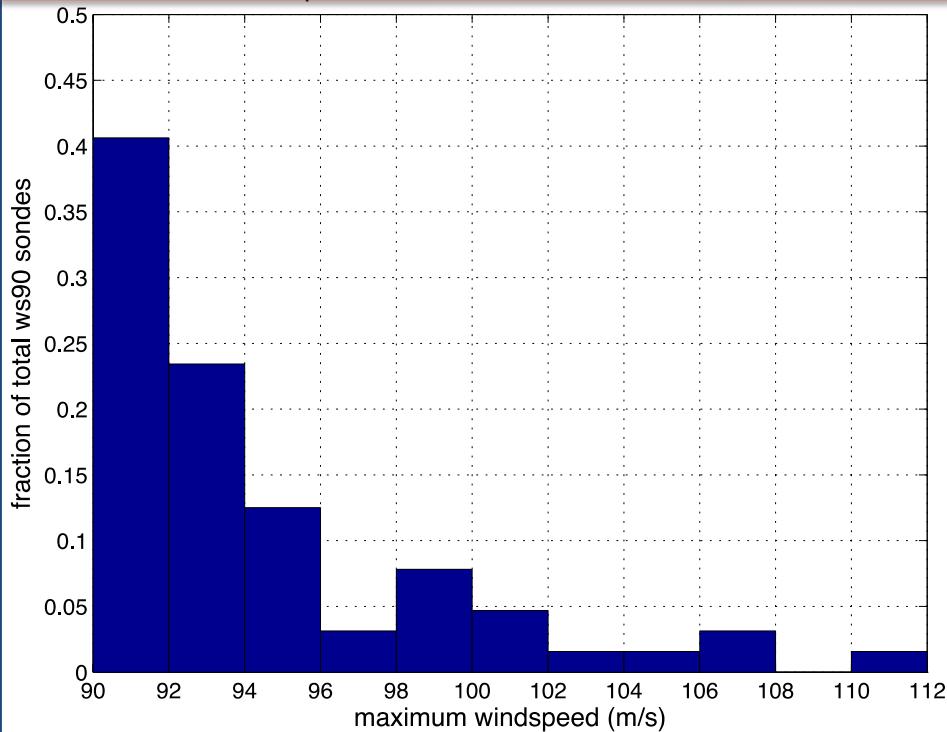


Simulated

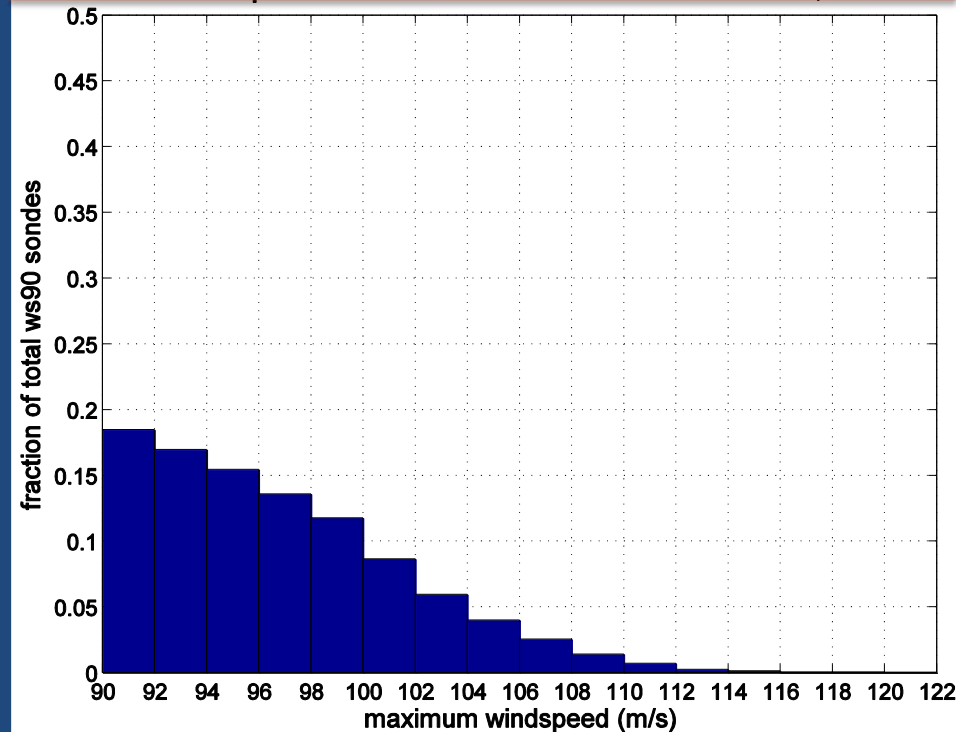


PDF of Max Wind Speed for sondes with $WS > 90$ m/s

Observed

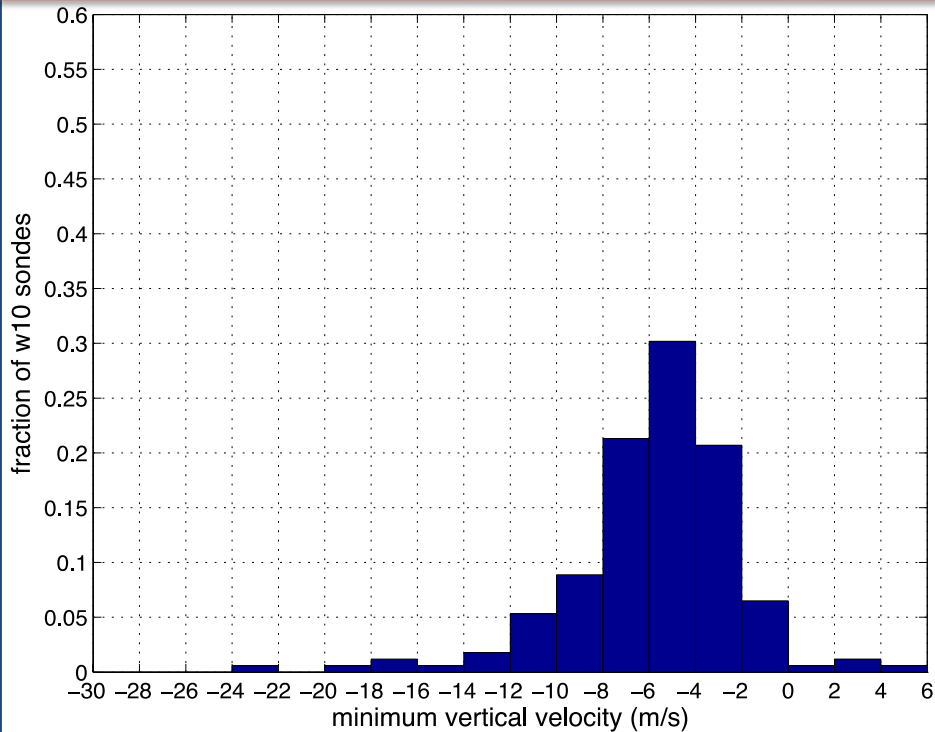


Simulated

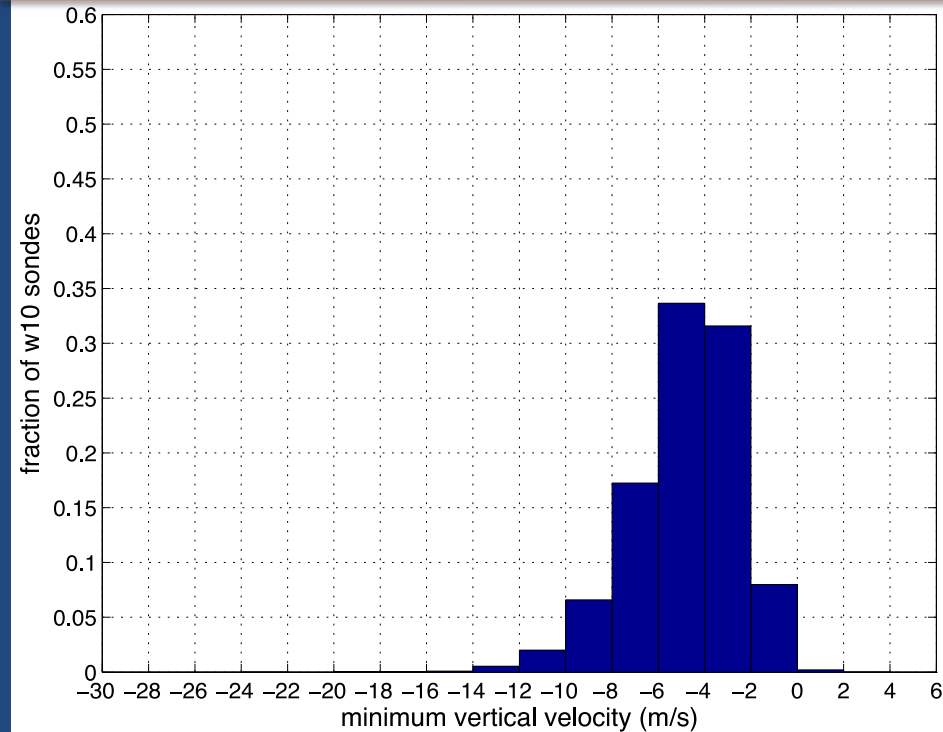


PDF of Min vertical velocity for sondes with $w > 10$ m/s

Observed

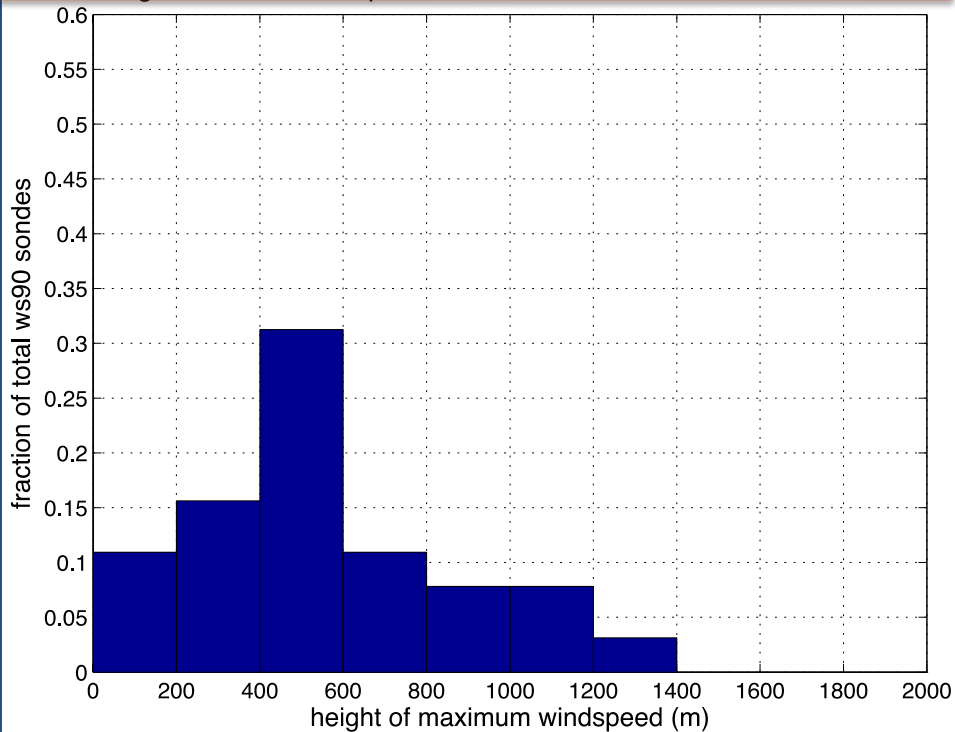


Simulated

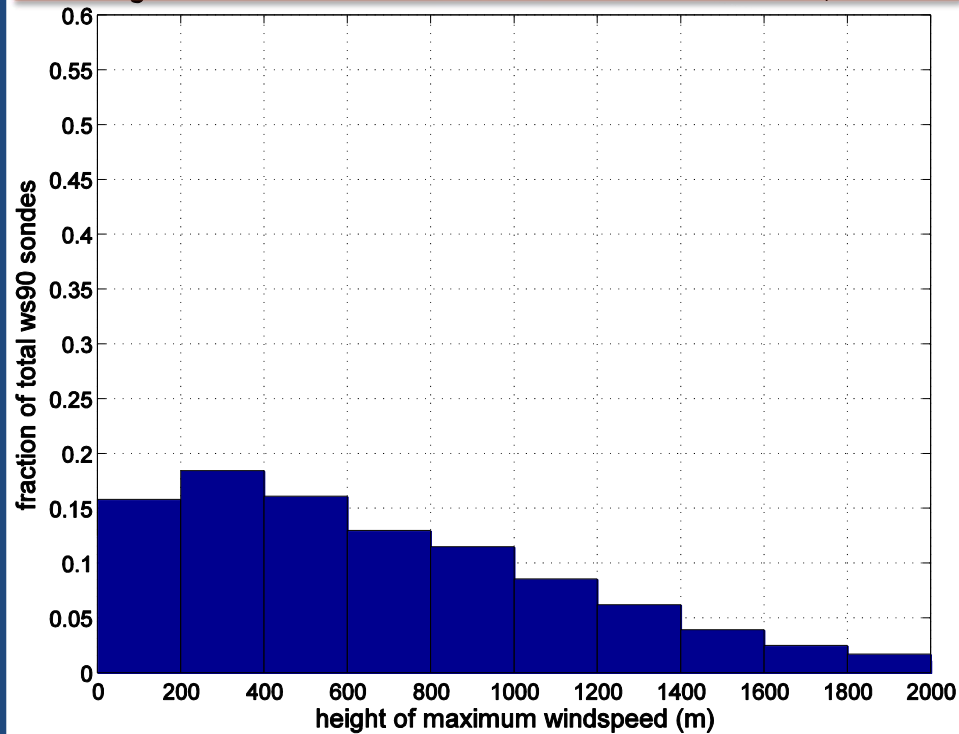


PDF of Height of Max WS for sondes with WS > 90m/s

Observed (WS > 90m/s)

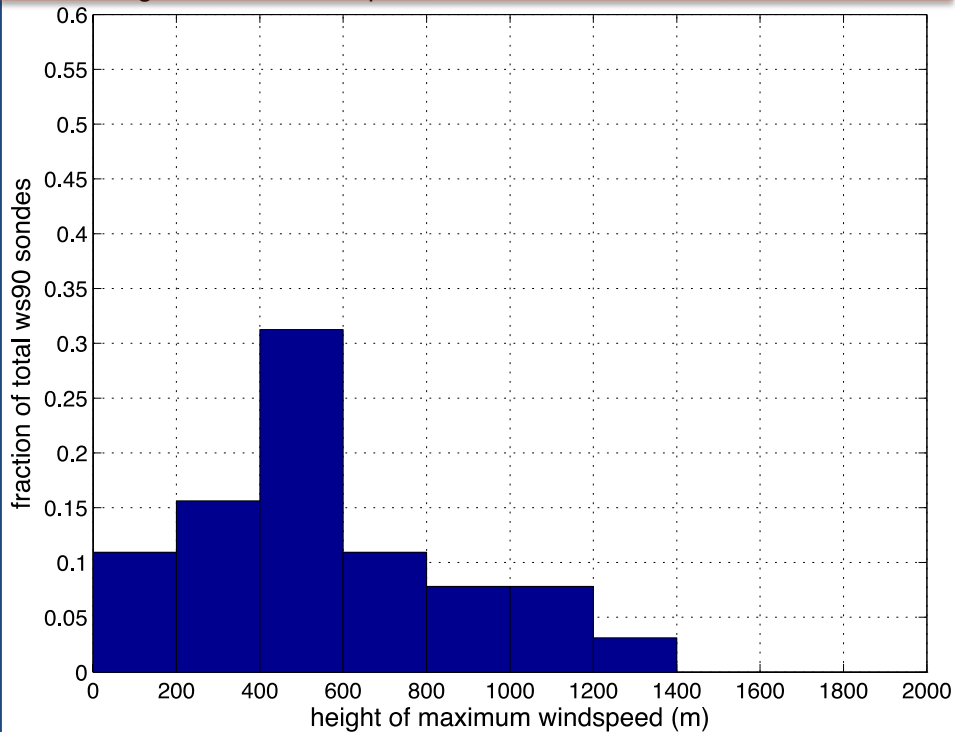


Simulated

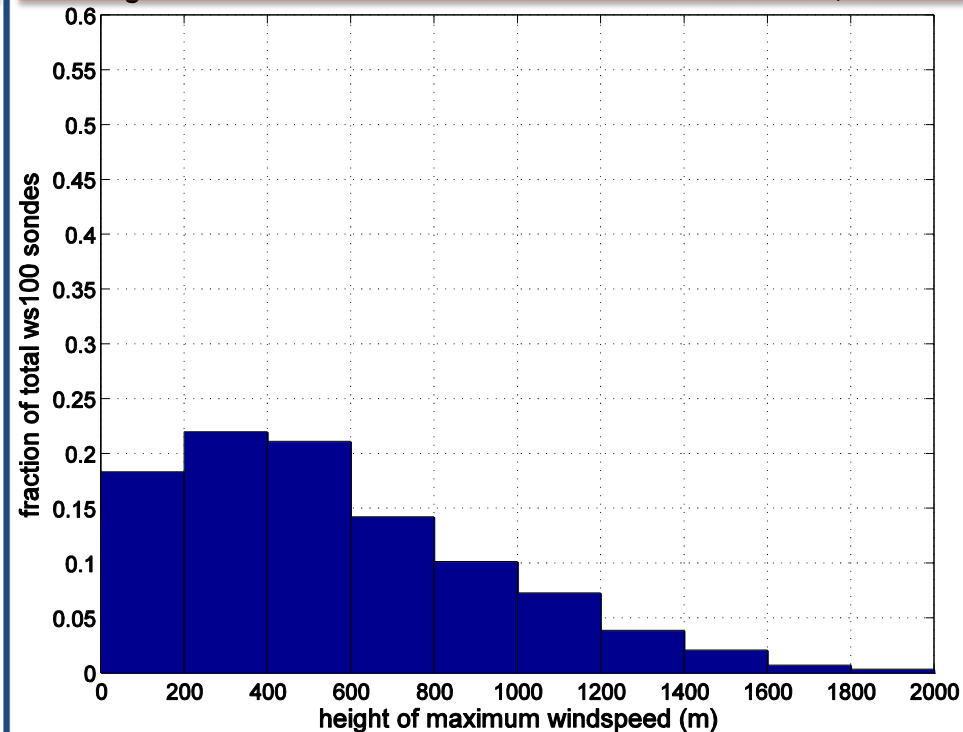


PDF of Height of Max WS for sondes with WS>100 m/s

Observed (WS>90m/s)

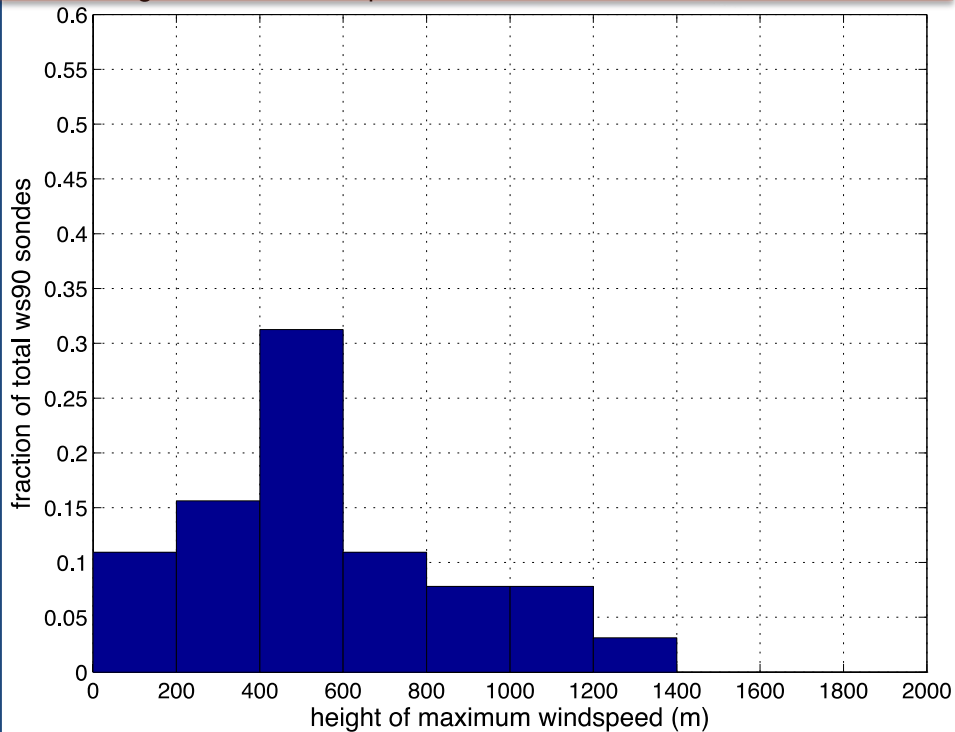


Simulated

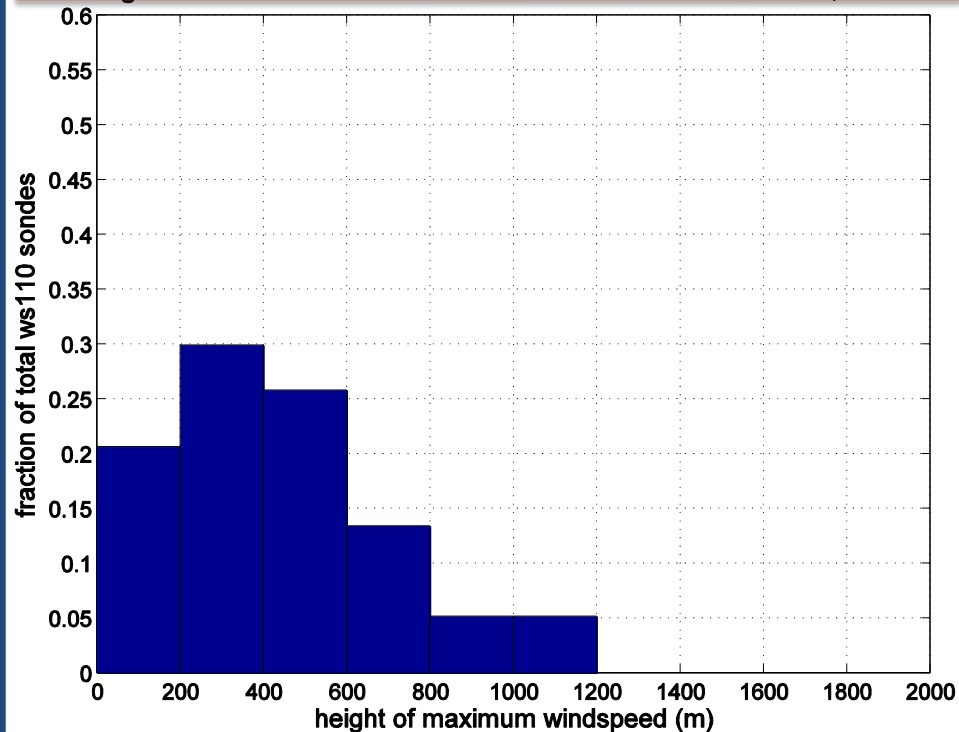


PDF of Height of Max WS for sondes with WS>110 m/s

Observed (WS>90m/s)

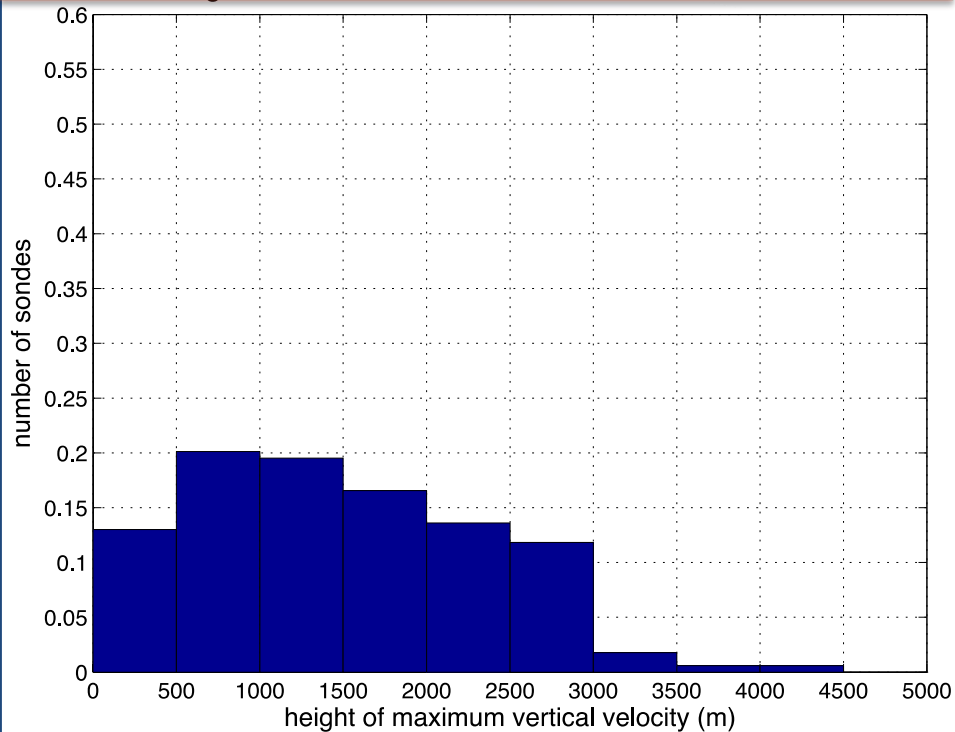


Simulated

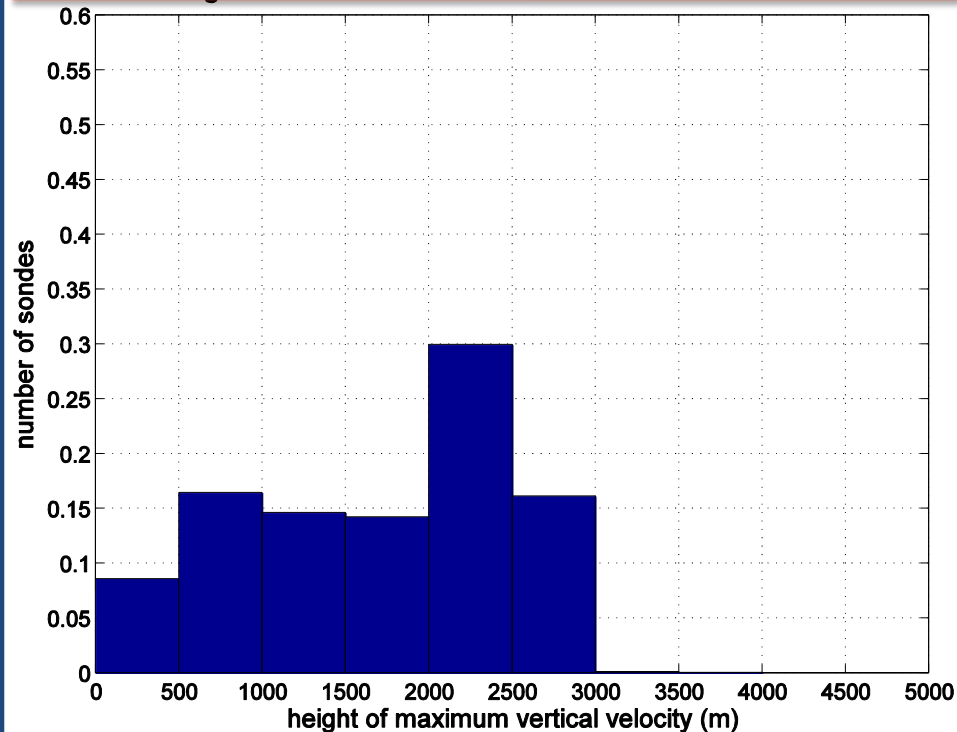


PDF of Height of Max w for sondes with $w > 10$ m/s

Observed

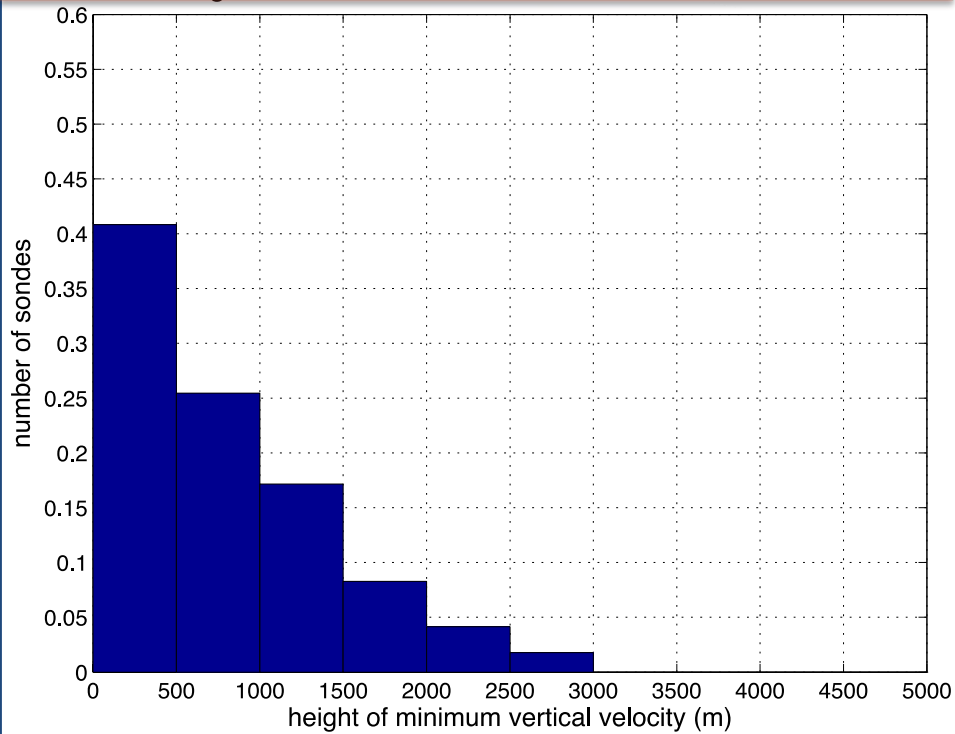


Simulated

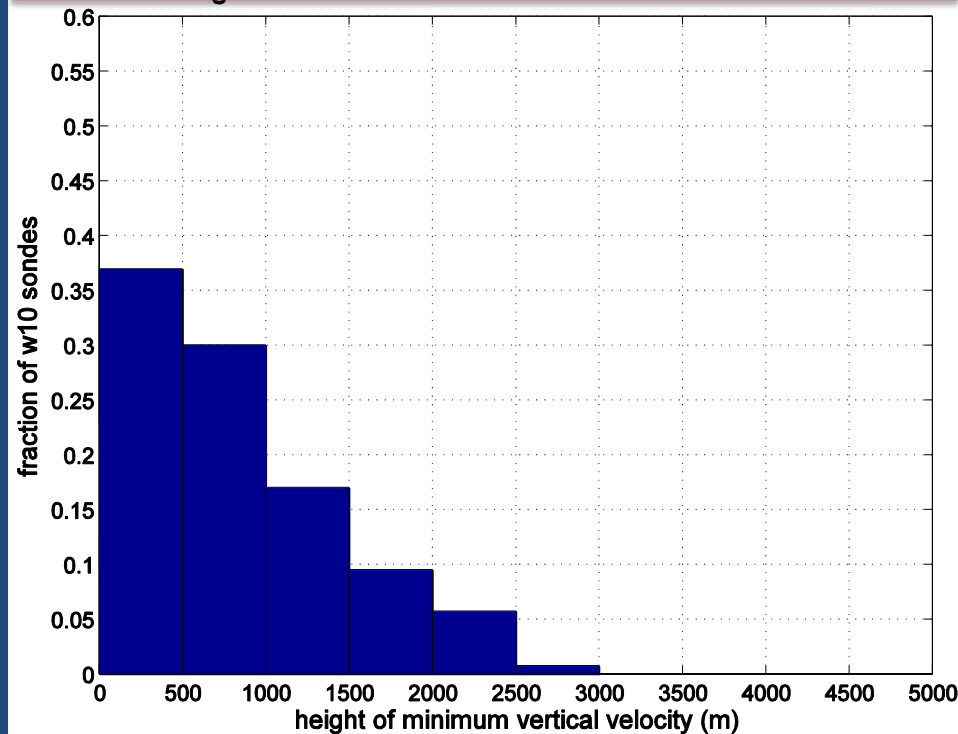


PDF of Height of Min w for sondes with $w > 10$ m/s

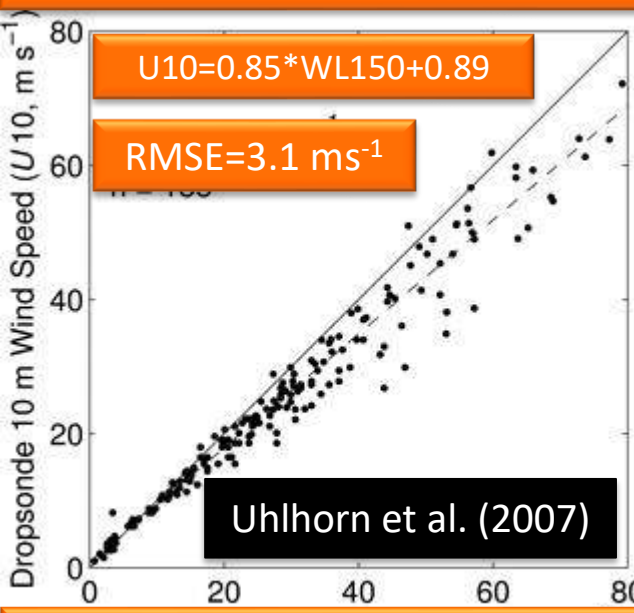
Observed



Simulated

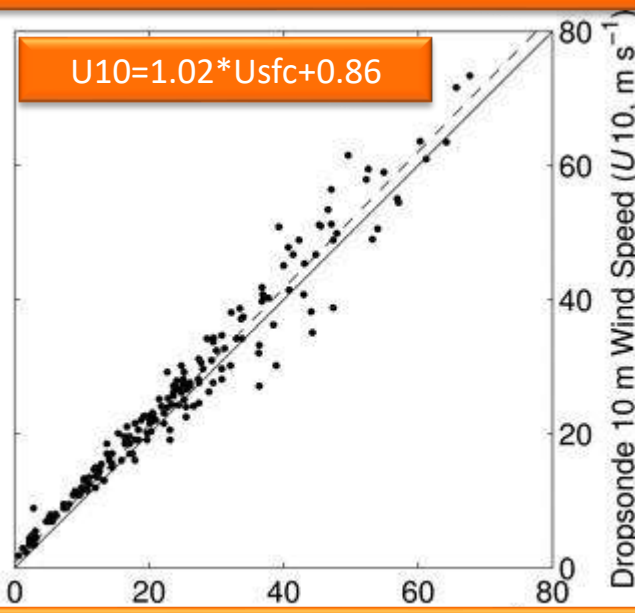


WL150 vs. 10-m Windspeed, Observed



WL150 (m/s)

USfc vs. 10-m Windspeed, Observed



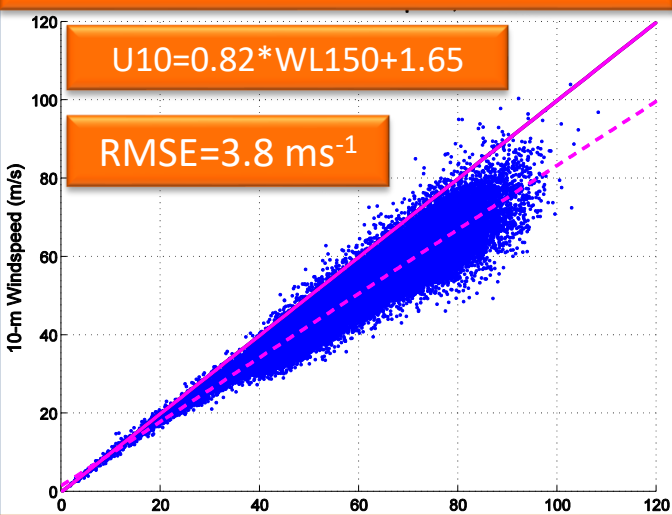
WL150 reduced to surface (m/s)

WL150: Mean wind speed in lowest 150 meters.

Usfc: Estimate of surface wind speed from WL150.

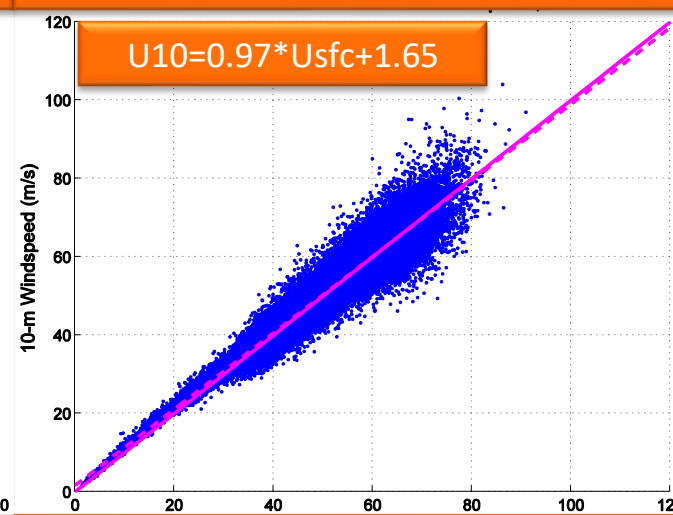
U10: Actual instantaneous 10-meter wind speed.

WL150 vs. 10-m Windspeed, Simulated



WL150 (m/s)

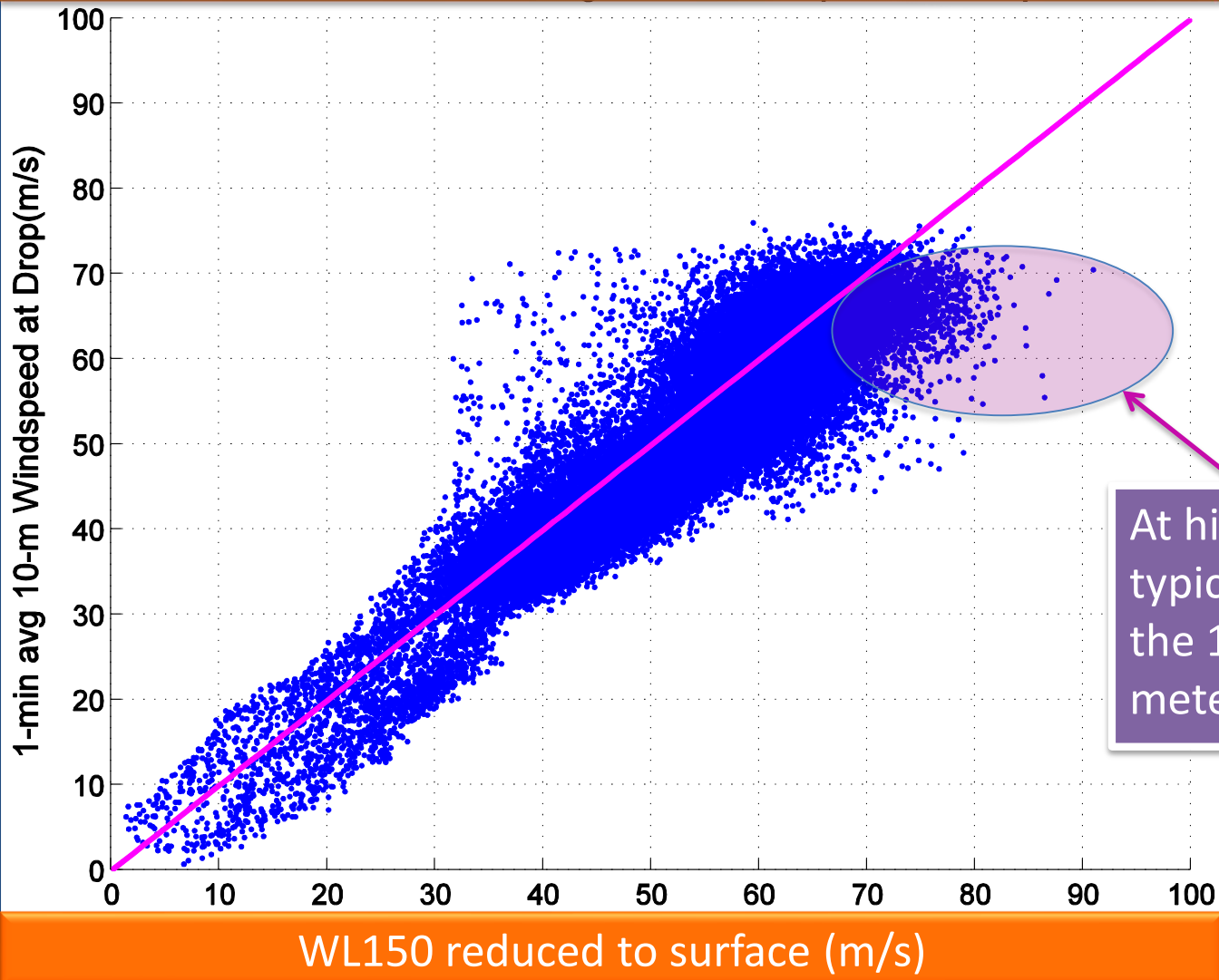
USfc vs. 10-m Windspeed, Simulated



WL150 reduced to surface (m/s)



USfc vs. 1-minute mean 10-m Windspeed, Simulated



At high wind speeds, U_{sfc} is typically an *overestimate* of the 1-minute average 10-meter wind speed.

A legend box containing a vertical magenta line and a horizontal magenta rectangle, with the text '1:1 Line' next to the rectangle.

Using Parcel Trajectories to Explore Dynamics

- Calculate backward and forward trajectories for air parcels that enter an extreme low-level updraft.
- We can learn where such parcels originate and how the wind speed changes along the trajectory path.

Is Buoyancy Important for Forcing the Acceleration of Low-Level Updrafts?

We can determine the total vertical acceleration, and partition it into two parts:

1. A “**thermodynamic**” acceleration, associated with buoyancy
2. A “**dynamic**” acceleration, associated with the perturbation pressure gradient force.

Is Buoyancy Important?

Momentum equation (neglecting Coriolis and diffusion):

$$\frac{dw}{dt} = -c_p \theta_v \frac{\partial \pi'}{\partial z} + B$$

$$\pi = \left(\frac{p}{p_0} \right)^{\frac{R_d}{c_p}}$$

$$B = g \left(\frac{\theta - \theta_0}{\theta_0} + 0.61(q_v - q_{v0}) - q_h \right)$$

θ : Potential Temperature

$\theta_0(z)$: Base State Potential Temperature

q_v : Water Vapor Mixing Ratio

q_h : Hydrometeor Mixing Ratio

Note: p is the perturbation pressure, after a hydrostatic basic state has been removed.

We can decompose pressure into a buoyant and a dynamic part:

Momentum equation (neglecting Coriolis and diffusion):

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0} \nabla p + B \vec{k} - \vec{v} \cdot \nabla \vec{v}$$

Multiplying by density and taking divergence:

$$\frac{\partial}{\partial t} (\nabla \cdot \rho_0 \vec{v}) = -\nabla^2 p + \frac{\partial}{\partial z} (\rho_0 B) - \nabla (\rho_0 \vec{v} \cdot \nabla v)$$

We can decompose pressure into a buoyant and a dynamic part:

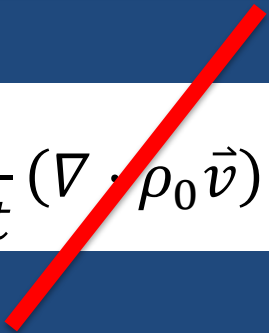
Momentum equation (neglecting Coriolis and diffusion):

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0} \nabla p + B \vec{k} - \vec{v} \cdot \nabla \vec{v}$$

$$\rho_0 = \rho_0(z)$$

Anelastic approximation

Multiplying by density and taking divergence:


$$\frac{\partial}{\partial t} (\nabla \cdot \rho_0 \vec{v}) = -\nabla^2 p + \frac{\partial}{\partial z} (\rho_0 B) - \nabla (\rho_0 \vec{v} \cdot \nabla v)$$

$$\nabla \cdot \rho_0 \vec{v} = 0$$

Divergence of 3D Mass Flux is Zero

We can decompose pressure into a buoyant and a dynamic part:

Momentum equation (neglecting Coriolis and diffusion):

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_0} \nabla p + B \vec{k} - \vec{v} \cdot \nabla \vec{v}$$

Multiplying by density and taking divergence:

$$\frac{\partial}{\partial t} (\nabla \cdot \rho_0 \vec{v}) = -\nabla^2 p + \frac{\partial}{\partial z} (\rho_0 B) - \nabla (\rho_0 \vec{v} \cdot \nabla \vec{v})$$

$$\nabla^2 p = F_B + F_D$$

$$F_B = \frac{\partial}{\partial z} (\rho_0 B)$$

$$F_D = -\nabla (\rho_0 \vec{v} \cdot \nabla \vec{v})$$

We can decompose pressure into a buoyant and a dynamic part:

$$\nabla^2 p = F_B + F_D$$

$$F_B = \frac{\partial}{\partial z} (\rho_0 B)$$

$$F_D = -\nabla(\rho_0 \vec{v} \cdot \nabla \vec{v})$$

Buoyancy Source

Dynamic Source

$$p = p_B + p_D$$

$$\nabla^2 p_B = F_B$$

$$\nabla^2 p_D = F_D$$

p_B and p_D can be solved for numerically, from which the pressure gradient forces, PGB and PGD can be found.

$$PGB = -\frac{1}{\rho} \frac{\partial p_B}{\partial z}$$

$$PGD = -\frac{1}{\rho} \frac{\partial p_D}{\partial z}$$

We can decompose pressure into a buoyant and a dynamic part:

$$\frac{dw}{dt} = (B + \text{PGB}) + \text{PGD}$$

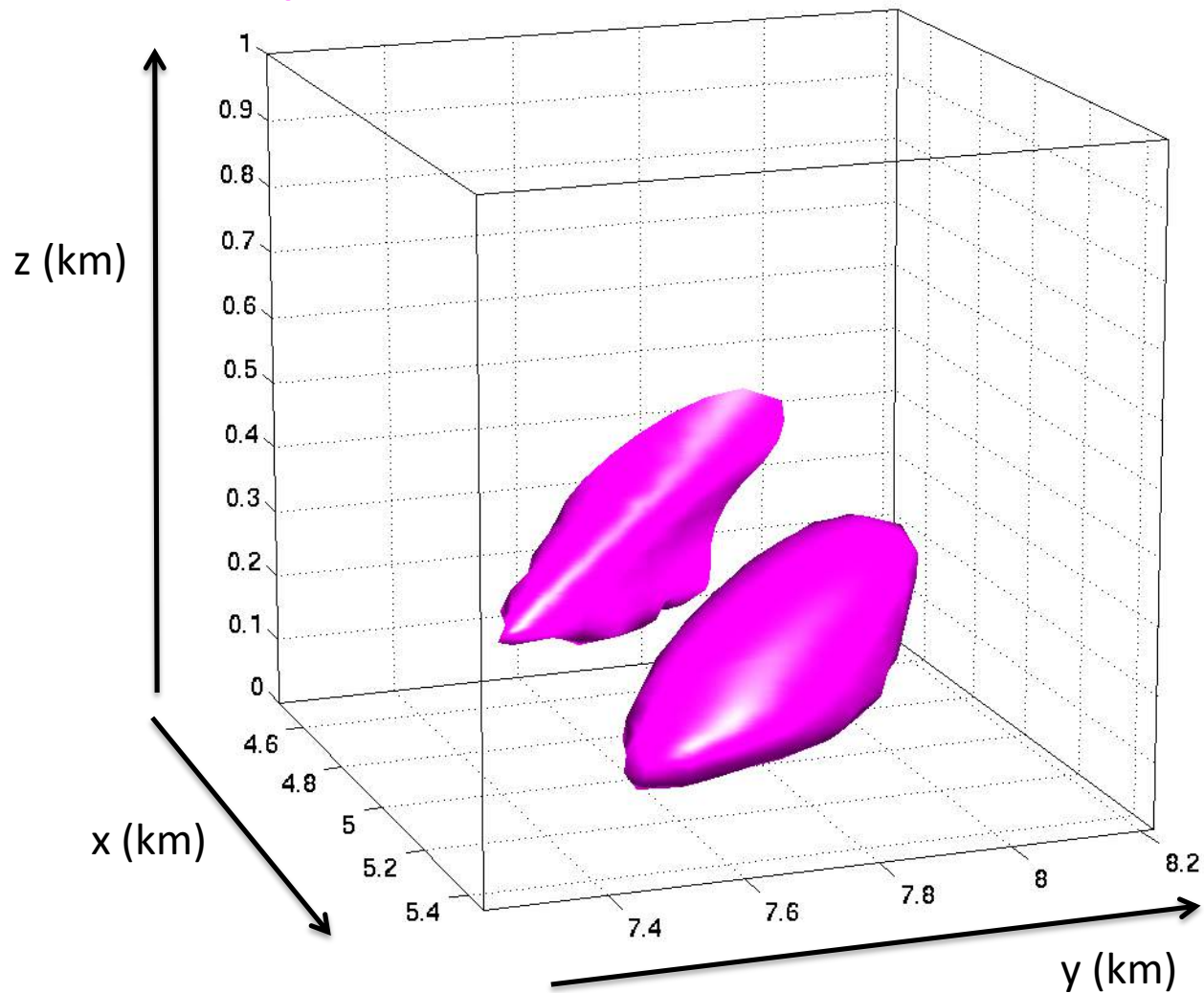
$$\text{PGB} = -\frac{1}{\rho} \frac{\partial p_B}{\partial z}$$

$$\text{PGD} = -\frac{1}{\rho} \frac{\partial p_D}{\partial z}$$

- B and PGB are each strongly dependent on the choice of reference state, but their sum is not.
- So we combine B and PGB into one term, and compare this to PGD.

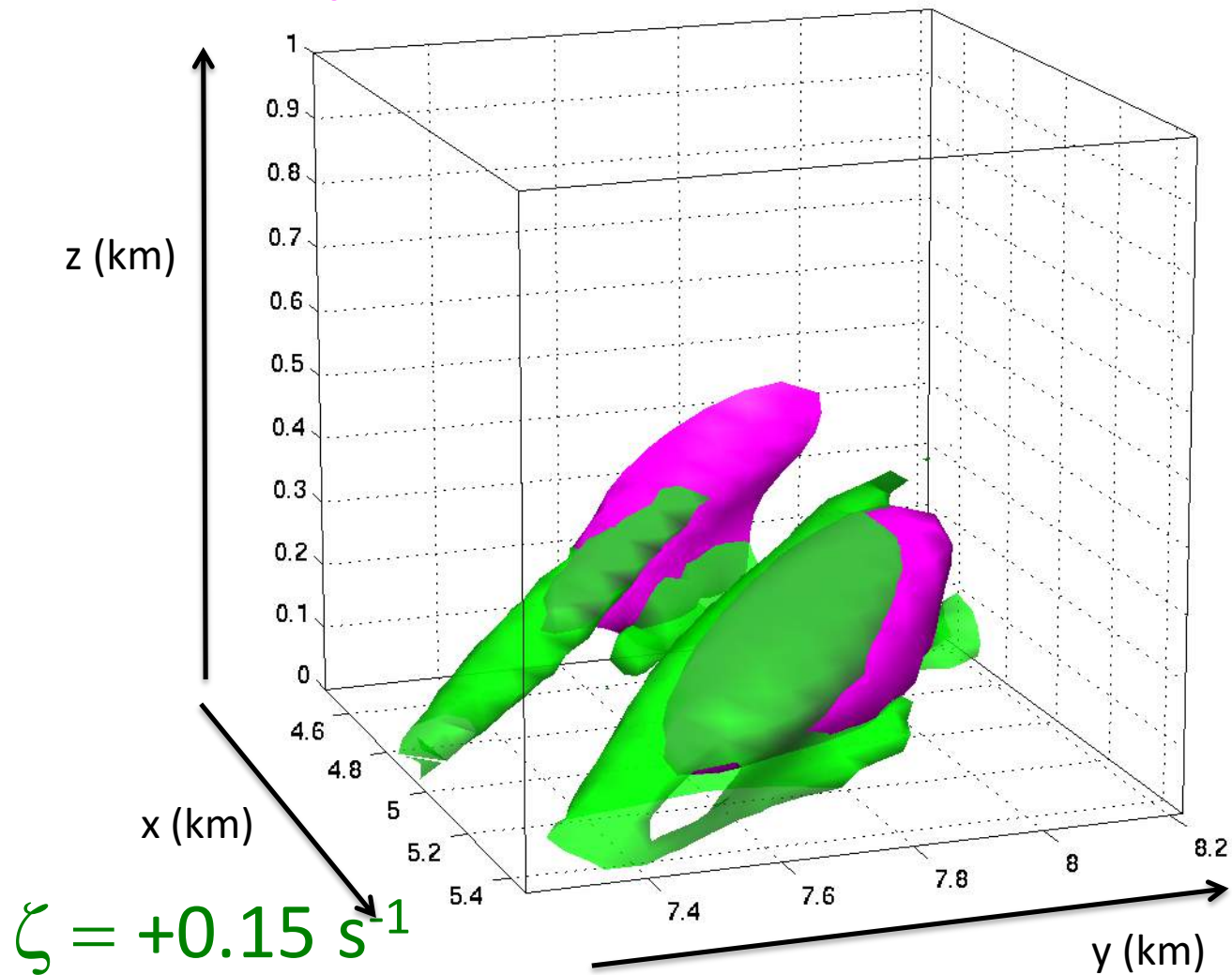
Examine updraft in a 1x1x1 km cube

$w = +12$ m/s



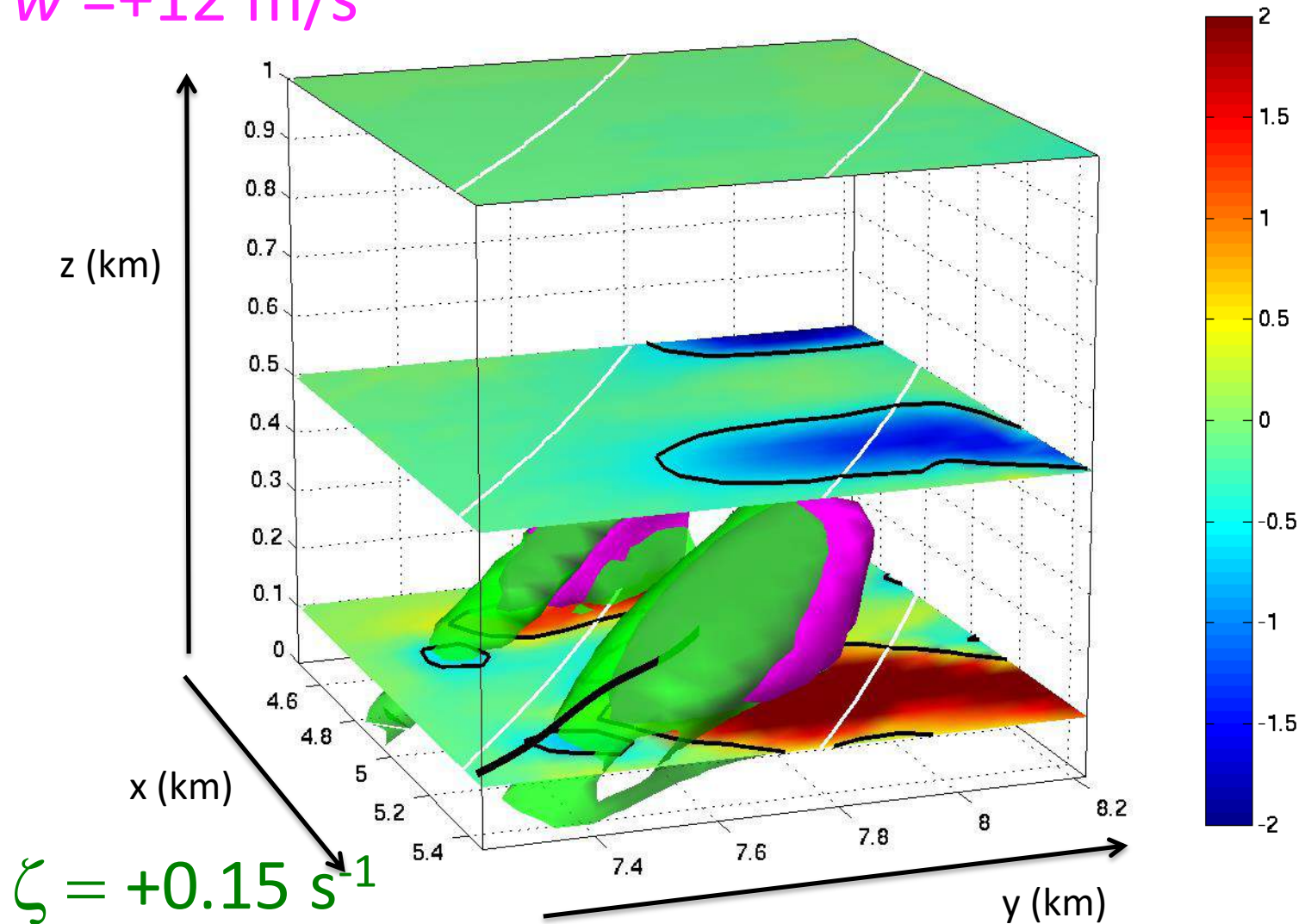
Add Vorticity Isosurface

$w = +12 \text{ m/s}$



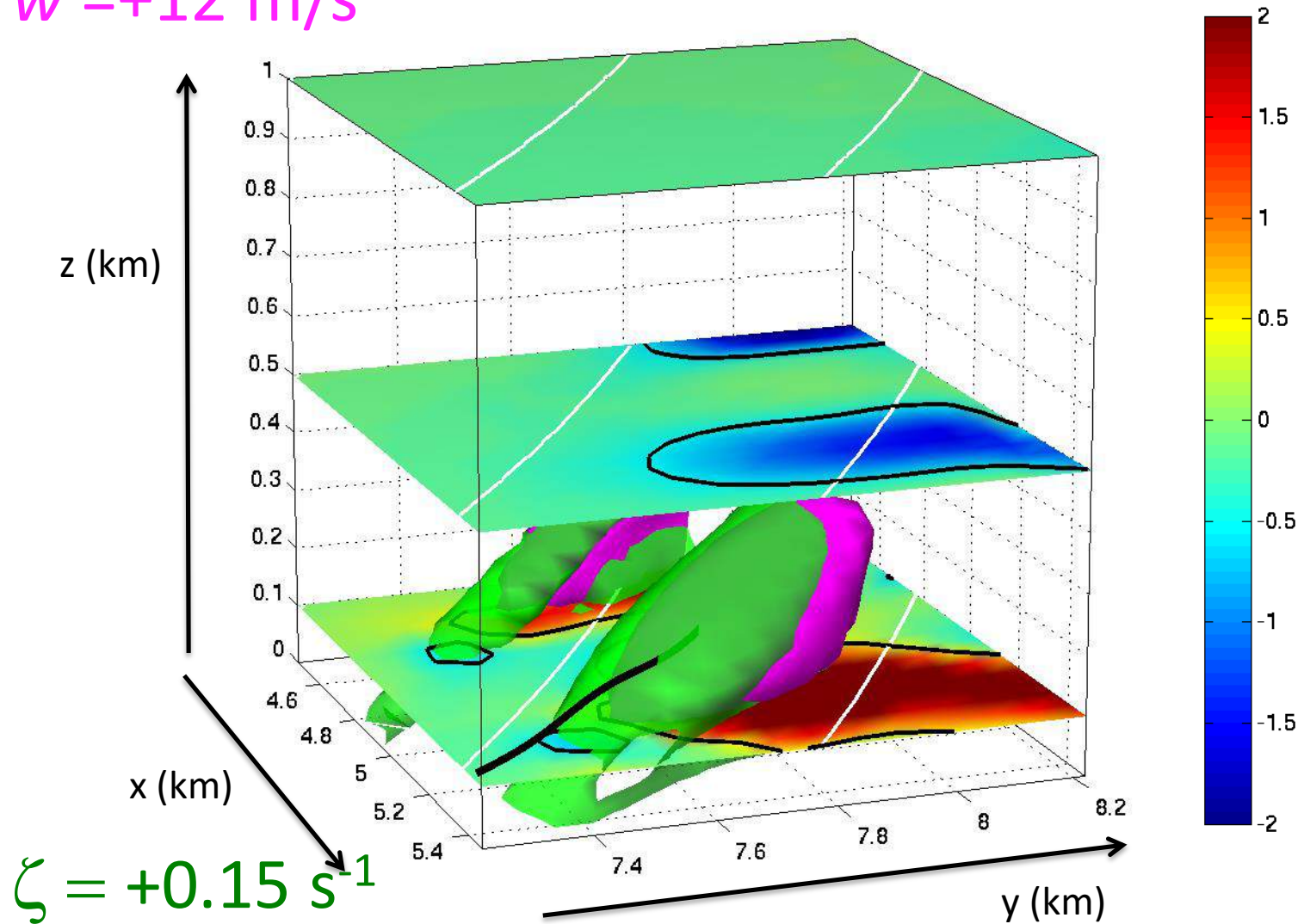
Total Acceleration (ms^{-2} ; +/- 0.5 black)

$w = +12 \text{ m/s}$



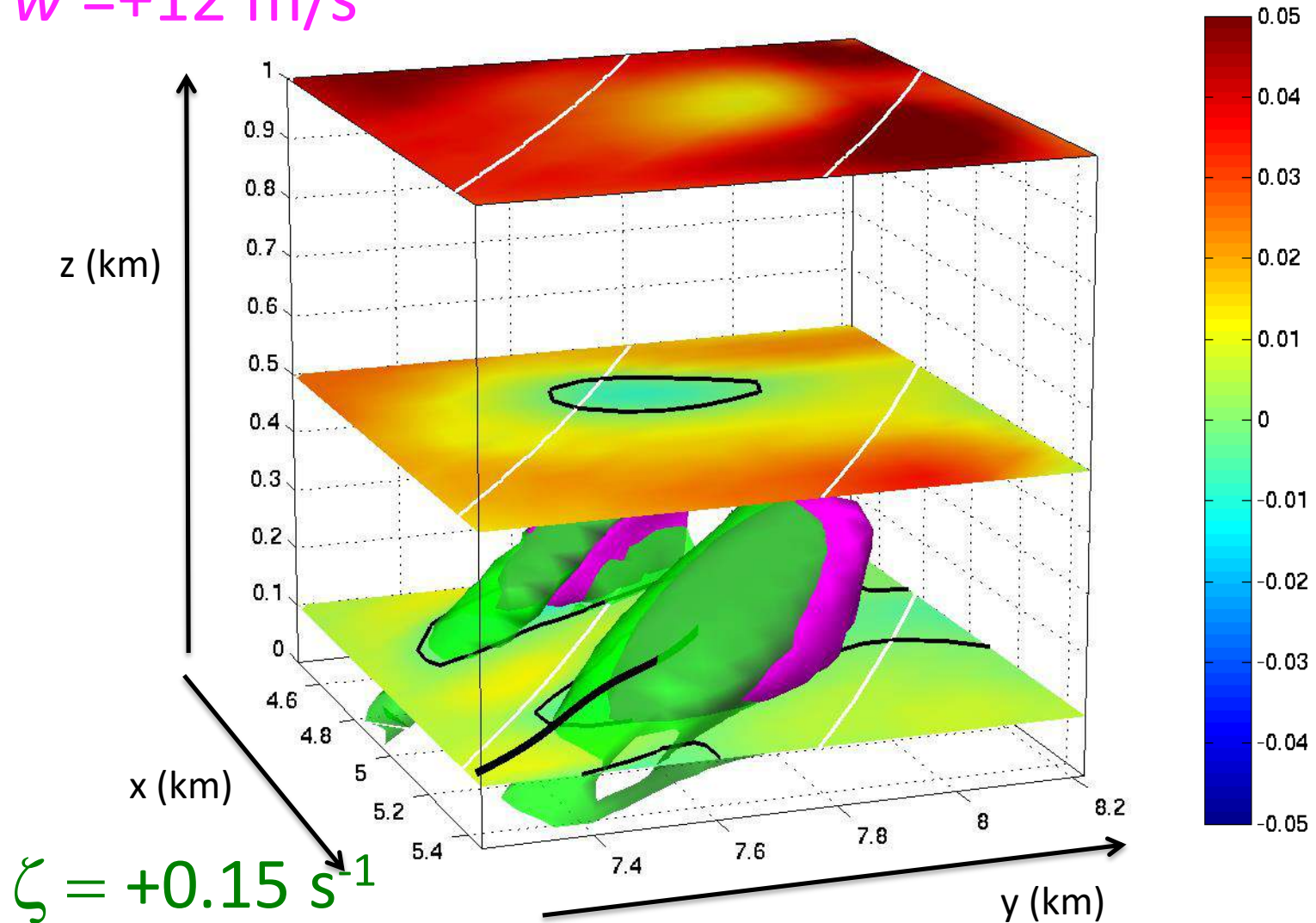
Dynamic Acceleration (ms^{-2} ; +/- 0.5 black)

$w = +12 \text{ m/s}$



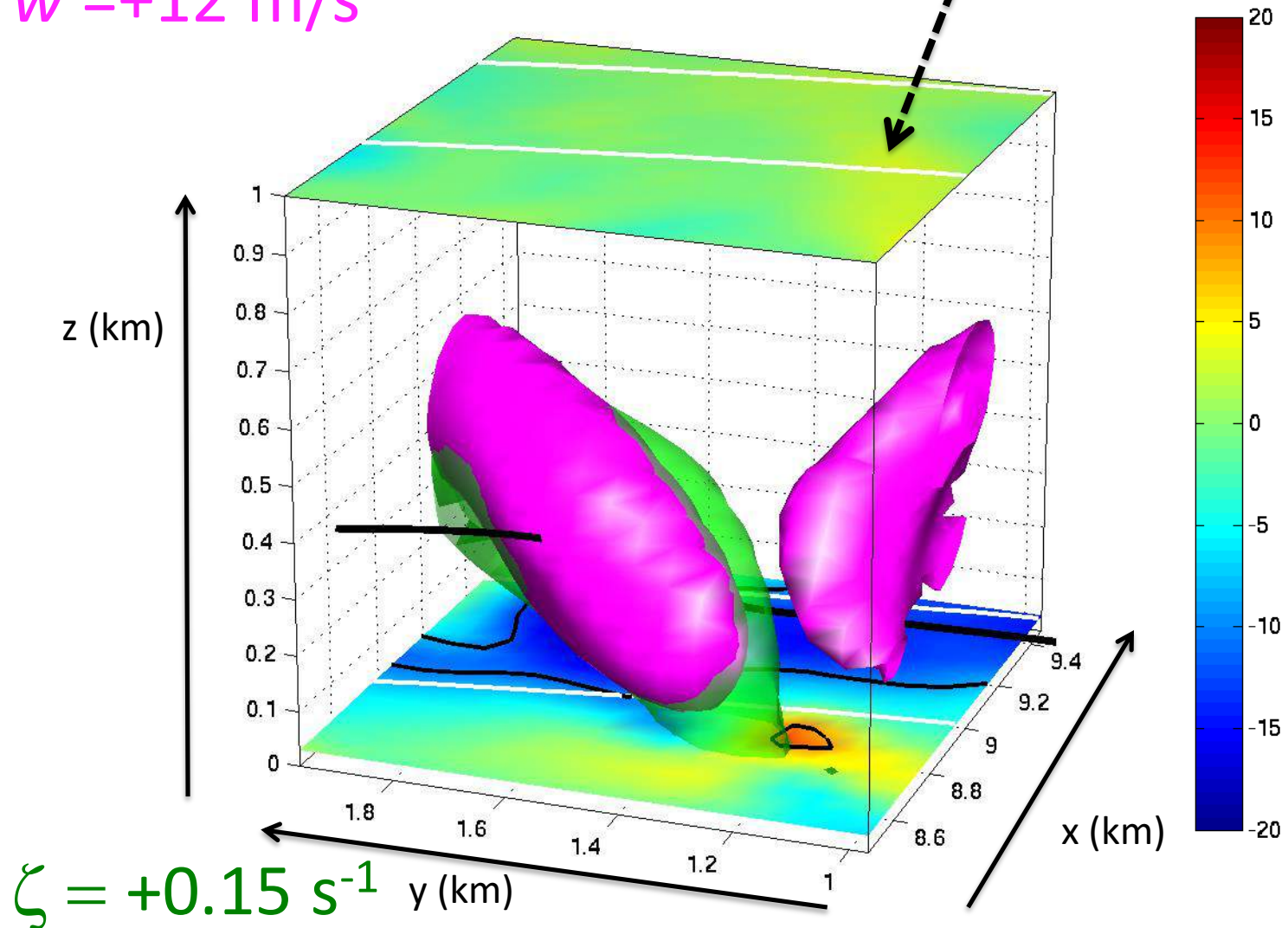
Buoyant Acceleration (ms^{-2} ; 0 black)

$w = +12 \text{ m/s}$



Slices of Perturbation Vr (ms⁻¹; +/- 10 black)

$w = +12$ m/s



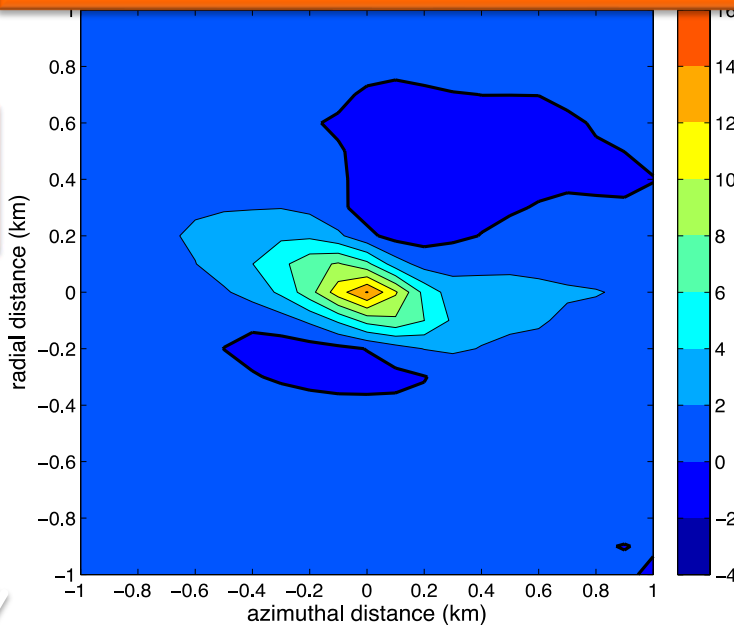
Is there a *systematic* relationship between the updraft, horizontal winds, and vorticity?

We can examine composite fields from many updrafts:

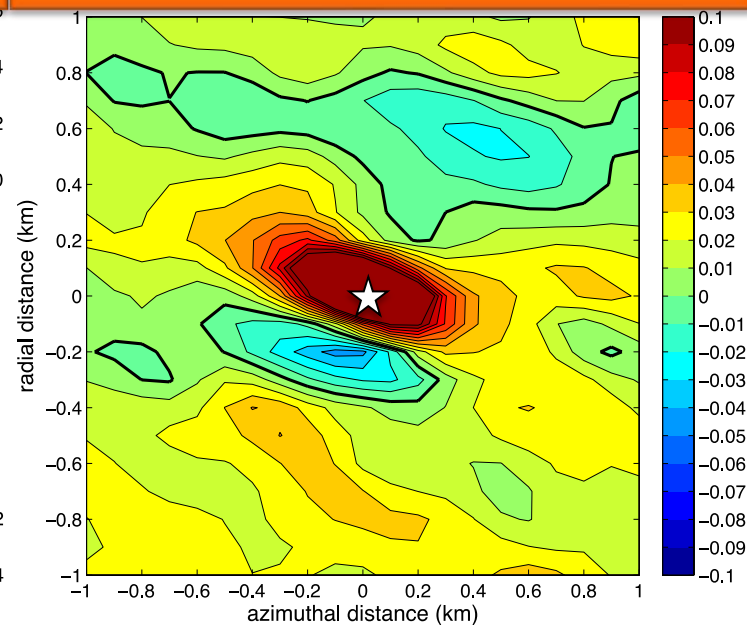
1. At a given level, find all points where $w \geq 12$ m/s.
2. Interpolate to cylindrical coordinates.
3. Take a 2x2 km box (in radius/azimuth) around each point.
4. Average all such boxes.

$z=100$ m

Vertical Velocity



Vertical Vorticity

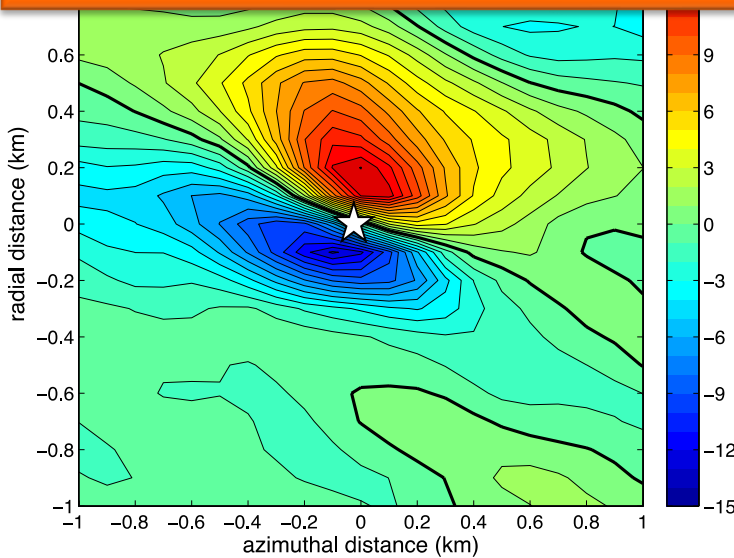


Downstream
(Cyclonic)

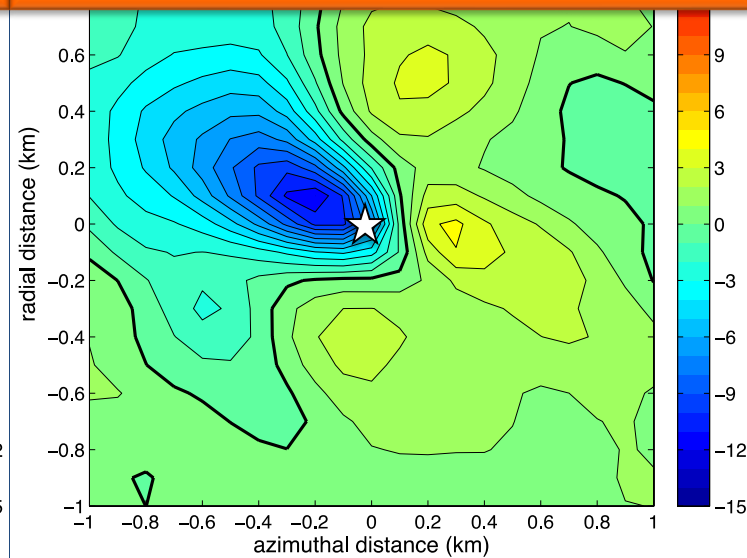


Towards Center

Perturbation V_t

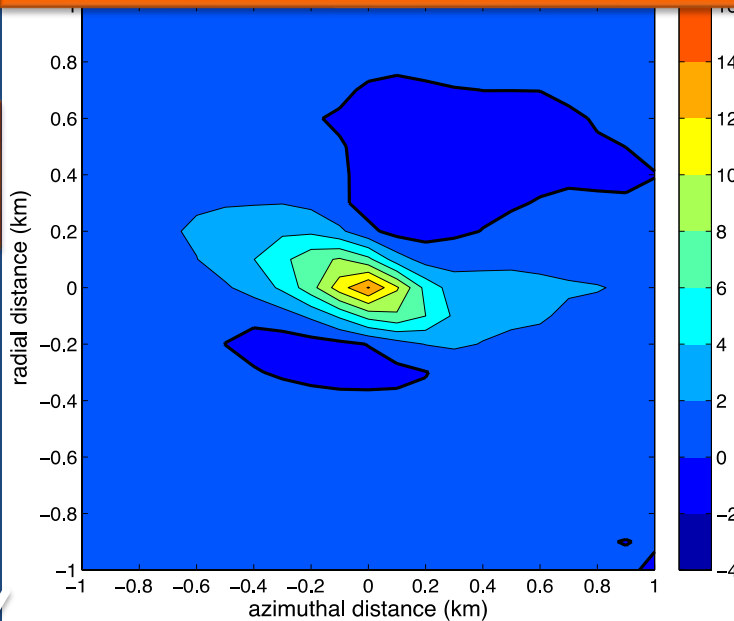


Perturbation V_r

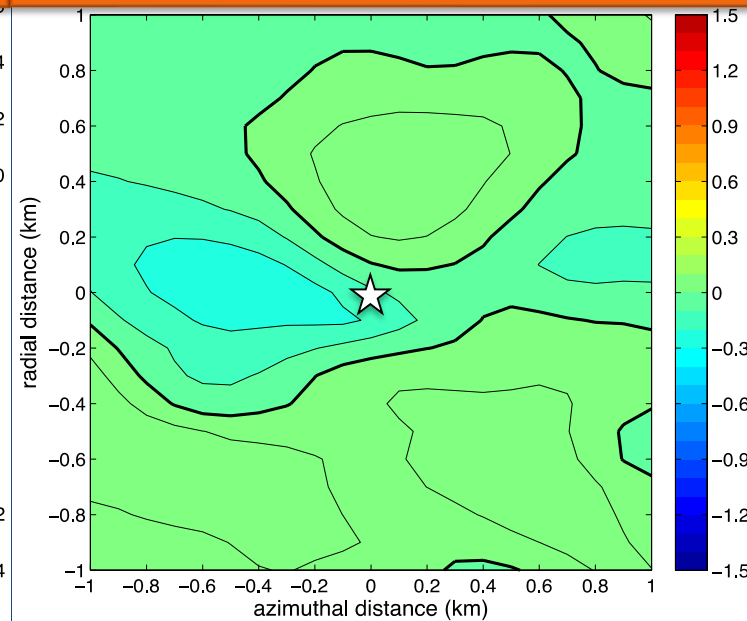


$z=100$ m

Vertical Velocity



Perturbation θ_v

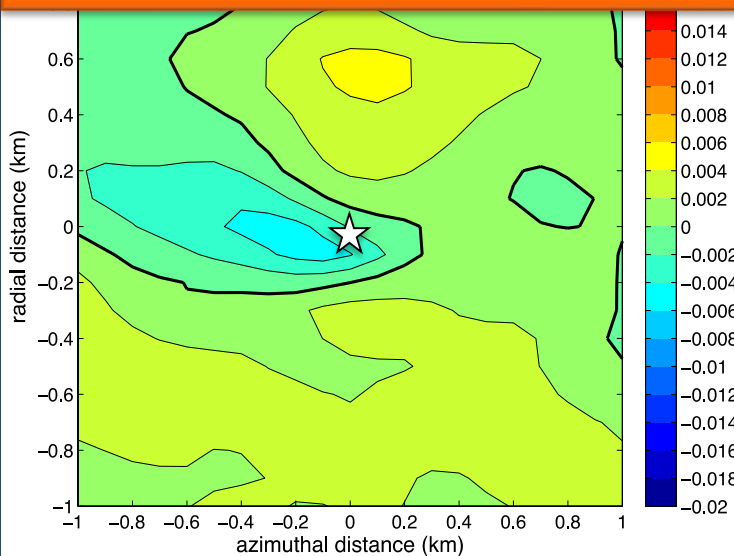


Downstream
(Cyclonic)

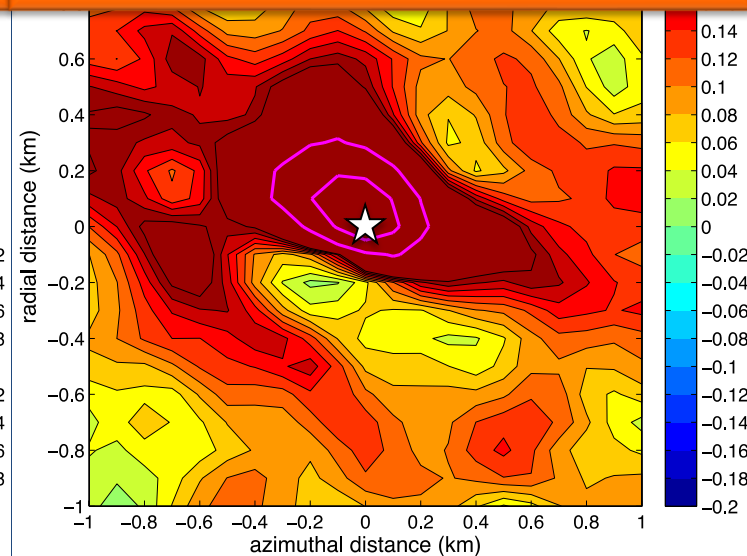


Towards Center

Buoyant Acceleration

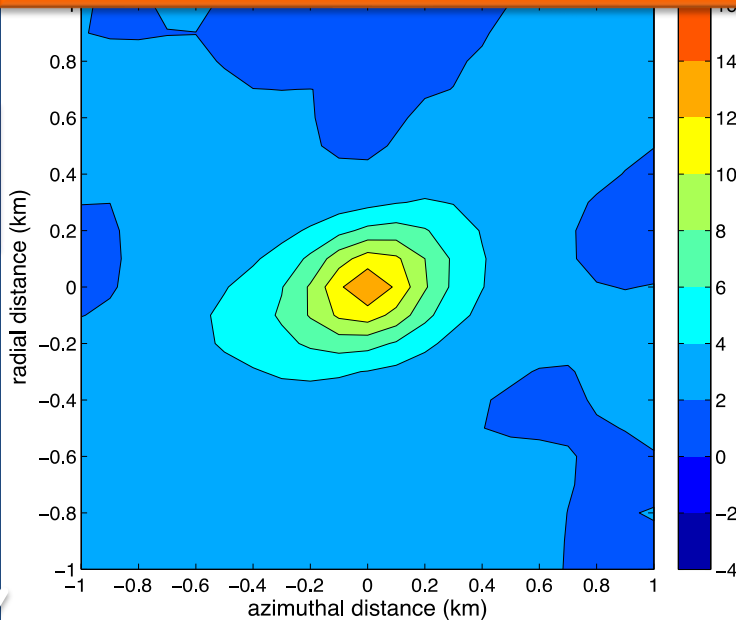


Dynamic Acceleration

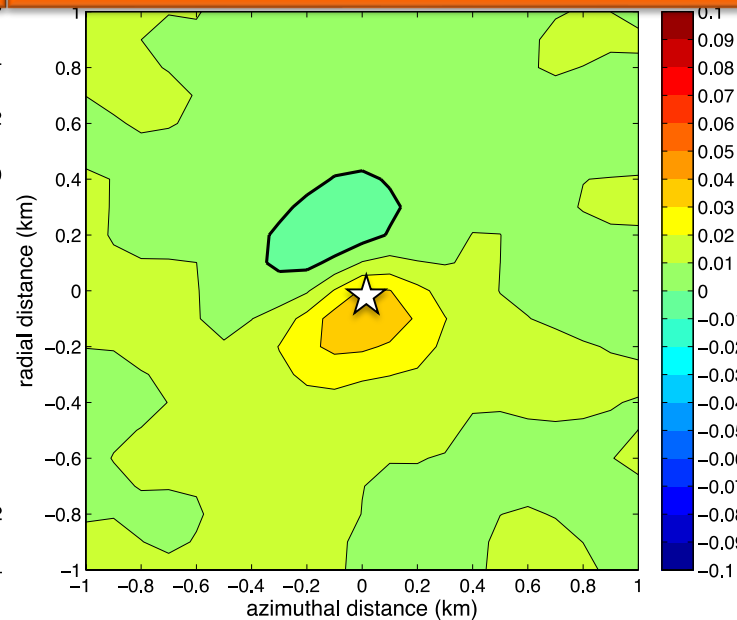


Z=1000 m

Vertical Velocity



Vertical Vorticity

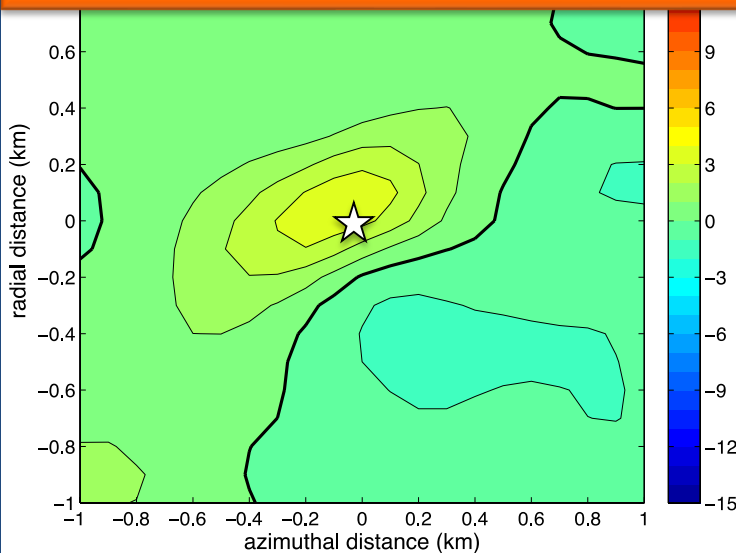


Downstream
(Cyclonic)

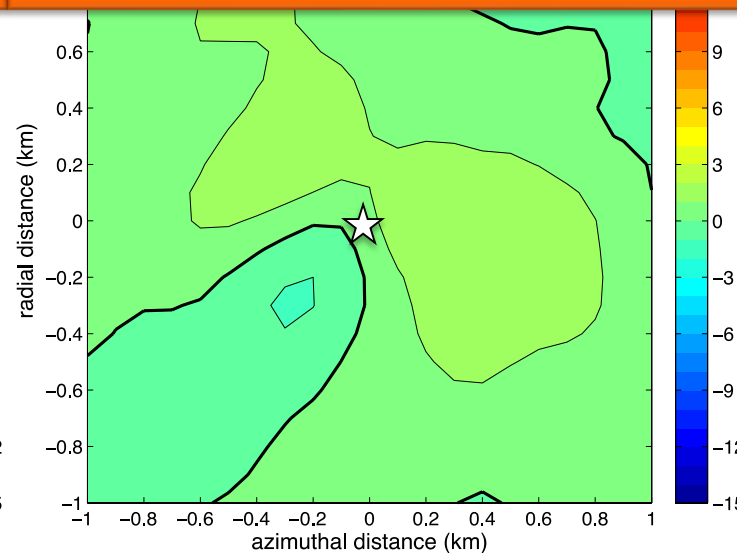


Towards Center

Perturbation V_t

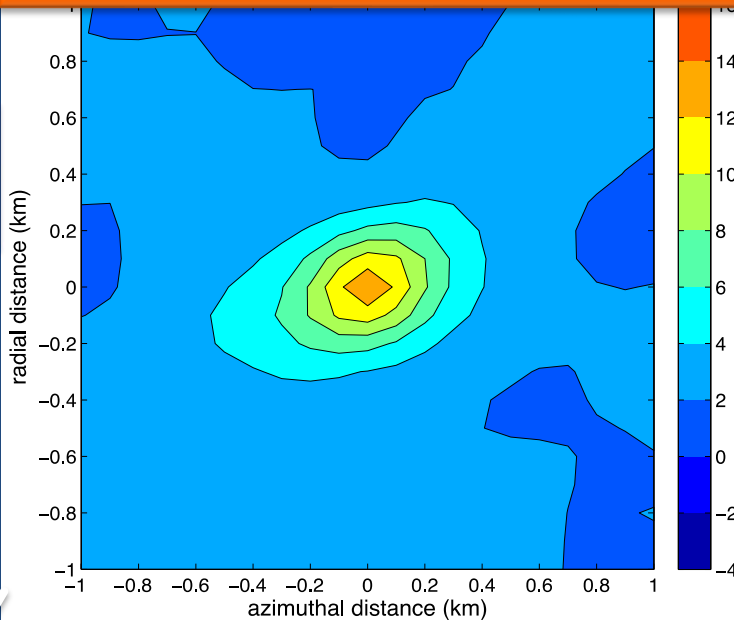


Perturbation V_r

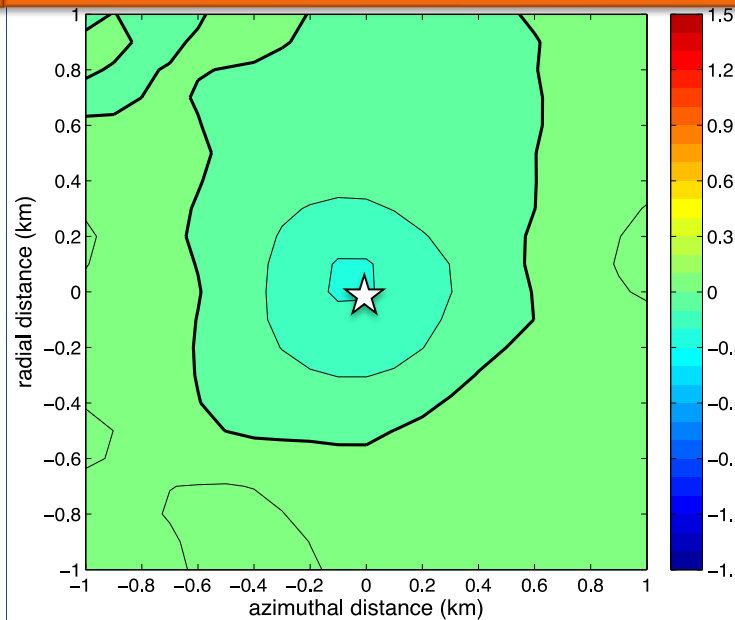


Z=1000 m

Vertical Velocity



Perturbation θ_v

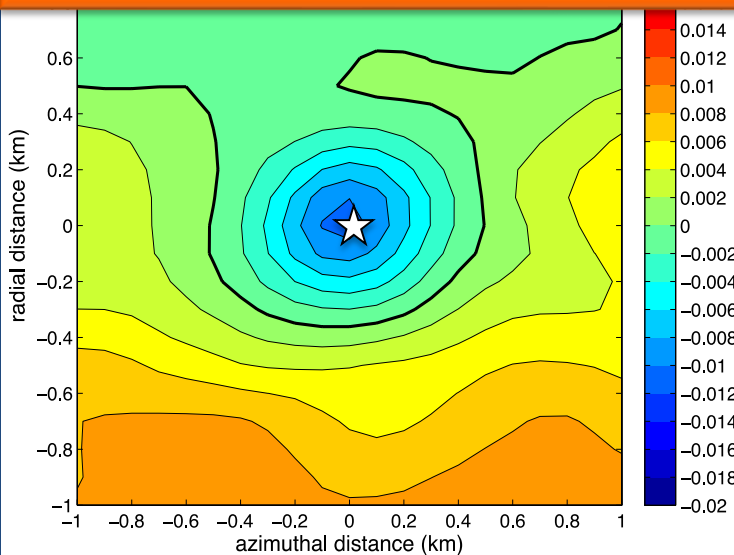


Downstream
(Cyclonic)



Towards Center

Buoyant Acceleration



Dynamic Acceleration

